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A New SIR-based Model for Influenza Epidemic

Kang-Hung Yang, Jin-Yuan Hsu

***Abstract*—**In recent years, several severe large-scale influenza outbreaks happened in many countries, such as SARS in 2005 or H1N1 in 2009. Those influenza Epidemics have greatly impacts not only on people's life and health, but medical systems in different countries. Although severe diseases are more experienced, they are not fully controlled. Governments have different policies to control the spreads of diseases. However, those policies have both positive and negative social or economical influence on people and society. Therefore, it is necessary and essential to develop an appropriate model for evaluations of policies. Consequently, a proper measure can be implemented to confront the diseases. The main goal of this study is to develop a SIR-based model for the further evaluations of the candidate policies during the influenza outbreaks.

***Keywords*—**SIR, influenza, systems dynamic model, H1N1

I. Introduction

WITH the tendency of globalization, the distances and the relationships between countries become closer. People have more work or traveling opportunities outside their own countries. However, that provides a way for spread of infectious diseases. Governments take the epidemic prevention issues in very serious manners and employ a lot of measures to prevent negative impacts on economy and society during the diseases outbreaks. According to the statistics report of WHO in 2004, the infectious disease was the top 2 cause of the death, which included AIDS, malaria, pulmonary tuberculosis, SARS, influenza epidemic, and etc.. Influenza epidemic almost happen every several decades, which characteristics are not similar to seasonal influenza, i.e. the virus can be contagious among different species, and during the infection, the virus might be mutated to a new virus; examples are H5N1 in 2003 and H1N1 in 2009. To face those serious influenza epidemics, governments take certain prevention measure to face the potential outbreaks in the future. While implanting a prevention measure, a government decision maker has to consider some external factors, such as social acceptance, economic impacts, resource limitations, and etc.. However, it is not easy to estimate the final results by conjecture if some prevention policies are adopted to deal with the influenza epidemic. Consequently, a suitable simulation model will be useful tool. The two popular models are compartmental models and agent based modes.

The development history of the agent based models has been more then 30 years, which can model a complicated social system. However, because of lacking the real data and large computer resources in the past, some breakthroughs have been achieved recently [1].

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The core of the compartmental models is a set of ordinary differential equations, which analyze group behaviors (or status) of people and the spread of an influenza. Comparing to the agent based models, compartmental models are good for quick estimations and less requirements of computer resources, and however, they can not predict individual's situations [2].

The aim of this paper is to develop a new SIR-based model, which is named as SELMAHRD (see Section 3), for a policy maker to have a quick scan of some disease prevention measures during the outbreaks of an influenza epidemic. The primary measures include the pulse vaccination and the segregation

The embryo of the SIR model[1] was established by W.O. Kermack and A.G. McKendrick [3]. The model defines 3 types of people groups, which includes susceptible people, infected people, and recovered people. In the model, there are some assumptions and limitations as follows; the total population is fix, people death due to diseases are not considered, and age and population structure is not included as well. Based on the prototype SIR model, a lot of researchers applied it to discuss the effectiveness of the policies or to analyze some important factors that will influence the behaviors of the epidemic. [4] introduced a homotopy parameter, p, into SIR model and solved the model analytically to discuss the effects of the constant vaccination strategy. [5] modified the original SIR model to analyze susceptible agents during interactions with infectious neighbors and the mechanism of how those susceptible agents to become infectious. Others related reference are [6], [7], [8], and etc.

SEIR is also a SIR-based model. The difference between SIR and SEIR is a compartment, Exposed, added into the model. [9], [10] and [11] applied SEIR model to analyze the control treatments for the epidemic.

If we further think about deeply what are considerations that can be provided as necessary information for policy making, SIR, or SEIR model might not provide sufficient information, example, death number, hospitalized people, and so on.

Before this study, two different models, i.e., SEIRD and SEAHR, are developed by [2] and [12]. However, SEIRD considered the death compartment without the hospitalized compartment whiles SEAHR considered the hospitalized compartment but without the death compartment. Therefore, the goal of this study is to combine the death and hospitalized compartments into a SIR-based model so that the model will provide information as more as possible for a policy maker to evaluate the policies during the epidemic outbreaks. The following sections are organized as follows. Section 2 explains the briefs of SIR, SEIRD, and SEIAHR models. Section 3 describes the main model of this study, i.e., SELMAHRD model. Section 4 proceeds to the model validations. Section 5 demonstrates some results with or without different policies. Section 6 concludes this study in brief.

[1] Another name of SIR model is Kermack-Mcermack model.

II. Some SIR-Based Models

A. Definitions of the symbols

This section lists all the symbols that will be used for determine all models in Section 2 and Section 3.

$S(t)$: Numbers of susceptible people at time t

$E(t)$: Numbers of exposed people at time t

$I(t)$: Numbers of infected people at time t

$R(t)$: Numbers of recovered people at time t

R_∞ : The limiting recovered population size, that is , $\lim_{t\to\infty} R(t) = R_\infty$

$D(t)$: Numbers of death people at time t

$N(t)$: Total number of people in the system at time t

$A(t)$: Numbers of asymptomatic people at time t

$H(t)$: Numbers of hospitalized people at time t

$L(t)$: Numbers of latent people at time t

$M(t)$: Numbers of symptomatic people at time t

α : Transmission rate

α_I : The ratio that latent people become symptomatic people

β : Contact rate

β_I : transmission probability

β_{II} :effective transmission probability

a : A control item to

δ_1 : A control item ratio to reduce latent people

δ_2 : A control item ratio to reduce symptomatic people

δ_3 : A control item ratio to reduce asymptomatic people

σ : The velocity that exposed people become latent people

σ_2 : The velocity that symptomatic people become hospitalized people

γ : Recovery rate

R_0 : Basic reproduction number

μ_I : Death rate

μ_1 : The velocity that exposed people become latent people

μ_2 : The velocity that latent people become symptomatic people

υ : The ratio that people gets a vaccination

i : Velocity of immunity after people gets a vaccination

r_1 Death rate of symptomatic people

r_2 Death rate of hospitalized people

γ_2 : Removal rate of infected people become recovered people

γ_3 : Removal rate of asymptomatic people become recovered people

γ_4 : Removal rate of hospitalized people become recovered people

ρ_1 : The ratio of symptomatic people become death people

ρ_2 : The ratio of symptomatic people become hospitalized people

p_1 : The ratio of hospitalized people become death people

B. SIR

SIR stands for Susceptible, Infected, and Recovered, which was developed by W.O. Kermack and A.G. McKendrick in 1927. This model is the earliest mathematical compartmental model for epidemic researches. The conceptual compartments and equations can be illustrated as Figure 1.

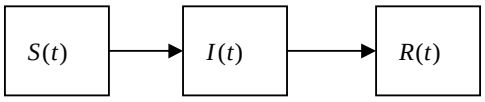

Fig. 1 SIR Model

$$\frac{dS(t)}{dt} = -\beta I(t)S(t) \quad (1)$$

$$\frac{dI(t)}{dt} = \beta I(t)S(t) - \gamma I(t) \quad (2)$$

$$\frac{dR(t)}{dt} = \gamma I(t) \quad (3)$$

$$\frac{dS(t)}{dt} + \frac{dI(t)}{dt} + \frac{dR(t)}{dt} = 0 \quad (4)$$

$$N(t) = S(t) + I(t) + R(t) \quad (5)$$

$$R_0 = \frac{\beta}{\gamma} \quad (6)$$

R_0 is an indicator to measure the dynamics of the epidemic almost in all models. In SIR mode, $R_0 > 1$ means the epidemic continues whiles $R_0 < 1$ indicates that the epidemic will diminish gradually.

C. SEIRD

SEIRD stands for Susceptible, Exposed, Infected, Recovered, and Death. The differences from SIR model are extra two compartments, namely, exposed and death compartments. Figure 2 shows the concept model of SEIRD.

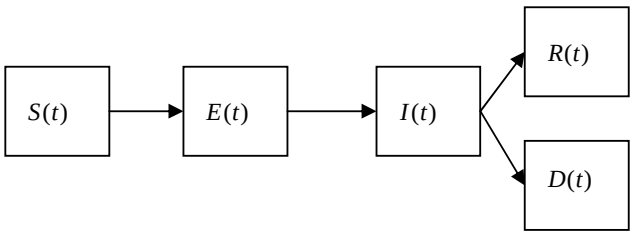

Fig. 2 SEIRD Model

The follows list the governing equations of SEIRD.

$$\frac{dS(t)}{dt} = -\beta\alpha\frac{[E(t) + I(t)]S(t)}{N(t)} \quad (7)$$

$$\frac{dE(t)}{dt} = \beta\alpha\frac{[E(t) + I(t)]S(t)}{N(t)} - \sigma E(t) \quad (8)$$

$$\frac{dI(t)}{dt} = -\mu_I I(t) + \sigma E(t) \quad (9)$$

$$\frac{dR(t)}{dt} = \gamma I(t) \quad (10)$$

$$\frac{dD(t)}{dt} = \mu_I I(t) \quad (11)$$

$$\frac{dS(t)}{dt} + \frac{dE(t)}{dt} + \frac{dI(t)}{dt} + \frac{dR(t)}{dt} + \frac{dD(t)}{dt} = 0 \quad (12)$$

$$N(t) = S(t) + E(t) + I(t) + R(t) + D(t) \quad (13)$$

D.SEIAHR

SEIAHR is a model that includes an asymptomatic compartment and a hospitalized compartment, which provide policy intervention information, i.e. number of hospitalized people, for a policy maker to do the decision making to deal with epidemics. Figure 3 shows the conceptual model of the SEIAHR.

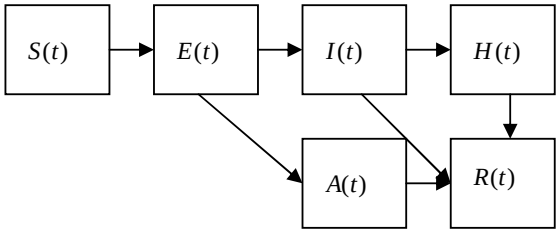

Fig. 3 SEIAHR model

The follows list the detail equations to describe the SEAHR model. Equation (22) is the basic reproduction number derived from SEIAHR model.

$$\frac{dS(t)}{dt} = -\frac{\beta_I S(t)[I(t) + \delta_3 A(t)]}{S(t) + E(t) + I(t) + A(t)} \frac{\beta}{1 + a[H(t) + R(t)]} \quad (14)$$

$$\frac{dE(t)}{dt} = \frac{\beta_I S(t)[I(t) + \delta_3 A(t)]}{S(t) + E(t) + I(t) + A(t)} \frac{\beta}{1 + a[H(t) + R(t)]} - \mu E(t) \quad (15)$$

$$\frac{dI(t)}{dt} = \alpha_I \mu E(t) - (\sigma_2 + \gamma_2) I(t) \quad (16)$$

$$\frac{dA(t)}{dt} = (1 - \alpha_I)\mu E(t) - \gamma_3 A(t) \quad (17)$$

$$\frac{dH(t)}{dt} = \sigma_2 I(t) - \gamma_4 H(t) \quad (18)$$

$$\frac{dR(t)}{dt} = \gamma_2 I(t) + \gamma_3 A(t) + \gamma_4 H(t) \quad (19)$$

$$\frac{dS(t)}{dt} + \frac{dE(t)}{dt} + \frac{dI(t)}{dt} + \frac{dA(t)}{dt} + \frac{dH(t)}{dt} + \frac{dR(t)}{dt} = 0 \quad (20)$$

$$N(t) = S(t) + E(t) + I(t) + A(t) + H(t) + R(t) \quad (21)$$

$$R_0 = \frac{\beta\beta_I}{1 + R_\infty}\left[\frac{\alpha_I}{\sigma_2 + \gamma_2} + \frac{\delta_3(1 - \alpha_I)}{\gamma_3}\right] \quad (22)$$

III. SELMAHRD Model

This study tried to extend the ideas of the SEIRD and SEIAHR to develop a model that includes hospitalized and death compartments which can provide "future" information for a policy maker. Some common ways to alleviate or prevent the influence are isolating susceptible people (adults or students) and letting them at home, doing pulse vaccination, or etc..

The assumptions are similar to those of SIR model, which include total population is fix, population structure is not considered in the model, latent period will not change with time, and recovered people will not get infected again. Based on those assumptions, the conceptual model is shown in Figure 6.

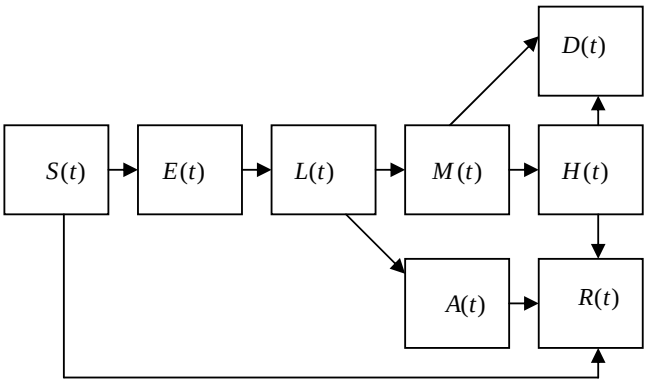

Fig. 4 SELMAHRD model

In the model, two new idea ideas are different from SEIRD and SEIAHR. First, Infected ($I(t)$) are subdivided into two compartments, that is, Latent ($L(t)$) and Symptomatic ($M(t)$) because nowadays, such as H1N1, some people may actually infect other people without any symptom, but later on their symptoms come out. Second, in order to include the "vaccination", an arrow from Susceptible ($S(t)$) to Recovered ($R(t)$) is put to represent the people who get vaccination will not get infection from epidemic.

The detail equations are listed in the following.

$$\frac{dS(t)}{dt} = -\frac{\beta_{II}\beta S(t)(1-\upsilon)[\delta_1 L(t) + \delta_2 M(t) + \delta_3 A(t)]}{N(t)} - \upsilon i S(t) \quad (23)$$

$$\frac{dE(t)}{dt} = \frac{\beta_{II}\beta S(t)(1-\upsilon)[\delta_1 L(t) + \delta_2 M(t) + \delta_3 A(t)]}{N(t)} - \mu_1 E(t) \quad (24)$$

$$\frac{dL(t)}{dt} = \mu_1 E(t) - [\alpha\mu_2 + (1-\alpha)]L(t) \quad (25)$$

$$\frac{dM(t)}{dt} = \alpha\mu_2 L(t) - [\rho_1 r_1 + \rho_2 r_2 + (1 - \rho_1 - \rho_2)\gamma_2]M(t) \quad (26)$$

$$\frac{dA(t)}{dt} = (1-\alpha)L(t) - \gamma_3 A(t) \quad (27)$$

$$\frac{dH(t)}{dt} = \rho_2\sigma M(t) - [p_1 r_2 + (1 - p_1)\gamma_4]H(t) \quad (28)$$

$$\frac{dD(t)}{dt} = \rho_1 r_1 M(t) + p_1 r_2 H(t) \quad (29)$$

$$\frac{dR(t)}{dt} = (1 - \rho_1 - \rho_2)\gamma_2 M(t) + \gamma_3 A(t) + (1 - p_1)\gamma_4 H(t) \quad (30)$$

$$N(t) = S(t) + E(t) + L(t) + M(t) + A(t) + H(t) + D(t) + R(t) \quad (31)$$

$$\frac{dS(t)}{dt} + \frac{dE(t)}{dt} + \frac{dL(t)}{dt} + \frac{dM(t)}{dt} + \frac{dA(t)}{dt} + \frac{dD(t)}{dt} + \frac{dH(t)}{dt} + \frac{dR(t)}{dt} = 0 \quad (32)$$

Followed by the deductions in [10], the basic reproduction number can be derived as the following equation.

$$R_0 = \frac{\beta_{II}\beta(1-\upsilon)\delta_1}{\alpha\mu_2 + (1-\alpha)} \quad (33)$$

IV. Model Validation

In order to examine the validity of the SELMAHRD model, PowerSim is used for the development of the model and the symptomatic, hospitalized, and death of H1N1 of 2009 in Taiwan were used for the comparison with the model outputs. All initial values are listed in Table 1. All parameters are determined by try-and-error method, and the parameter values are within the range that previous researches suggested.

TABLE I
Parameter Setting For SELMAHRD Model

$S(0)$	22300929 people	υ	0.21%
$E(0)$	0 people	i	14/day
$L(0)$	0 people	μ_1	1/3/day
$M(0)$	1 people	μ_2	1/day
$A(0)$	0 people	α_{II}	67%
$H(0)$	0 people	ρ_1	0.2%
$R(0)$	0 people	ρ_2	3.3%
$D(0)$	0 people	p_1	0.1%
β_{II}	25%	r_1	0.1/day
β	10 people /day	r_2	0.1/day
δ_1	50%	γ_2	0.1/day
δ_2	10%	γ_3	1/3/day
δ_3	75%	γ_4	0.2/day

Figure 5 to Figure 6 show the comparison results between the real data and simulation data.

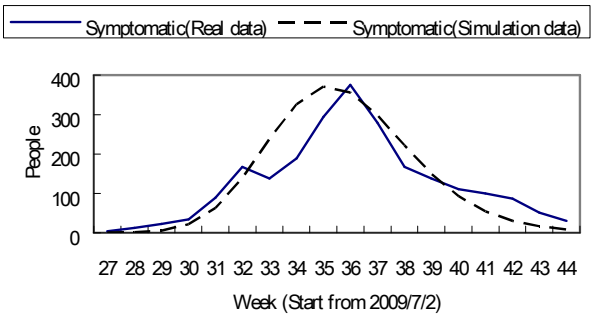

Fig. 5 Symptomatic results comparison

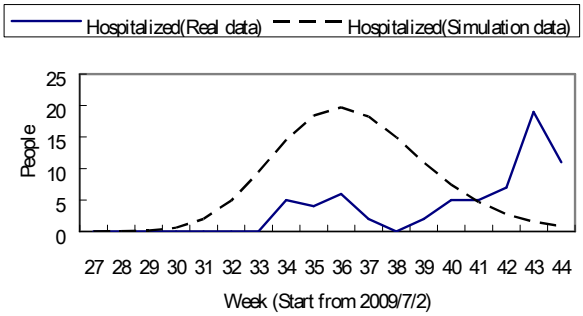

Fig. 6 Hospitalized results comparison

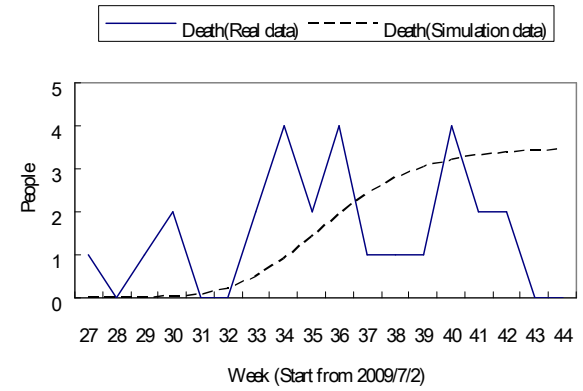

Fig. 7 Death results comparison

The comparison of symptomatic results between real data and model forecasts are very similar. The overall relative error percentage is 45.5%, the smallest error happens at week 36, only 5% difference, which is the peak value during the H1N1 epidemic period. From the Figure 6, one observation is that the real data seems to delay comparing to the simulation results. From the news history, when H1N1 happened in Mexico for a period of time, there was no H1N1 patent in Taiwan yet. Most people didn't pay H1N1 very much until few patents were reported. These might be a possible reason that the simulation data are ahead of the real data a little bit. But that inference needs more evidence. For the death comparisons, since the number is very small, therefore, it is hard to judge anything. The only thing that can be observed there is a tread explaining more death might be happen in the future.

V. Scenario Analysis For The Policies

Since the model is still under development, the scenario results are preliminary just for the big picture of what-if analyses. Two different policy measures are considered. The first measure is to encourage susceptible people or enforce symptomatic people to stay at home until they are safe for other people, here; this scenario is named as "Self-management" scenario. And the second measure is to encourage people to be vaccinated. This study chose the parameters that related to the policies, and change the values to examine any possibility to

decrease H1N1 epidemic. The parameters that relate to the first measure are δ_1, δ_2 and δ_3 and the parameter that relates the second measure is υ.

A. Self-management scenario

Figure 8 shows results of self-management scenario with different times. The original parameters settings are $\delta_1 = 50\%$, $\delta_2 = 10\%$, $\delta_3 = 75\%$ and the new parameter setting are $\delta_1 = 45\%\%$, $\delta_2 = 9\%$, $\delta_3 = 67.5\%$.

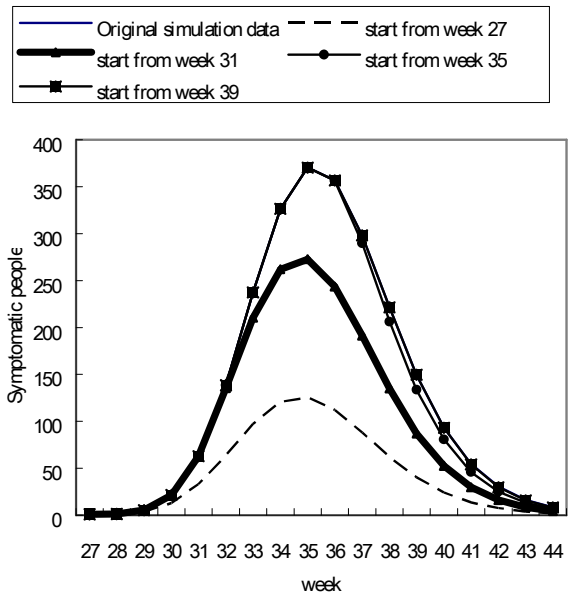

Fig. 8 Self-management policy starting from different time

Equation (23) and (24) explain the implications of three parameters. Once a person has doubted himself infected or he or she has got infected, lower values of the parameters mean that less people become latent, symptomatic or asymptomatic people. Therefore, self-management policy can be implemented by decreasing the values of δ_1, δ_2, and δ_3.

Figure 8 shows that the earlier implements of the policy cause the less symptomatic people. And if the policy is implemented after two months of the outbreak, this policy is not a good solution anymore.

B. Vaccination scenario

The parameter setting for the vaccination scenario is $\upsilon = 20\%$ instead of the original setting $\upsilon = 0.21\%$. Figure 9 shows vaccination starts from different time. Very similar to the self-management scenario, early intervention of the scenario will be more effective than late intervention of the measure.

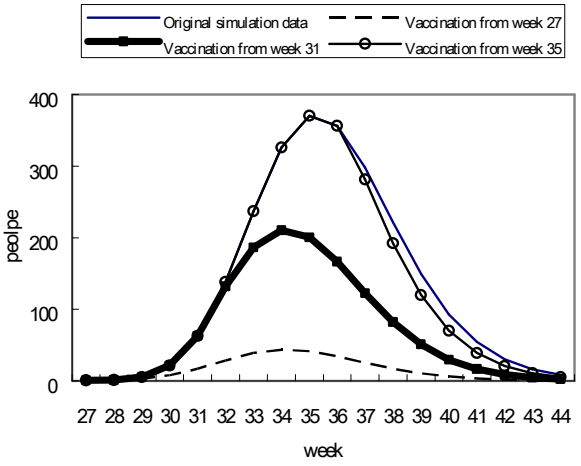

Fig. 9 Vaccination policy starting from different time

VI. Conclusions

Now a day, epidemic outbreaks seem to happen more frequently, and for each time, the situation is getting worse. People have to face the tough challenges each time to conquer the eco- and bio- disaster. This study established a SIR-based compartmental model and used it to discuss the effectiveness of two common policy measures. Since the model is still under development, the results are might not be so strong, but the model has shown its potential that can be used for more complicated what-if analyses. In the following researches, the parameters have to be determined more strictly to get more convincing results. Different type of epidemics can be tested by this model to see the capability of this model.

Acknowledgment

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