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Study of Stress Wave Propagation with NHDMOC

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***Abstract*—**MOC (method of cell) is a new method of investigating wave propagating in material with periodic microstructure, and can reflect the effect of microstructure. Wave propagation in periodically laminated medium consisting of linearly elastic layers can be treated as a special application of this method. In this paper, it was used to simulate the dynamic response of carbon-phenolic to impulsive loading under certain boundary conditions. From the comparison between the results obtained from this method and the exact results based on propagator matrix theory, excellent agreement is achieved. Conclusion can be made that the oscillation periodicity is decided by the thickness of sub-cells. In the end, the NHDMOC method, which permits studying stress wave propagation with one dimensional strain, was applied to study the one-dimensional stress wave propagation. In this paper, the ZWT nonlinear visco-elastic constitutive relationship with 7 parameters, NHDMOC, and corresponding equations were deduced. The equations were verified, comparing the elastic stress wave propagation in SHPB with, respectively, the elastic and the visco-elastic bar. Finally the dispersion and attenuation of stress wave in SHPB with visco-elastic bar was studied.

***Keywords*—**MOC, NHDMOC, visco-elastic, wave propagation

I. Introduction

With the application of composites as structural materials, research of numerical method on the dynamical response of composites becomes more and more important. Various approximate theories have been proposed to predict the dynamic response of a layered medium. The effective stiffness theory by Sun, Achenbach, and Hermann[1] is one of them. It is commonly accepted that a perfect method could be adopted to treat actual composites as a homogenous continuous medium with microstructure as viewed from micromechanics, and can reflect the influence of microstructure of composites on the overall response. His document is a template for *Word (doc)* versions. If you are reading a paper version of this document, so you can use it to prepare your manuscript.

A few workers, such as Sun, Achenbach, and Hermann, who expanded the displacement within the sub-layers into a limited-order series firstly, worked up MOC. Drumheller and Bedford[2] were the first to introduce stress continuity conditions and give expression to stress and displacement boundary conditions and formalize the homogenizing technique, referred to as 'smoothing operation'. Aboudi[3] made a pivotal contribution to MOC by introducing an approximation for the momentum conservation relation by relations of successive orders of sub-layer averaged stress moments. The prominent versatility of MOC is the compatibility with arbitrary constitutive relations. Clements[4] and his co-workers carried on research in numerical simulation of plate-impact experiment with his version of MOC, called as 'Non-homogenized Dynamic Method of Cells (NHMOC)', and obtained inspiring results. Xu and co-workers[5,6,7]has studied stress wave propagation in SHPB with MOC and NHDMOC.

In this paper, the MOC was applied to simulate the problem of material dynamic response to impulse loading or impaction under certain boundary condition. Comparison was made between the results obtained from this method and the exact results based on propagator matrix technique. Finally, we studied the attenuation of stress wave induced by triangular pulse loading. NHDMOC was utilized to study the stress wave propagation in the visco-elastic material.

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II. Basic Theory

In this section, a brief introduction of MOC and NHDMOC and propagator matrix technique was carried out respectively.

MOC Theory

According to the MOC theory, the periodically laminated materials which contain representative volume elements (RVE) or cell can be transformed into a homogeneous higher order continuum with microstructure. In MOC, the unit representative cell consists of two layers, called as sub-cells which were labeled by α =1 and α =2, representing matrix layer and fiber respectively. Accordingly, the thickness of each sub-layer was denoted by d_1, d_2, mass density by ρ_1 and ρ_2. Considering plane wave propagation in the x_1 direction normal to the layers, two local coordinates, $\bar{x}_1^{(\alpha)}$ ($\bar{x}_1^{(\alpha)}$=1,2), were introduced with origins located at the center of each layer, their positions were denoted by $x_1^{(\alpha)}$ in Fig.1.

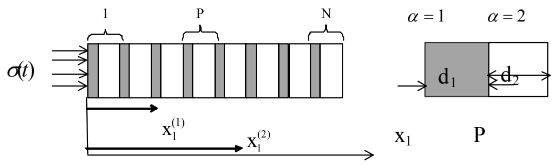


Fig. 1 Sketch of a periodically bi-laminated composite and a representative cell

The MOC uses an expansion of the particle displacement in terms of the Lerngdre polynomials as

$$u^{\alpha,p}(\bar{x}_p,t) = \sum_{l=0}^{N} U_l^{\alpha,p}(t) P_l(2\bar{x}_p / d_\alpha) \tag{1}$$

It is considered that dynamic effects do not contribute to the micro-structural response unless a second or higher order treatment is performed, so N=2 was adopted to calculate the stress wave in periodically laminated materials which contain representative volume elements (RVE) or cell. At the interfaces of the layer (or RVE) p the displacement and traction are continuous, so

$$\mathrm{u}_1^{(1,P)}(+\mathrm{d}_{1,P}/2,\mathrm{t}) = \mathrm{u}_1^{(2,P)}(-\mathrm{d}_{2,P}/2,\mathrm{t}) \tag{2}$$

$$\sigma_{11}^{(1,P)}(+\mathrm{d}_{1,P}/2,\mathrm{t}) = \sigma_{11}^{(2,P)}(-\mathrm{d}_{2,P}/2,\mathrm{t}) \tag{3}$$

The strain component in layer p , is given by

$$\varepsilon_{11}^{(\alpha,\mathrm{p})} = \partial \mathrm{u}_1^{(\alpha,\mathrm{p})} / \partial \overline{\mathrm{x}}_1^{(\alpha)} \tag{4}$$

The motion equation for each sub-cell can be written as

$$\frac{\partial}{\partial \overline{x}_p}[\sigma^{(\alpha,p)}(\overline{x}_p,t) + q^{(\alpha,p)}(\overline{x}_p,t)] = \rho_\alpha \ddot{u}^{(\alpha,p)}(\overline{x}_p,t) \tag{5}$$

The addition of artificial viscosity term $q^{(\alpha,p)}$ is used to reduce the unphysical, high frequency oscillation.

$$q^{(\alpha,p)}(\overline{x}_p,t) = \eta_\alpha d_\alpha \sqrt{E_L^{(\alpha)} \rho_p}\, \dot{\varepsilon}^{(\alpha,p)}(\overline{x}_p,t) \tag{6}$$

In which, $\dot{\varepsilon}^{(\alpha,p)}$ and $E_L^{(\alpha)}$ stands for strain rate and elastic modulus in sub-cell (α,p) , viscosity coefficient η_α is independent of thickness d_α , taken as 0.01 mostly.

Based on Lagrange motion equation, the momentum conservation equation was converted to a hierarchy of relations for successive orders of cell-averaged stress moments as equation(7). In which m=0,1,2,3 correspond to the different order stress-moment equation.

$$\begin{aligned} &\sigma^{(\alpha,p)}(d_\alpha/2,t) - (-1)^m \sigma^{(\alpha,p)}(-d_\alpha/2,t) \\ &- m(2/d_\alpha)^m \int_{-d_\alpha/2}^{d_\alpha/2} \sigma^{(\alpha,p)}(\overline{x}_p,t)\overline{x}_p^{m-1} d\overline{x}_p + \\ &m(2/d_\alpha) \int_{-d_\alpha/2}^{d_\alpha/2} \frac{\partial q^{(\alpha,p)}(\overline{x}_p,t)}{\partial \overline{x}_p} \overline{x}_p^{m} d\overline{x}_p \\ &= \rho_\alpha (2/d_\alpha)^m \int_{-d_\alpha/2}^{d_\alpha/2} \ddot{u}^{(p)}(\overline{x}_p,t)\overline{x}_p^{m} d\overline{x}_p \end{aligned} \tag{7}$$

Because the stress-moment equations derive from the general momentum conservation relation, theoretically, arbitrary constitutive relations can be used for the component materials of the laminate. In fact, only few types of constitutive relations have been successfully adopted into MOC.

As same as other hydrodynamic-based methods, the boundary and initial conditions under pulse loading or impact must be considered to build up the close-form equations.

Together with the sub-cell continuity conditions of stress and displacement at interior and boundary sub-cells, such as free or rigid boundary, and the equation of motion in each layer, the approximate moment expansion of the momentum conservation relation was used to establish a system of 6M algebraic equations in the second time derivatives of the field variables of the cells. This system can be represented in the form

$$\mathbf{A}\ddot{\mathbf{Q}} = \mathbf{R} \tag{8}$$

where $\mathbf{A}$ is the matrix of constant coefficients concerning the thickness and mass density of materials, and $\mathbf{Q}$ is the compact column matrix composed of cell coefficients

$$\ddot{\mathbf{Q}}(\mathrm{t}) = \left(\ddot{\mathrm{U}}_0^{(1,1)}(\mathrm{t}), \ddot{\mathrm{U}}_0^{(2,1)}(\mathrm{t}), \ddot{\mathrm{U}}_1^{(1,1)}(\mathrm{t}), \ddot{\mathrm{U}}_1^{(2,1)}(\mathrm{t}), \ddot{\mathrm{U}}_2^{(1,1)}(\mathrm{t}), \ddot{\mathrm{U}}_2^{(2,1)}(\mathrm{t}), \cdots, \ddot{\mathrm{U}}_2^{(2,\mathrm{N})}(\mathrm{t})\right)^{\mathrm{T}} \tag{9}$$

Column matrix $\mathbf{R}$ is composed of items without time derivative in the system of equations. Generally assume that the composite is at rest at time t=0 prior to the loading. The system of equations can be solved from a step-by-step procedure in time by introducing the finite difference form

$$\mathbf{Q}(t+\Delta t) = (\Delta t)^2 \mathbf{A}^{-1}\mathbf{R}(t) + 2\mathbf{Q}(t) - \mathbf{Q}(t-\Delta t) \tag{10}$$

A. NHDMOC theory

Different to earlier MOC methods, each sub-cell was treated as an entire cell in NHDMOC method by which the stress wave can be studied in non-periodically laminated materials, shown in Fig.2.

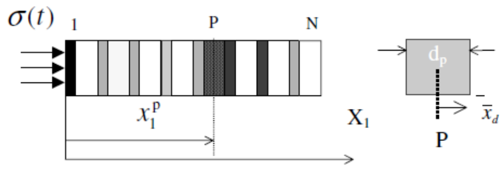


Fig. 2 Schematic of non-periodically laminated materials in NHDMOC.

The particle displacement in terms of truncated Lengdre polynomials is written as

$$u^{\alpha,p}(\overline{x}_p,t) = \sum_{l=0}^{N} U_l^{\alpha,p}(t) P_l(2\overline{x}_p/d_\alpha) \tag{11}$$

Also equation (8) can be established just like MOC as

$$\mathbf{A}\ddot{\mathbf{U}}(t) + \mathbf{B}\dot{\mathbf{U}}(t) = \mathbf{R}(t) \tag{12}$$

$$\begin{aligned} \mathbf{U}(t) = [&U_0^{(1)}(t), U_1^{(1)}(t), U_2^{(1)}(t), U_0^{(2)}(t), U_1^{(2)}(t), U_2^{(2)}(t), \\ &\ldots\ldots, \\ &U_0^{(N)}(t), U_1^{(N)}(t), U_2^{(N)}(t)]^T \end{aligned} \tag{13}$$

The first and second time derivatives in equation (12) can be represented as differences：

$$\begin{aligned} \mathbf{U}(t+\Delta t) = \Delta t^2 [\mathbf{A} + \frac{\Delta t}{2}\mathbf{B}]^{-1}\mathbf{R}(t) + 2[\mathbf{A} + \frac{\Delta t}{2}\mathbf{B}]^{-1}\mathbf{A}\mathbf{U}(t) \\ - [\mathbf{A} + \frac{\Delta t}{2}\mathbf{B}]^{-1}[\mathbf{A} - \frac{\Delta t}{2}\mathbf{B}]\mathbf{U}(t-\Delta t) \end{aligned} \tag{14}$$

Before solving the system of equation (10) and (15), the values of the cell coefficients at t=0 and $t = \Delta \mathrm{t}$ must be determined. Assumed that the composite is at rest at time t=0 prior to the loading, so cell coefficients at time $t = 0$ equal to zeros. Only those coefficients at $t = \Delta \mathrm{t}$ are left unknown. While this can be done by utilization of the loading condition on the boundary or impact interface. When the laminated composite is subjected to an external stress $\sigma(t)$, the relationship between the velocity of particle $\dot{u}_1$ and the loading stress pulse $\sigma(t)$ under planar stain can be written as

$$\dot{u}_1(\bar{x}_1^{(1,1)},t)=\frac{c_L^{(1,1)}}{c_{11}^{(1,1)}}\sigma\left(t-\frac{\bar{x}_1^{(1,1)}+d_1/2}{c_L^{(1,1)}}\right) \qquad (15a)$$

or

$$\dot{u}_1(\bar{x}_1^{(1)},t)=\frac{c_L^{(1)}}{c_{11}^{(1)}}\sigma\left(t-\frac{\bar{x}_1^{(1)}+d_1/2}{c_L^{(1)}}\right) \qquad (15b)$$

where $c_L^{(1)}$ and $c_L^{(1,1)}$ is the corresponding longitudinal sound speed and $c_{11}^{(1,1)}$ and $c_{11}^{(1,1)}$ is the elastic material constant, such as the material properties of laminated tungsten carbide particle strengthened carbon-phenol composite listed in Table 1. And in the simulation of impact of flyer, the initial cell coefficients can be determined through the similar analysis. About the choice of Δt, one can refer to the work by B. E. Clements [4].

TABLE I
MATERIAL PROPERTIES , THICKNESS AND MASS DENSITY OF CARBON FABRIC AND PHENOLIC RESIN

	c_{11} (GPa)	ρ (Kg/m3)	d (m)
Phenolic resin	3.569	1.3×10^3	6×10^{-5}
Carbon fabric	21.15	1.9×10^3	1×10^{-4}

B. Propagator matrix technique

The propagator matrix technique[8] bases on the momentum equation, the constitutive equation and three-dimension Fourier transform. The key of this technique is to obtain the "propagator matrix", which is used to transfer the displacement-stress vector from one position to another.

Considering a right-handed Cartesian coordinate system x_1, x_2, x_3 with x1-direction normal to the layers, using the subscript 1,2,3 to denote the component along the direction of axes x_1,x_2,x_3, respectively. Suppose each component of the composite is linearly elastic, the Hook's law could be written as

$$\sigma_{ij}=c_{ijkl}e_{kl}=c_{ijkl}\frac{\partial u_k}{\partial x_l} \qquad (16)$$

σ_{ij}, e_{ij} are the components of the stress tensor σ and the strain tensor e respectively, while c_{ijkl} are stiffness coefficients, and the summation convention is assumed with i,j,k,l=1,2,3. The momentum equation is written as

$$\rho\frac{\partial^2 u_i}{\partial t^2}=\frac{\partial \sigma_{ij}}{\partial x_j} \qquad (17)$$

Taking the 3-D Fourier transform

$$G(k_2,k_3,\omega)=\iiint_{\infty} g(y,z,t)\exp[i(\omega t-k_2y-k_3z)]dydzdt \qquad (18)$$

of equation (16) and (17), and after some manipulation, the following formula can be obtained

$$\frac{\partial \mathbf{V}(x_1)}{\partial x_1}=i\omega\mathbf{A}(x_1)\mathbf{V}(x_1) \qquad (19)$$

where k_2, k_3 are the component of wave-number vector $\vec{k}$ in x_2, x_3 direction respectively, i is the imaginary unit, $\mathbf{V}$ is the joint displacement-stress vector defined as

$$\mathbf{V}(x_1)=(\mathbf{u},-\boldsymbol{\sigma}/i\omega) \qquad (20)$$

and $\mathbf{A}$ is a 6×6 matrix, referred as the system matrix. Equation (18) could be solved to get

$$\begin{aligned}\mathbf{V}(x_1)&=\mathbf{D}diag(e^{i\omega(x_1-x_{10})q_1},e^{i\omega(x_1-x_{10})q_{21}},\ldots,e^{i\omega(x_1-x_{10})q_6})\mathbf{D}^{-1}\mathbf{V}(x_{10})\\&=\mathbf{P}(x_1,x_{10})\mathbf{V}(x_{10})\end{aligned} \qquad (21)$$

where $\mathbf{V}(x_{10})$ is the displacement-stress vector at reference position $x_1=x_{10}$, q_i (i=1,2,...,6) are the eigenvalues of the system matrix $\mathbf{A}$, and $\mathbf{D}$ is the eigenvector matrix of $\mathbf{A}$ (i.e. each column of $\mathbf{D}$ is an eigenvector). Matrix $\mathbf{P}$ transfers the displacement-stress vector at $x_1=x_{10}$ to the position x_1 and is referred as the propagator matrix. If there are more than one layer between the position x_1 and x_{10} ,equation (9) is also correct but with the following definition of $\mathbf{P}$

$$\mathbf{P}(x_1,x_{10})=\mathbf{P}(x_1,x_{1i})\mathbf{P}(x_1,x_{1i-1})\cdots\mathbf{P}(x_1,x_{1i0}) \qquad (22)$$

III. APPLICATION AND RESULTS

A. Stress pulse

Assume the laminated tungsten carbide particle strengthened carbon-phenol composite was loaded along the x_1 direction by an external pulse described as σ (t), used by Aboudi [3].

$$\sigma(t)=A\frac{\left[t^3H(t)-3(t-\zeta)^3H(t-\zeta)+3(t-2\zeta)^3H(t-2\zeta)-(t-3\zeta)^3H(t-3\zeta)\right]}{6\zeta^3} \qquad (23)$$

where H(t) is the Heaviside function, ξ is time parameter which could be chosen as 2 Δt or greater, and A is an amplitude factor. The different value of ξ can be set to obtain different loading curve, such as.Fig.3.

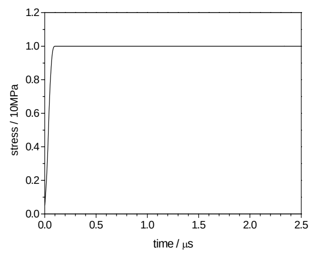


Fig. 3 The profile of incident stress pulse when ξ is set as 4Δ t

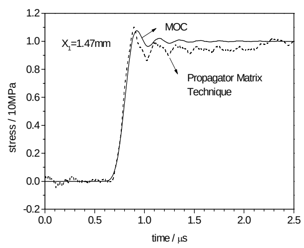


Fig. 4 Comparison of two results when stress pulse (ξ =4Δ t) incidence.

B. Results and discussion

MOC was applied to the WC/C/Ph of semi-infinite extent under the loading stress pulse(shown in Fig.3). The stress wave profiles obtained with the MOC and propagator matrix technique at x_1=1.47mm were given in Fig.4.

From Fig.4, we can see when the stress wave arrives at the position x_1=1.47mm, the time calculated with these two theories is the same. For higher-frequency stress impulse (such as the pulse with ξ =4Δ t) loading, we can observe the obvious ringing with relatively regular periodicity and the trend of attenuation with time increasing in Fig.4. When the induced wave arrives at position x_1=1.47mm, stress rises to a maximum, greater than 10MPa, then went down to a minimum, greater than 9MPa. From the attenuation trend, we can conclude that the stress will stay at the same amplitude value of the loading pulse in the end.

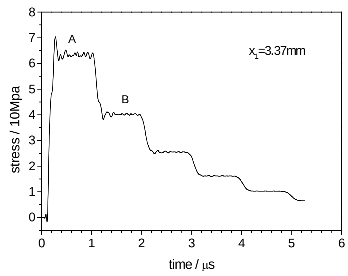


Fig. 5 Stress wave profile obtained with MOC at x_1=3.37mm in the carbon-fabric sub-layer.

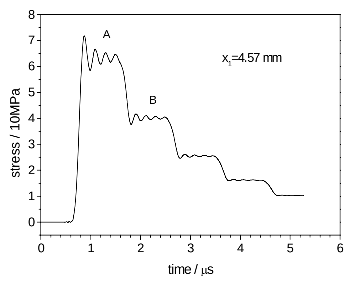


Fig. 6 Stress wave profile obtained with MOC at x_1=4.57mm in the WC particle-phenol sub-layer

The MOC was also applied to simulate the problem of flyer impacting the WC/C/Ph composite target of semi-infinite extent. A LY12 aluminum alloy disk of 3mm thickness was used as a flyer plate, its initial velocity was of 20m/s. We define t=0 to be the time just at the instant of impact. Fig.5 and Fig.6 is the stress wave profile obtained with MOC in the carbon-fabric sub-layer and in the WC particle-phenolic resin mixture sub-layer, respectively. Both in Fig.5 and in Fig.6, we can observe a series of stress plateaus. About 0.23μ s after the impact, the stress wave arrives at the carbon-fabric sub-layer center located at x_1=3.37mm. The first plateau, labeled as A, is induced by the loading wave. When the backward propagating compression wave reaches the free surface of flyer, a release wave is reflected back toward the impact interface and transmitted into the target, then the stress in the target is reduced by the release wave. And so on, a series of release waves are transmitted into the WC/C/Ph composite target, which is the reason of a series of stress plateaus. The duration time of the stress plateau are the same, about 1μ s, which is determined by the flyer thickness.

C. Comparison of numeric result with theory

In order to verify this method, results of this work was compared with theoretical results of stress wave propagation in elastic SHPB bar, in which two same aluminum alloy bar (ρ_0 =2784 kg/m3, E_0 =71.4 GPa) impacting at speed of 20m/s was considered, the maximum theoretical value of stress is 141Mpa, shown in Fig.7. As concerned as the value and the arrival time and width of stress wave, theoretical result are in good agreement with the result of this method. It can be seen that, as expected, neither wave attenuation nor dispersion is found, this confirms the validity of the split Hopkinson elastic bar technique which is based on the undistorted characteristics of elastic wave propagation[9].

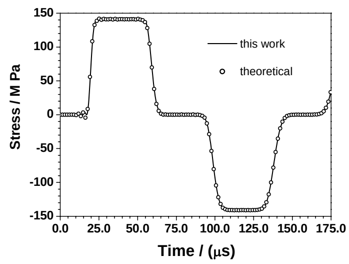
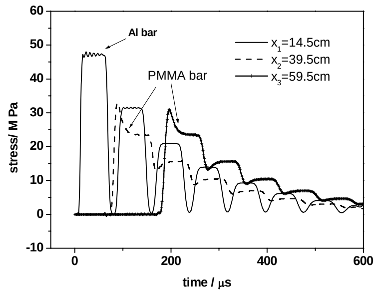


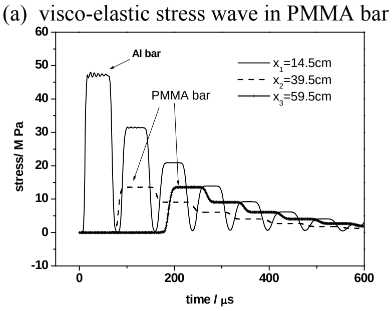
Fig. 7 Comparison of numerical results and theoretical results

D. Stress wave propagation in elastic bar with NHDMOC

As the development of SHPB experimental technique, visco-elastic material with low acoustic impedance such as PMMA ($\rho_0 C_0$ =1MPa·s/m) is used as pressure bar[10] to study mechanical behavior of low impedance materials like nylon and foams. The key to application of the split Hopkinson bar technique lies in the method for determining the stresses and particle velocities at two interfaces of specimen[11]. When visco-elastic SHPB setup was used to study behavior of low impedance materials, the shifting of recorded pulse depends on the solution of wave propagation in a visco-elastic bar, such as the inverse calculation method by Rota.L[12] and he famous Ray Theory. The main difficulty in Ray theory lies in the compatibility with constitutive equation with complicated formula, the calculation in Ref.[13]was carried out with constitutive relation in which only one high-frequency Maxwell element was considered.In this paper the NHDMOC method, which permits studying stress wave propagation with one dimensional strain, was applied to study the one-dimensional stress wave propagation. Combined with ZWT non-linear visco-elastic constitutive equations, both high-frequency and low-frequency Maxwell can be considered. Strain curves of two position in visco-elastic PMMA pressure bar impacted by aluminum bar at speed of 20 m/s was plotted in Fig. 8. As theoretically expected, the dispersion and attenuation of wave is increasing with increasing propagation distance in PMMA bar, which is in good agreement with theory and experiment in L.L. Wang's work.



(a) visco-elastic stress wave in PMMA bar



(b) apparent stress wave in PMMA bar

Fig. 8 stress wave obtained with NHDMOC

IV. Conclusion

Based on the NHDMOC method and the ZWT nonlinear visco-elastic constitutive relationship, new stress analysis method was applied to study the one-dimensional strain and stress wave propagation. The method was verified from the comparison of elastic stress wave propagation in laminated composite and SHPB with elastic bar and visco-elastic bar respectively. Finally the dispersion and attenuation of stress wave in SHPB with visco-elastic bar was studied, the importance and necessity of visco-elastic wave calculation for split Hopkinson polymer bars is further confirmed, and the NHDMOC method is verified to be successful and convenient in the analysis of visco-elastic wave propagating in polymer bar.

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