

# Stochastic Mixed 0-1 Integer Programming Applied to International Transportation Problems under Uncertainty

Y. Wu

**Abstract**—Today's business has inevitably been set in the global supply chain management environment. International transportation has never played such an important role in the global supply chain network, because movement of shipments from one country to another tends to be more frequent than ever before. This paper studies international transportation problems experienced by an international transportation company. Because of the limited fleet capacity, the transportation company has to hire additional trucks from two countries in advance. However, customer's shipment information is uncertain, and decisions have to be made before accurate information can be obtained. This paper proposes a stochastic mixed 0-1 programming model to solve the international transportation problems under uncertain demand. A series of experiments demonstrate the effectiveness of the proposed stochastic model.

**Keywords**—Global supply chain management, international transportation, stochastic programming.

## I. INTRODUCTION

THE world is a very different place than it was only a few years ago. More and more companies are seeking to achieve competitive advantages by expanding their operations to a global scale. This paper is motivated by the international transportation problems experienced by an international transportation company. The company has two warehouses located in two counties: one is in the low-cost country; the other is in the high-cost country. The company needs to transport goods warehoused in the low-cost country to the high-cost country in order to satisfy demand in the high-cost country. However, customer's shipment information is uncertain, and decisions have to be made before accurate information can be obtained. As international transportation involves a cross-bordering process, only trucks that have two licenses from two countries can operate in two countries. As the company has limited trucks with two licenses, they may need to hire additional trucks from two countries in the case of high demand. In this study, the demand from the high-cost country is defined as the stochastic parameter. International logistics has been drawn lots of attention by researchers and practitioners only a couple of years ago. Excellent surveys in the field of international logistics are presented in [1]–[3]. Reference [4]

presents the North America Trade Agreement (NAFTA) perspective on cross-border trucking transportation between the US, Canada, and Mexico. Reference [5] examines the possibility of expediting current port-of-entry processing of commercial trucks entering the US from Mexico.

## II. PROBLEM DESCRIPTION AND MODEL FORMULATION

In this paper, the company has its own truck  $i^0$  ( $i \in I^0$ ) with two licenses, and the unit trip cost is denoted by  $h^0$ . The company has two warehouses located in Mainland China and Hong Kong, respectively. When the demand in the Hong Kong market is higher than the company's transportation capacity, the company may rent a truck  $i$  ( $i \in I^1$ ) with a Chinese driving license; rent a truck  $i$  ( $i \in I^2$ ) with a Hong Kong's driving license; and rent a truck can  $i$  ( $i \in I^3$ ) with two countries' licenses. Because the boundary restricts of two countries, only those truck with two countries' licenses can directly cross the border from Mainland china to Hong Kong. All trucks have their maximum loading capacity, represented by  $C_i$  ( $i \in I^0 \cup I^1 \cup I^2 \cup I^3$ ). Let  $h_1$  represent the unit cost for renting a truck with a Chinese driving license,  $h_2$  represent the unit cost for renting a truck with a Hong Kong's driving license, and  $h_3$  represent the unit cost for renting a truck with two licenses from both countries. It is also assumed that the unit inventory cost in the Mainland China's warehouse is denoted by  $c^1$ , and the unit inventory cost in the Hong Kong's warehouse is  $c^2$ . In day  $t$  ( $t \in T$ ),  $s_t$  volumes of products are required to be transported by renting trucks from the Mainland China's warehouse. In this study, demand  $d_{ts}$ , representing the volumes of products, is defined as the stochastic parameter. Each scenario  $s$  with probability  $p_s$ , where  $s \in S$  and  $\sum_{s \in S} p_s = 1$ , provide one possible course for future demand. If the demand is not satisfied, a shortage cost incurs. The unit shortage cost is denoted by  $c^3$ . Let  $w_0^1$  and  $w_0^2$  denote the initial volume of products stored in the Mainland China and Hong Kong's warehouses, respectively. The company has three options for transporting products internationally:

- to use company-owned fleet, which can directly transport products from the Mainland China's warehouse to the Hong Kong's warehouse.

Y. Wu is with the School of Management, University of Southampton, University Road, Highfield, Southampton, UK, SO17 1BJ (Phone: +44 (0) 23 80598711; fax: +44 (0) 23 80593844; e-mail: y.wu@soton.ac.uk).

- to rent trucks with two countries' licenses, which can directly transport products from Mainland China to Hong Kong; or
- to rent trucks with China's license first, switching to trucks with Hong Kong's license on the border. This option incurs a trans-shipment cost on the border point, because it involves unloading and loading process. The unit trans-shipment cost is denoted by  $c^0$ .

The company needs to determine an optimal transportation plan to minimize the total transportation cost under uncertain demand. The decision variables include:

$$x_{it} = \begin{cases} 1 & \text{if truck } i \text{ operates on day } t \\ 0 & \text{otherwise} \end{cases}, i \in I^0 \cup I^1 \cup I^2 \cup I^3,$$

$$t \in T;$$

$$X_{it} = \text{volume of products loaded by truck } i \text{ on day } t, i \in I^0 \cup I^1 \cup I^2 \cup I^3, t \in T;$$

$$w_t^1 = \text{inventory of volume of products in the Mainland China's warehouse on day } t, t \in T;$$

$$w_{ts}^2 = \text{inventory of volume of products in the Hong Kong's warehouse on day } t \text{ for each scenario } s, t \in T, s \in S;$$

$$w_{ts}^3 = \text{shortage of volume of products in the Hong Kong's warehouse on day } t \text{ for each scenario } s, t \in T, s \in S;$$

The international transportation problems can be formulated as the following stochastic mixed 0-1 integer programming model:

$$\min \sum_{t \in T} \sum_{i \in I^0} h^0 x_{it} + \sum_{t \in T} \sum_{i \in I^1} h^1 x_{it} + \sum_{t \in T} \sum_{i \in I^2} h^2 x_{it} + \sum_{t \in T} \sum_{i \in I^3} h^3 x_{it} + \sum_{t \in T} \sum_{i \in I^0} c^0 * X_{it} + \sum_{t \in T} c^1 w_t^1 + \sum_{s \in S} \sum_{t \in T} p_s (c^2 w_{ts}^2 + c^3 w_{ts}^3) \quad (1)$$

s.t.

$$\sum_{i \in I^0 \cup I^1 \cup I^2 \cup I^3} X_{it} + w_{t-1,s}^2 = d_{ts} + w_{ts}^2 - w_{ts}^3, t \in T, s \in S \quad (2)$$

$$s_t + w_{t-1}^1 = \sum_{i \in I^0 \cup I^1 \cup I^2 \cup I^3} X_{it} + w_t^1, t \in T \quad (3)$$

$$\sum_{i \in I^0} X_{it} = \sum_{i \in I^1} X_{it}, t \in T \quad (4)$$

$$X_{it} \leq C_i x_{it}, i \in I^0 \cup I^1 \cup I^2 \cup I^3, t \in T \quad (5)$$

$$x_{it} \in \{0,1\}, X_{it} \geq 0, i \in I^0 \cup I^1 \cup I^2 \cup I^3, t \in T \quad (6)$$

$$w_t^1 \geq 0, t \in T \quad (7)$$

$$w_{ts}^2, w_{ts}^3 \geq 0, t \in T, s \in S \quad (8)$$

The first term in objective function (1) is the trip cost of the company-owned trucks. The second, third and fourth terms are the unit cost of hiring China's trucks, Hong Kong's trucks, and trucks with two licenses. The fifth term is the trans-shipment cost. The sixth term is the warehousing cost in the Mainland China's warehouse. The last term is the expected value of the warehousing and cost shortage in the Hong Kong's warehouse. Constraint (2) is a stochastic constraint. It ensures that, on day t, demand has to be satisfied in each scenario by the sum of the initial inventory in the Hong Kong warehouse, the total amount of the products that

arrive in the Hong Kong warehouse on day t and any shortage, minus surplus goods at the end of day.

Constraint (3) ensures that, on day t, the total volume of the products received plus the products already stored in the Hong Kong warehouse is equal to the total volume of the products required by the markets plus products stored in the Hong Kong's warehouse, minus shortage of volume of products in the Hong Kong's warehouse. Constraint (4) ensures that, on day t, the total volume of the products available to transport plus the initial products stored in the Mainland China warehouse is equal to the sum of volume of the products transported to the border by the trucks with Chinese license and volume of the products directly transported to Hong Kong by trucks with two licenses, minus the volume of stored in the Mainland China warehouse at the end of day. Constraint (5) ensures that, for every truck, the loading volume of products cannot exceed its capacity limit. Constraints (6) and (7) determine variable types.

### III. COMPUTATIONAL RESULTS

Under the company's current logistics strategy, three company-owned trucks (V1, V2, V3) with the capacity of 450 are used to deliver the products between the Mainland China warehouse and Hong Kong warehouse; three China-hire trucks (V4, V5, V6,) with the capacity of 250 operate between the Mainland China warehouse and the border; three Hong Kong-hire trucks (V7, V8, V9) with the capacity of 450 operate between the border and the Hong Kong's warehouse; and two Hong Kong trucks (V10, V11) with the capacity of 450 operate between the Mainland China's warehouse and Hong Kong's warehouse. The unit trip cost for the company's owned trucks is \$200. However, the unit hiring cost for China's trucks, Hong Kong's trucks, and the trucks with two licenses are \$300, \$1000, and \$1200, respectively. It is also assumed that all trucks have the same loading capacity of 500. The unit inventory cost is \$1 in the Mainland China's warehouse and \$4 in the Hong Kong's warehouse. The unit shortage cost is \$12. The unit trans-shipment cost is \$0.5. Table I shows the supply to the Mainland China's warehouse over the next week.

TABLE I  
SUPPLY

Day	Mon	Tue	Wed	Thu	Fri	Sat
Quantity	2500	2300	2800	2600	2400	2500

This paper considers the shipment demand as the stochastic parameters, whose values depend on the future economic situation. As economic conditions are uncertain decision makers can only capture the realizations of future economic conditions. It is assumed that the future economic situation will fit into one of three possible situations – good, fair and bad – with associated probabilities. Let  $s_1$  represent a good economy with probability  $p_1, p_1 = \Pr\{s_1\}$ ;  $s_2$  represents a fair economy with probability  $p_2, p_2 = \Pr\{s_2\}$ ; and  $s_3$  represents a

bad economy with probability  $p_3$ ,  $p_3 = \Pr\{s_3\}$ . Table II shows the demand information for each scenario in Hong Kong.

TABLE II  
UNCERTAIN DEMAND

Scenario	Quantity	Mon	Tue	Wed	Thu	Fri	Sat
$s_1$	$p_1=0.1$	2600	2400	2900	2700	2500	2600
$s_2$	$p_2=0.2$	2500	2300	2800	2600	2400	2500
$s_3$	$p_3=0.7$	2400	2200	2700	2500	2300	2400

Table III shows the computational results including a loading plan for all trucks, a trans-shipping plan, and warehousing plan and a shortage plan. The related costs are also shown in Table 2. The total weekly logistics cost under uncertain demand is \$28,715. If the future economy is good, it means demand will be high. The company will adopt the strategy for scenario 1. If the future economy is fair, it means demand will be medium. The company will adopt the strategy for scenario 2. Similarly, if the future economy is bad, it means demand will be low. The company will adopt the strategy for scenario 3. Therefore, the company is able to response to the different situations that might happen in the future, while minimizing the whole transportation cost between two countries.

TABLE III  
INTERNATIONAL TRANSPORTATION PLAN AND RELATED COST

	Mon	Tue	Wed	Thu	Fri	Sat	Cost
Company-owned trucks	V1 (450)	V1 (450)	V1 (450)	V1 (450)	V1 (450)	V1 (450)	\$3,600
	V2 (450)	V2 (450)	V2 (450)	V2 (450)	V2 (450)	V2 (450)	
	V3 (450)	V3 (450)	V3 (450)	V3 (450)	V3 (450)	V3 (450)	
Hired trucks from China			V4 (200)	V4 (200)			\$1,500
	V4 (150)		V6 (250)	V5 (250)			
Hired trucks from HK	V8 (150)		V8 (450)	V8 (450)			\$3,000
Inventory in China	V10 (450)	V10 (450)	V10 (450)	V10 (450)	V10 (450)	V10 (450)	\$14,400
	V11 (450)	V11 (450)	V11 (450)	V11 (450)	V11 (450)	V11 (450)	
Inventory in HK	$s_1$			100			\$1,060
	$s_2$			200	150		
	$s_3$						
Shortage in HK	$s_1$	200	200	200		250	\$2,880
	$s_2$	100	100	100		50	
	$s_3$					250	

#### IV. CONCLUSION

This paper studies the international transportation problems. A stochastic mixed 0-1 integer programming model is formulated to deal with the uncertain information and short shipment notice. The computational results show that the company is able to achieve quick response to the changing demand while minimizing the total transportation cost. Further research will examine on simultaneously transporting goods from high-cost country back to low-cost country, which should substantially improve the fleet's efficiency.

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**Y. Wu** received her PhD in the Control Theory and Control Engineering in the Northeast University, Shenyang, China, in March 1999. Her research interests are supply chain and logistics, port operations, air cargo forwarding, production planning and scheduling, stochastic programming. She is currently a lecturer at the School of Management, University of Southampton, UK. She had been a Senior Research Associate at the Department of Management Sciences, City University of Hong Kong. Before joining the academic profession, she worked in a large electrical company as a Senior Engineer in computer science.