

# Complex Energy Signal Model for Digital Human Fingerprint Matching

Jason Zalev, and Reza Sedaghat

**Abstract**—This paper describes a complex energy signal model that is isomorphic with digital human fingerprint images. By using signal models, the problem of fingerprint matching is transformed into the signal processing problem of finding a correlation between two complex signals that differ by phase-rotation and time-scaling. A technique for minutiae matching that is independent of image translation, rotation and linear-scaling, and is resistant to missing minutiae is proposed. The method was tested using random data points. The results show that for matching prints the scaling and rotation angles are closely estimated and a stronger match will have a higher correlation.

**Keywords**—Affine Invariant, Fingerprint Recognition, Matching, Minutiae.

## I. INTRODUCTION

THIS technique involves comparing sets of extracted fingerprint minutiae that have been converted into an equivalent complex energy signal representation. Two types of minutiae, an endpoint and a bifurcation are shown in Fig. 1. Minutiae may be efficiently extracted from digital fingerprint images by using [1, 5]. In addition, our representation requires that an additional reference point be extracted called the *origin*. Several methods for extracting reference points from fingerprint images are given in [2, 3, 4]. The orientations of minutiae with respect to this origin can be compared between two fingerprints with far less computational complexity than would otherwise be possible. It is a simple matter to locate the position of each minutia in polar coordinates with respect to this origin. A matching technique described in [2] also uses such a reference point but an identifier based on Gabor filters is used for comparison.

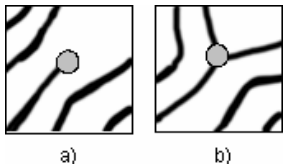


Fig. 1 Types of minutiae: a) an endpoint; b) a bifurcation

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## II. MINUTIAE SETS

The *minutiae-set* for a given fingerprint  $t$  will be defined as:

$$S_t \equiv \{(r_1, \theta_1)_t, (r_2, \theta_2)_t, \dots, (r_k, \theta_k)_t \mid k \in \mathbb{Z}^+\} \quad (1)$$

where  $k$  is the *length* of the minutiae and is written as  $\|S_t\|$ . A particular minutiae  $\beta_n = (r_n, \theta_n)$  has an associated radius  $r_n$  and an angle  $\theta_n$  that signifies its position in polar coordinates with respect to the origin. Any other minutiae properties including the minutiae type are not used this procedure. For convenience, the subscript  $t$  may be omitted when the fingerprint a minutia belongs to is known. We may also move subscripts inside and outside the brackets; for example,  $(r_n, \theta_n)_t$  is the same as  $(r_{nt}, \theta_{nt})$ .

It should be noted that the set of minutiae between two *matching* fingerprints will differ. For our definition, if two minutiae-sets *match* they belong to the same *equivalence class* and are considered *equal*. This is written as  $S_\sigma = S_\tau$  and will later be defined in Equation 12. Note that if two fingerprints are equal this does not necessarily mean that they are *equivalent*, which is denoted as  $S_\sigma \equiv S_\tau$  (Equation 4). Two matching minutiae sets are shown in Fig. 2.

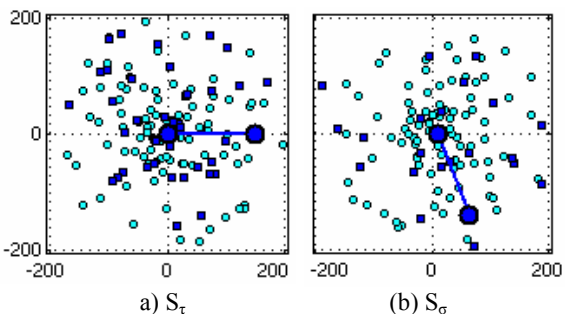


Fig. 2 Minutiae-sets  $\tau$  and  $\sigma$  illustrate an actual matching. There are 99 matching minutiae (small circles), and 84 total minutiae that do not match (small squares). The rotation of each fingerprint is also shown (line between large circles). The origin of  $\sigma$  (position of large circle near center) is offset by  $\|d\| = 10$  pixels in the positive x-axis direction. The rotation  $\phi$  is  $291^\circ$  and the scaling factor  $\alpha=0.95$

Given two fingerprints  $\tau$  and  $\sigma$  and the minutiae-sets  $S_\tau$  and  $S_\sigma$ , a matching algorithm must be made to incorporate the following allowances: i) a finite number of minutiae that are present in  $S_\tau$  may be missing from  $S_\sigma$ ; ii) a finite number of minutiae that are missing from  $S_\tau$  may be present in  $S_\sigma$ ; iii) The sets  $S_\tau$  and  $S_\sigma$  may be rotated about the origin by a *phase-rotation angle* known as  $\phi$ ; iv) The sets  $S_\tau$  and  $S_\sigma$  may be linearly scaled by a *scaling-factor* of  $\alpha$ ; v) the entire fingerprint image may be offset by an *image-translation vector* of  $\vec{q}$ ; and vi) the positions of origins detected between two matching prints may be offset by an *origin-translation vector* of  $\vec{d}$ , which is independent of the value of  $\vec{q}$ . In addition, each minutia may have the classification bifurcation or endpoint, and each minutia may also have a direction. The minutiae classification and direction are not incorporated into our matching criteria, but may be validated by adding a further post-matching stage.

In our algorithm the value of  $\vec{q}$  is of no concern, since the position of each minutia is calculated with respect to the origin. Hence, even if the entire fingerprint image is translated excessively, the effects from the translation offset  $\vec{q}$  will cancel out. This is because the position of the origin is extracted based on the features of fingerprint topology and will move approximately with the entire translation of the image. Nevertheless, the origin extraction process is not perfect and this causes the origin-translation vector  $\vec{d}$  to exist. It is recommended that the method of origin extraction be chosen as to keep the vector  $\vec{d}$  to a minimum.

For each minutiae-set  $S_i$ , an *ordering* will also be defined by:

$$\begin{aligned} \forall \beta_i, \beta_j \in S_i \text{ where } \beta_i = (r, \phi)_i \text{ and } \beta_j = (r, \phi)_j \\ i < j \Rightarrow r_i \leq r_j \end{aligned} \quad (2),$$

where  $\beta_i$  and  $\beta_j$  denote the  $i^{\text{th}}$  and  $j^{\text{th}}$  minutiae in the set. Hence, if the radius of  $\beta_i$  is less than the radius of  $\beta_j$  then  $\beta_i$  will be ordered before  $\beta_j$  in the set. In the case where  $\beta_i$  and  $\beta_j$  are the same distance from the origin, the minutiae with the smallest angle will be ordered first:

$$\begin{aligned} \forall \beta_i, \beta_j \in S_i \text{ where } \beta_i = (r, \phi)_i \text{ and } \beta_j = (r, \phi)_j \\ r_i = r_j \text{ and } i < j \Rightarrow \phi_i \leq \phi_j \end{aligned} \quad (3).$$

Two minutiae sets are equivalent if: i) they have the same length; and ii) if a minutia is in one set then it must be present in the other set with exactly the same coordinates. Specifically,

$$\begin{aligned} S_\tau \equiv S_\sigma \Leftrightarrow \\ \|S_\tau\| = \|S_\sigma\| = k \text{ and} \\ (r_n, \theta_n)_\tau = (r_n, \theta_n)_\sigma, n = 1, 2, \dots, k \end{aligned} \quad (4).$$

A *rotation* on a minutiae-set is done by subtracting an angle from the set. Thus,

$$S_i - \phi \equiv \{(r_{1i}, \theta_{1i} - \phi), (r_{2i}, \theta_{2i} - \phi), \dots, (r_{ki}, \theta_{ki} - \phi)\} \quad (5),$$

where  $k = \|S_i\|$ . *Scaling* can be done to a minutiae-set by multiplying the set by a positive real constant. Thus,

$$\alpha S_i \equiv \{(\alpha r_{1i}, \theta_{1i}), (\alpha r_{2i}, \theta_{2i}), \dots, (\alpha r_{ki}, \theta_{ki})\} \quad (6),$$

where  $k = \|S_i\|$ .

A pair of minutiae-sets  $S_\tau$  and  $S_\sigma$  is defined as a *perfect match* for some fingerprints  $\sigma$  and  $\tau$  if and only if:

$$\exists \phi, \alpha \in R : S_\tau \equiv \alpha S_\sigma - \phi. \quad (7).$$

In other words, a perfect match means that two sets of minutiae differ only by a scaling factor and in rotation. There can be no missing or extra minutiae between the sets in a perfect match. The origin-translation vector  $\vec{d}$  must also be zero.

### III. ENERGY SIGNAL REPRESENTATION

In digital signal processing, a complex signal  $g(x)$  is an *energy signal* if [6]

$$\int_{-\infty}^{\infty} \|g(x)\|^2 dx < \infty \quad (8).$$

The fingerprint *signature*  $P_{S_i}(x)$  for a fingerprint  $t$  is a complex-valued energy signal that is now defined by:

$$P_{S_i}(x) = \sum_{\forall (r, \theta) \in S_i} e^{i\theta} \gamma(x-r) \quad (9),$$

where  $\gamma(x)$  is a *blending function* used to model the *region of influence of a minutia*. Several blending functions that can be used are shown in Fig. 3. Fig. 4 and Fig. 5 show the fingerprint signatures corresponding to the minutiae-sets of Fig. 2, where a Gaussian blending function was used. Notice how the phase and magnitude plots of Fig. 4 do not correspond between the two signatures. This is because a signature is only calculated once per image and it does not take into account scaling. Accordingly, the blending function should be chosen so that the signatures generated between scaled images will still maintain a strong correlation.

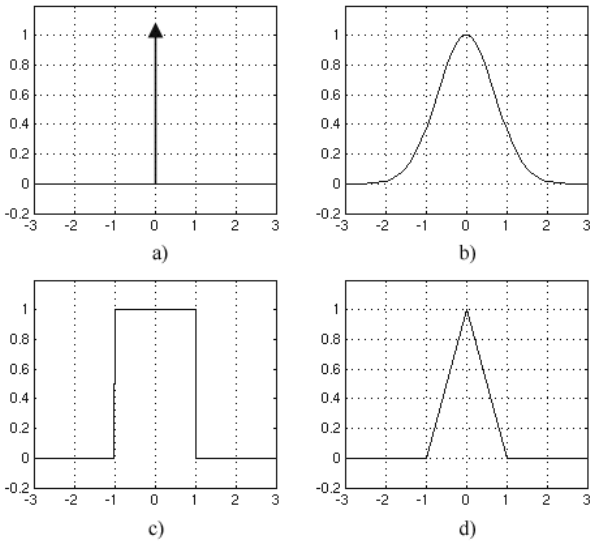


Fig. 3 Blending functions for  $\gamma(x)$  : a) the Dirac impulse function  $\delta(x)$ ; b) the Gaussian function  $\exp(-x^2 / T)$ ; c) a rectangular pulse; and d) a triangular pulse

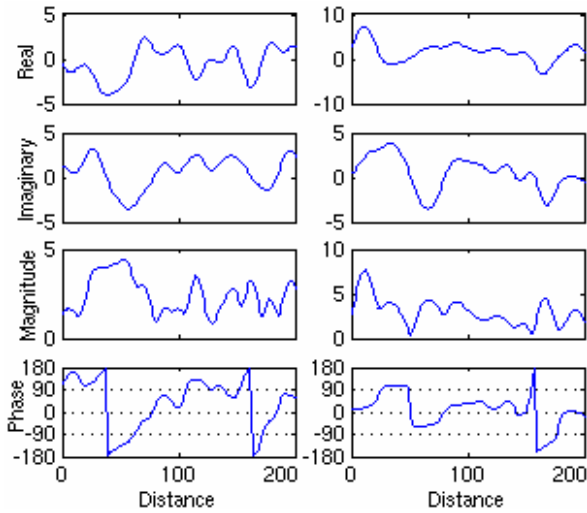


Fig. 4 Magnitude and phase plots for the fingerprint signatures  $P_{S_\tau}$  (left) and  $P_{S_\sigma}$  (right)

#### IV. CORRELATION ERROR

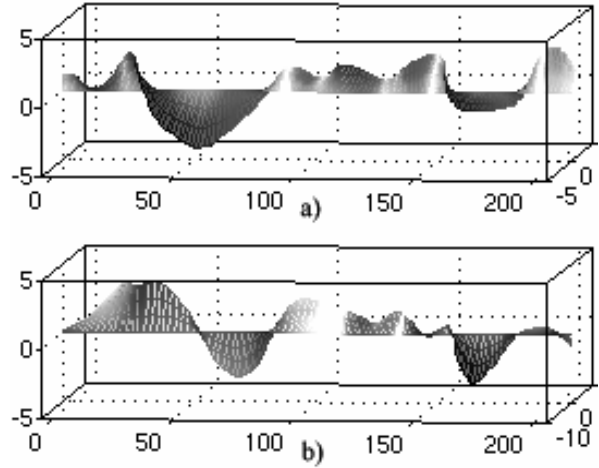


Fig. 5 Fingerprint signatures plotted in the complex x-z plane corresponding to minutiae-sets a)  $P_{S_\sigma}$  and b)  $P_{S_\tau}$ . The y-axis corresponds to the distance from the origin

The correlation error  $c(\phi, \alpha)$  is the Euclidean distance between the two signatures  $P_{S_\tau}(x)$  and  $P_{S_\sigma}(x)$  at a particular phase-rotation angle  $\phi$  and linear scaling factor  $\alpha$ . It is given by:

$$c(\phi, \alpha) = \int_{-\infty}^{\infty} \|P_{S_\tau}(x) - P_{(S_\sigma-\phi)}(\alpha x)\|^2 dx \quad (10).$$

The values of  $\phi$  and  $\alpha$  that minimize  $c(\phi, \alpha)$  will correspond to the actual phase-rotation and scaling factor between two matching fingerprints. Fig. 6 illustrates the correlation error between the minutiae-sets in Fig. 2. The *normalized correlation error*  $c_N(\phi, \alpha)$  is defined as

$$c_N(\phi, \alpha) = \frac{1}{\sqrt{\|S_\tau\|} \sqrt{\|S_\sigma\|}} c(\phi, \alpha) \quad (11),$$

and can be used to identify two matching fingerprints. The normalized correlation error is independent of the length of minutiae-sets.

#### V. MATCHING

Two minutiae-sets match if and only if they are equal. The condition for when two minutiae-sets are equal is defined as

$$S_\tau = S_\sigma \Leftrightarrow \exists \phi, \alpha \in R \text{ such that } c_N(\phi, \alpha) \leq \psi \quad (12),$$

where  $\psi$  is the *threshold matching constant*. An appropriate value for  $\psi$  can be selected based on the implementation

method used. When the normalized correlation error between two minutiae-sets is less than the threshold matching constant, this means that two fingerprints match. The values of  $\phi$  and  $\alpha$  that minimize the correlation error function correspond to the rotation, and the scaling factor between the two images. The value of  $\psi$  should be selected so that all fingerprints that match will be detected, but no false matches will be generated.

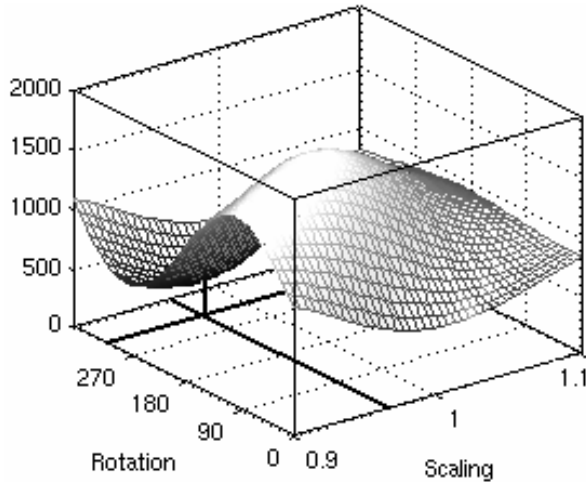


Fig. 6 Correlation error between  $\tau$  and  $\sigma$ . The minimum is located at ( $\phi=304.6, \alpha=0.9667$ ). In non-matching prints, the surface should be flatter and the minimum

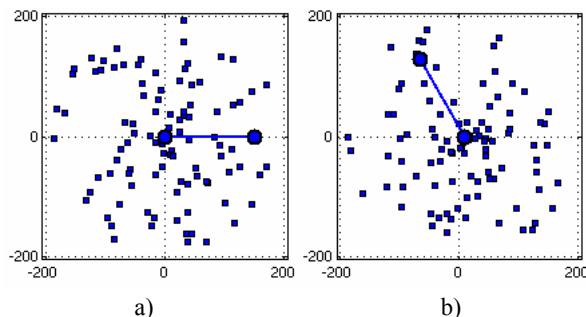


Fig. 7 Two non-matching minutiae-sets. The minutiae positions do not correspond between (a) and (b)

In our example, the minimum (shown in Fig. 6) is located at  $\phi=304.6, \alpha=0.9667$ . Thus, the estimated phase-rotation angle is accurate to 4.7%, and the scaling factor has an accuracy of 1.8% with respect to the original values of  $291^\circ$  and 0.95.

In Fig. 7 two non-matching minutiae-sets are shown, where the minutiae positions are all random. The correlation error for these two prints is shown in Fig. 8. It can be seen that the error surface is flatter and that the minimum value is higher than for the matching case of Fig. 6.

In addition, it was found that minutiae located closer to the origin are much more susceptible to error from a translated origin  $\vec{d}$  due to the properties of Euclidian geometry. For this reason any minutia located closer than approximately 20

percent of the image size with respect to the origin is dropped from the calculation of correlation error.

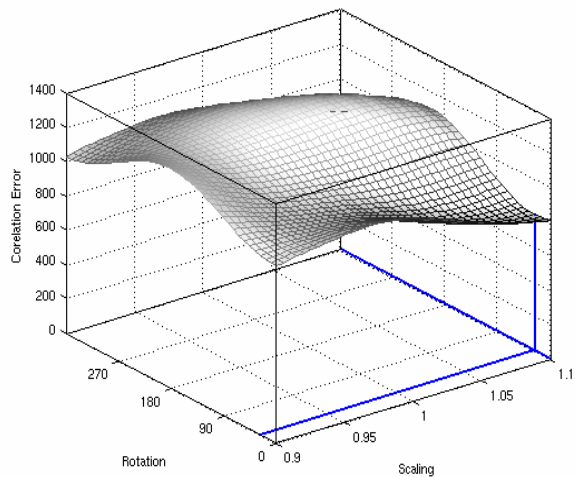


Fig. 8 Correlation error between the two non-matching prints in Fig. 7. The minutiae in Fig. 7a are not related to the minutiae in Fig. 7b, so the correlation curve is flatter and the minimum value is greater than in Fig. 6

## VI. CONCLUSION

We have reduced the problem of minutiae matching from comparing points in the plane to the problem of finding a correlation between two complex-valued energy signals. This process allows fingerprints to be matched independent of linear-scaling, angular rotation and translation and is also resistant to extra or missing minutiae between the two fingerprints.

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