

# Investigating the Effect of Uncertainty on a LP Model of a Petrochemical Complex: Stability Analysis Approach

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**Abstract**—This study discusses the effect of uncertainty on production levels of a petrochemical complex. Uncertainty or variations in some model parameters, such as prices, supply and demand of materials, can affect the optimality or the efficiency of any chemical process. For any petrochemical complex with many plants, there are many sources of uncertainty and frequent variations which require more attention. Many optimization approaches are proposed in the literature to incorporate uncertainty within the model in order to obtain a robust solution. In this work, a stability analysis approach is applied to a deterministic LP model of a petrochemical complex consists of ten plants to investigate the effect of such variations on the obtained optimal production levels. The proposed approach can determinate the allowable variation ranges of some parameters, mainly objective or RHS coefficients, before the system lose its optimality. Parameters with relatively narrow range of variations, i.e. stability limits, are classified as sensitive parameters or constraints that need accurate estimate or intensive monitoring. These stability limits offer easy-to-use information to the decision maker and help in understanding the interaction between some model parameters and deciding when the system need to be re-optimize. The study shows that maximum production of ethylene and the prices of intermediate products are the most sensitive factors that affect the stability of the optimum solution.

**Keywords**—Linear programming, Petrochemicals, stability analysis, uncertainty

## I. INTRODUCTION

OPTIMIZATION of production levels is an essential task to maximize company profit margins and to remain in the competitive market especially with high fluctuating in the prices of raw materials and products in addition to variations in the supply and demand. Many models of petrochemical processes and optimization techniques were proposed to handle this task [1]; however, it was recognized that optimization results can suffer from the existence of uncertainty in model parameters due to inaccurate estimates of some parameters. In addition, variations in input data, such as supply and cost of raw materials, can easily affect the process profitability. Many studies were conducted to understand the effect of such uncertainty or variation on the optimized model. There are two general approaches for dealing with the existence of uncertainty in optimization problem: incorporating the uncertainty directly into the optimization

problem formulation, such as stochastic programming, and analysis of the effect of uncertainty on the optimal solution, post-optimality analysis.

Many techniques of stochastic and fuzzy programming were proposed to handle the effect of variation in process parameters in order to determine a robust operation conditions [2]. In the other hand, applications of post-optimality analysis have received less attention especially in refinery industry. Post-optimality analysis can help the decision maker to determine how much actual values of parameters may differ from the estimates used in the model before the optimal results become irrelevant. Generally, post-optimality analysis can provide the decision-maker with valuable information about sensitive parameters and constraints. In this project, we showed how the post-optimality analysis, mainly stability analysis, can be conducive to the decision maker in any process industry. A stability analysis technique, modified tolerance approach, is applied to the petrochemical complex in order to demonstrate the use of such analysis and to determine sensitive parameters that need accurate estimate or intensive monitoring. Moreover, the approach computes the stability limits (allowable variation ranges) of coefficients of objective function and right-hand-side of LP model to help the decision maker to maintain efficient plant operation.

## II. CASE STUDY: PETROCHEMICAL COMPLEX WITH TEN PLANTS

The petrochemical complex that is used as a case study consists of ten plants: an ethylene plant, a caustic soda and chlorine plant, an ammonia plant, an urea plant, an ethylene dichloride (EDC) plant, a vinyl chloride monomer (VCM) plant, a polyvinyl chloride (PVC) plant, and three different polyethylene plants (LDPE, HDPE, and LLDPE). The raw material for this complex is ethane which used to produce ethylene. Small quantities of some side-product hydrocarbons, such as propane, butane, and gasoline, are also produced from the ethylene plant. Produced ethylene can be delivered as a main raw material to four plants: LLDPE, LDPE, HDPE, and EDC, and/or exported based on demand and economics. The operation of the VCM plant depends on the production and price of EDC and its production can be exported or used to produce PVC. Detailed description of the plants and different technologies used in processing are presented in Meyers [3]; and prices of raw materials and products are obtained from ICIS [4].

The optimization model of petrochemicals production is formulated as a LP problem:

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$$\begin{aligned}
 \max \quad & c^T x \\
 s.t \quad & A_{ineq} x \leq b_{ineq} \\
 & A_{eq} x = b_{eq} \\
 & x \geq 0
 \end{aligned}$$

where  $x \in \mathbb{R}^n$  presents the production flow rate per day. The objective coefficients vector,  $c$ , presents the price of products and/or raw materials, right hand side vector,  $b$ , may presents the demand and supply, and matrix  $A$  usually describes plant specifications such as production yields.

The objective function of this problem is to maximize the net profit defined as the difference between sales revenue and total production costs for each product as follows:

$$\begin{aligned}
 \text{Max net profit} = & (\text{sales revenue} - \text{cost of raw materials} \\
 & - \text{operating costs})
 \end{aligned}$$

The problem constraints include inequality, equality and non-negativity constraints. The equality constraints mainly represent the mass balance around each plant and the inequality constraints represent the limitations of the process or the products, e.g., plant capacity and supply of raw materials.

### III. RESULTS AND DISCUSSION

The petrochemical complex problem was solved using Matlab and the maximum profit was found to be 977,000 dollars per day. Optimal solutions of process variables and production levels are shown in Table 1. All productions of EDC and CVM are transported to the next plant to produce a more valuable product, PVC. Moreover, HDPE is not produced because is relatively non-profitable comparing with LLDPE and LDPE.

Table II shows the allowable range of price changes for three products within which obtained optimal production levels remain optimum. Other useful information can be obtained from the stability limits. Such information can help the decision maker to understand the interaction between production levels and other factors (e.g., prices); and determine when the process need to be re-optimized.

In contrast, individual stability limits for individual variations is greater than those for simultaneous variations since the obtained stability range for each parameter decreases with increasing the number of uncertain parameters. Presented limits of simultaneous variation were obtained using same different weighting factors as in Table 2; however, different weighting factors can be used based on the relative important or frequent change of each parameters. Analysis and results obtained for variations in ethane and ethylene prices and in RHS coefficients (e.g. supply and demand of materials and capacity constraints) will be presented and discussed in the extended paper.

TABLE I  
OPTIMAL PRODUCTION LEVELS OF MAIN VARIABLES

Component	Production rate	
NG used	743.3	1000 kmol/day
NH <sub>3</sub> exported	50.0	
NH <sub>3</sub> to urea	90.0	
Urea	38.2	
Ethylene	62.0	
EDC exported	0	ton/day
VCM exported	0	
PVC exported	346.0	
NaOH	300.0	
LLDPE	624.7	
HDPE	0	
LDPE	550.0	

TABLE II  
STABILITY LIMITS FOR SIMULTANEOUS VARIATIONS IN PRODUCTS PRICES AT DIFFERENT WEIGHTING FACTORS

Simultaneous stability limits			
Variables	$k_i$	\$/ton	
		Lower	Upper
LLDPE	1	-52.67	12.50
LDPE	1	-12.50	Inf
HDPE	1	-1280	52.67
LLDPE	2	-70.36	16.66
LDPE	1	-8.33	Inf
HDPE	1	-1280	35.18

### IV. CONCLUSION

This study presents the application of post-optimality analysis to a simplified process engineering problem formulated as LP problem, in order to investigate the effect of uncertainty or variation in model parameters, mainly objective coefficients on the optimal production levels. Modified tolerance approach was used to compute the allowable variation limits, for individual and simultaneous variations, within which the operation levels remain optimum. The main objective of the obtained results is to supply the decision maker in the plant with useful and easy-to-use information that can help to understand the interaction between production levels and other factors (e.g., prices) and to enable effective use of sensitivity information such as Lagrange multipliers.

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