

# Design of an M-Channel Cosine Modulated Filter Bank by New Cosh Window Based FIR Filters

Jyotsna Ogale\* Alok Jain#

**Abstract**—In this paper newly reported Cosh window function is used in the design of prototype filter for  $M$ -channel Near Perfect Reconstruction (NPR) Cosine Modulated Filter Bank (CMFB). Local search optimization algorithm is used for minimization of distortion parameters by optimizing the filter coefficients of prototype filter. Design examples are presented and comparison has been made with Kaiser window based filterbank design of recently reported work. The result shows that the proposed design approach provides lower distortion parameters and improved far-end suppression than the Kaiser window based design of recent reported work.

**Keywords**—Window function, Cosine modulated filterbank, Local search optimization.

## I. INTRODUCTION

COSINE modulated filterbank is a special sub-class of the general  $M$ -channel filterbank as shown in Fig.1. In different areas of the digital signal processing such as equalization of wireless communication channel, sub-band coding, bio-signal processing, spectral analysis, adaptive signal processing, denoising, feature detection and extraction, these filter banks are extensively used [1-2]. In CMFB the analysis and synthesis filters are cosine modulated version of lowpass prototype filter. Therefore, the design of whole filterbank reduces to the design of prototype filter [3]. The CMFB design problem has been extensively studied in [3-5] and at the end the design problem is reduced to finding a linear phase prototype filter that provides a flat overall magnitude response. NPR type filterbank suffers with different types of distortions. Several optimization algorithms have been developed for minimization of these distortion parameters [3,6,7].

Recently a new window function has been reported by Kemal *et al.* [8], which is based on cosine hyperbolic function. Moreover, the authors have presented the application of the proposed window in FIR filter design. In this paper, its application in the form of design of CMFB is presented and comparison has been made with Kaiser Window based design of recently reported work [9].

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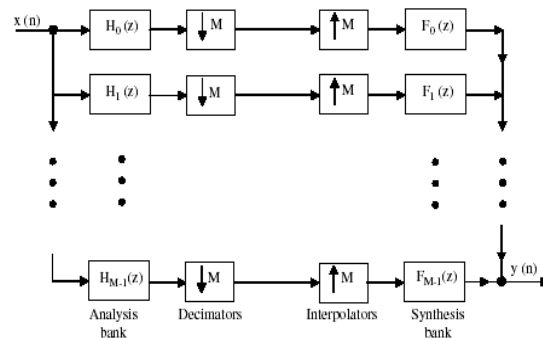


Fig. 1  $M$ -channel uniform filter bank.

## II. PROTOTYPE FILTER DESIGN

The impulse response coefficients of a causal  $(N-1)$  order linear phase FIR filter using window function is given by [1, 2]:

$$p(n) = w(n)p_i(n) \quad (1)$$

where,  $p_i(n)$  is the impulse response of the ideal lowpass filter and is expressed as:

$$P_i(n) = \frac{\sin\left(\omega_c \left(n - 0.5(N-1)\right)\right)}{\pi \left(n - 0.5(N-1)\right)} \quad (2)$$

$\omega_c$  is cut-off frequency of the ideal lowpass filter and  $w(n)$  is the window function. The prototype filter has been designed using newly reported Cosh window function. The time domain closed form expression is given by [8]:

$$w_c(n) = \begin{cases} \frac{\cosh\left(\alpha_c \sqrt{1 - \left(\frac{2n}{N-1}\right)^2}\right)}{\cosh(\alpha_c)} & |n| \leq \frac{N-1}{2} \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

The relationship between the window adjustable parameter ( $\alpha_c$ ) and the minimum stopband attenuation ( $A_s$ ) is given as [8]:

$$\alpha_c = \begin{cases} 0, & A_s < 20.8 \\ 0.2445(A_s - 20.8)^{0.4} \\ +0.1169(A_s - 20.8), & 20.8 \leq A_s < 50 \\ -8.722 \times 10^{-5} A_s^2 \\ +0.1335 A_s - 1.929, & 50 \leq A_s \leq 120 \end{cases} \quad (4)$$

The relationship between the normalized width of main lobe and stopband attenuation is required to find the minimum length of the filter which satisfies the given filter specifications. The normalized transition width is given by:

$$D_f = \begin{cases} 0, & A_s < 20.8 \\ 3.03 \times 10^{-4} A_s^2 \\ +0.05246 A_s - 0.2397, & 20.8 \leq A_s < 50 \\ -7.771 \times 10^{-6} A_s^2 \\ +0.07432 A_s - 0.5402, & 50 \leq A_s \leq 120 \end{cases} \quad (5)$$

The minimum filter length is calculated as:

$$N \geq \frac{D_f \omega_s}{\Delta \omega} + 1 \quad (6)$$

where,  $\omega_s$  is the stopband frequency and  $\Delta \omega$  is the transition bandwidth.

### III. PERFORMANCE PARAMETERS

Cosine modulation is the cost effective technique for  $M$ -channel filterbank. During the design phase, only the filter coefficients of prototype filter are required to optimize particularly in case of NPR filterbank. In NPR system, the PR conditions are relaxed by allowing small amount of errors. There are three types of errors occur at the reconstructed output, viz., amplitude ( $E_{pp}$ ), phase and aliasing ( $E_a$ ) [3].

The condition for NPR can be stated in terms of frequency response of the linear phase prototype filter. To obtain high quality reconstruction, the linear phase lowpass prototype filter  $P(e^{j\omega})$  must satisfy following two conditions as much as possible [5-7]:

$$|P(e^{j\omega})| \approx 0 \text{ for } |\omega| > \pi / M \quad (7)$$

$$T_0(e^{j\omega}) \approx 1, \text{ where } T_0(e^{j\omega}) = \sum_{k=0}^{2M-1} |P(e^{j(\omega-k\pi/M)})|^2 \quad (8)$$

The accuracy of the first approximation gives a measure of the aliasing error, while the accuracy of the second approximation gives a measure of the amplitude distortion. The phase error is eliminated by using linear phase prototype filter. However, other two distortion parameters can be minimized by applying suitable optimization technique. Traditional design approaches involve nonlinear and linear optimizations [7,9,10,11]. In this work, a single variable local

search optimization is used. The cutoff frequency of prototype filter is only variable parameter for minimizing the objective function. The other bandpass filters of analysis and synthesis sections are obtained by cosine modulation as given below [1,2]:

$$\begin{aligned} h_k(n) &= 2p(n) \cos \left[ \frac{\pi}{M}(k+0.5) \left( n - \frac{N}{2} \right) + (-1)^r \frac{\pi}{4} \right] \\ f_k(n) &= 2p(n) \cos \left[ \frac{\pi}{M}(k+0.5) \left( n - \frac{N}{2} \right) - (-1)^r \frac{\pi}{4} \right] \end{aligned} \quad (9)$$

for  $0 \leq k \leq M-1, 0 \leq n \leq N$

The input-output relationship for cosine-modulated filter bank is given by [1,2,7]:

$$Y(z) = T_0(z)X(z) + \sum_{l=1}^{M-1} T_l(z)X(zW_l^M) \quad (10)$$

where,  $W_M = e^{-j2\pi/M}$

$$T_0(z) = \frac{1}{M} \sum_{k=0}^{M-1} F_k(z)H_k(z)$$

is called the distortion transfer function and

$$T_l(z) = \frac{1}{M} \sum_{k=0}^{M-1} F_k(z)H_k(zW_l^M) \quad (11)$$

is the aliasing transfer function. These two are the major performance parameters.

### IV. OPTIMIZATION TECHNIQUE

In this work objective function given by Eq. (12) is used in optimization algorithm. Initially, input parameters, i.e., sampling rate, number of bands, passband and stopband frequencies, passband ripple and stopband attenuation of prototype filter are specified and determine the cutoff frequency, transition band and filter length. Initialize, different optimization pointers like step size, search direction, flag and initial (*error*) as well as expected minimum (*tol*) possible values of objective function. Inside the optimization loop, design the prototype lowpass filter and determine the bandpass filters for analysis and synthesis sections using cosine modulation. In optimization routine cutoff frequency is gradually changed as per the search direction and calculates the corresponding value of the objective function. Algorithm halts when it attains the minimum value of the objective function. The flowchart of optimization algorithm is given in appendix section and simulated on MATLAB 7.0.

$$\phi = \max \left| \left| P(e^{j\omega}) \right|^2 + \left| P(e^{j(\omega-\pi/M)}) \right|^2 - 1 \right| \quad (12)$$

$0 \leq \omega \leq \pi / M$

V. DESIGN EXAMPLES

In this section design of 8- and 16- channel CMFBs are present using Cosh and Kaiser window functions. The performance of newly reported Cosh window function is compared with Kaiser window function for same stopband attenuation as well as same filter order. Finally, obtained results are compared with recently reported work of Kha *et al.*[9]. In both examples the input parameters are deliberately chosen same as in reported work for the comparison of performance [9].

*Example 1:* An eight-channel cosine modulated filter bank is designed with Cosh and Kaiser window functions. In first case the stopband attenuation  $A_s$  is kept constant at 35.8 dB with stopband frequency  $\omega_s = 0.12\pi$  and filter length, amplitude distortion, aliasing distortion parameters are evaluated. The optimization algorithm is initialized with  $tol = 2.0E-04$ ,  $dir = 1$ ,  $step = 0.05$ ,  $ieror = 500$ . The magnitude responses of optimized prototype filters designed using both window functions and of filterbank are shown in Fig. 2. and Fig. 3, respectively. The plot of amplitude and aliasing distortions for Cosh window function is shown in Fig. 4 and Fig. 5, respectively. Table-1, shows the comparative performances for same stopband attenuation.

The variation of amplitude distortion with filter length for 8-channel CMFB is shown in Fig.6.

Similarly, in second case the filter order kept constant at 45 and filterbank are designed using Kaiser and Cosh window functions. The obtained value of performance parameters is given in Table-2.

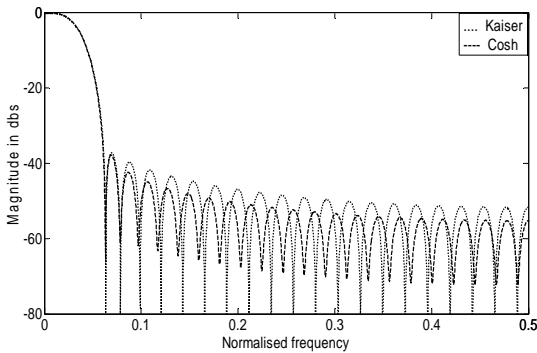


Fig. 2 Prototype filters frequency responses.

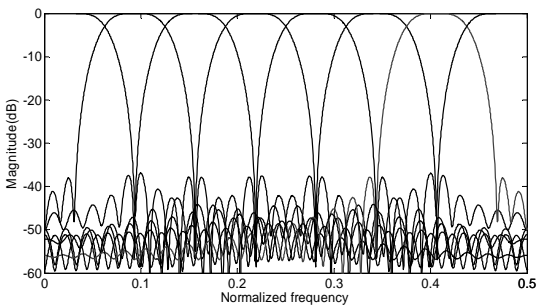


Fig. 3 Frequency response of 8-channel analysis filter bank

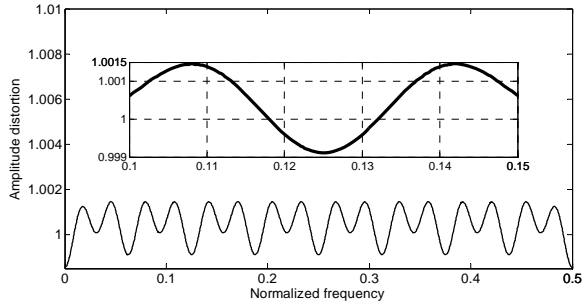


Fig. 4 Amplitude distortion magnitude plot; inset: Zoom plot showing one cycle between  $[0, \pi/M]$

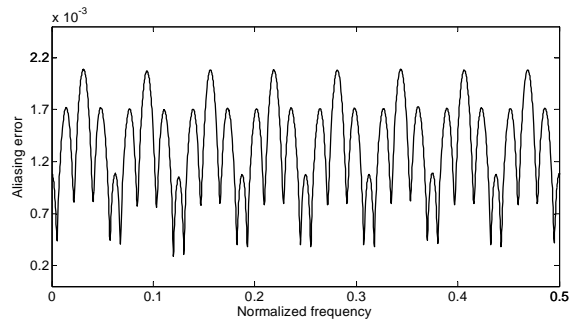


Fig. 5 Total aliasing distortion plot

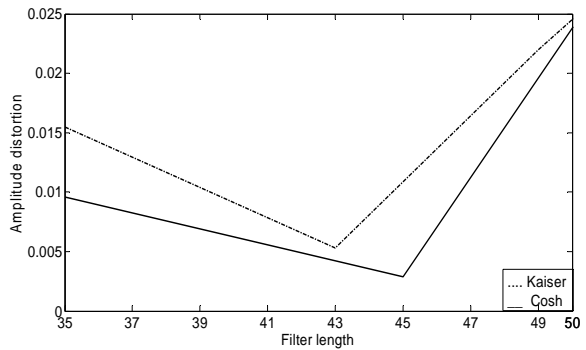


Fig. 6 Filter length vs amplitude distortion plot for 8-channel CMFB.

TABLE I PERFORMANCE COMPARISON AT  $A_s = 35.8$  dB FOR 8-CHANNEL FILTERBANK

S. No.	Window function	$N$	$\omega_s$	$E_{pp}$	$E_a$	$A_f$ dB	No. of iterations
1.	Kaiser	43	$0.12\pi$	$5.50 \times 10^{-3}$	$2.47 \times 10^{-3}$	50.10	38
2.	Cosh	45	$0.12\pi$	$2.00 \times 10^{-3}$	$2.01 \times 10^{-3}$	55.21	35

TABLE II PERFORMANCE COMPARISON AT  $N = 45$  FOR 8-CHANNEL FILTERBANK

S. No.	Window function	$A_s$ dB	$\omega_s$	$E_{pp}$	$E_a$	$A_f$ dB	No. of iterations
1.	Kaiser	38.6	$0.12\pi$	$2.19 \times 10^{-3}$	$5.15 \times 10^{-3}$	51.10	39
2.	Cosh	35.8	$0.12\pi$	$2.00 \times 10^{-3}$	$2.01 \times 10^{-3}$	55.21	35

*Example 2:* A 16-channel NPR type CMFB is designed. In first case the stopband attenuation is fixed at 45 dB (near end) for  $\omega_s = 0.0590\pi$  and other parameters are evaluated and shown in Table-3. The amplitude responses of optimized prototype filters and analysis filterbank are shown in Fig. 7 and Fig. 8, respectively. The plot of amplitude distortion ( $E_{pp}$ ) and aliasing distortion ( $E_a$ ) are shown in Fig. 9 and Fig. 10, respectively.

Similarly, for second case, the filter order is fixed at 97 at stopband frequency  $\omega_s = 0.0590\pi$ . The obtained different performance parameters are given in Table-4. The parameters are calculated for both window functions. The variation of amplitude distortion function with filter length for 16-channel CMFB is shown in Fig. 11. Variation of near end and far end attenuations with filter length is shown in Fig. 12.

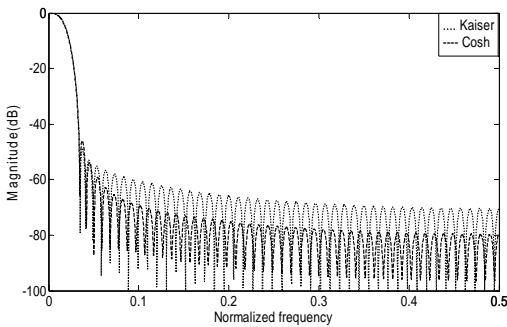


Fig. 7 Prototype filters frequency responses

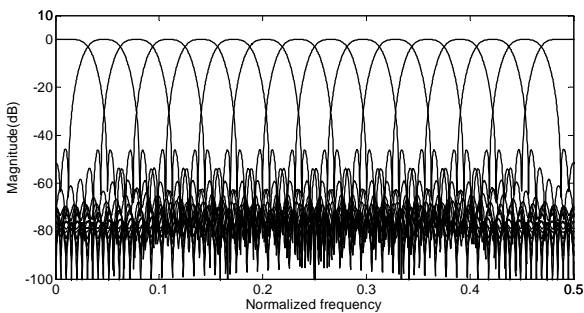


Fig. 8 Amplitude response of 16-channel analysis filterbank

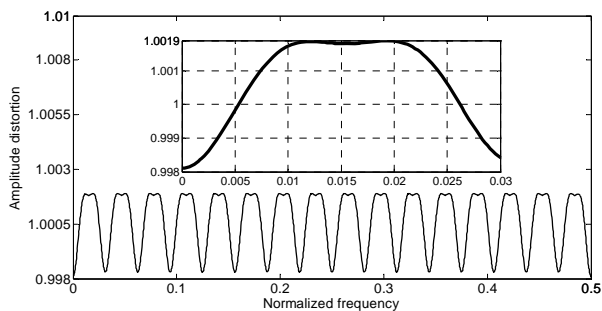


Fig. 9 Amplitude distortion magnitude plot; inset: Zoom plot showing one cycle between  $[0, \pi/M]$

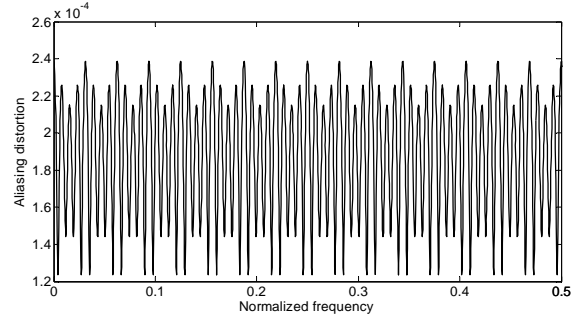


Fig. 10. Total aliasing distortion plot

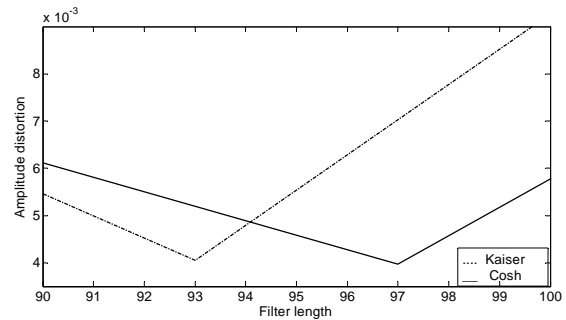


Fig. 11 Filter length vs. amplitude distortion plot for 16-channel CMFB.

TABLE III PERFORMANCE COMPARISON AT  $A_s = 45$  dB FOR 16-CHANNEL FILTERBANK

S. No.	Window function	N	$\omega_s$	$E_{pp}$	$E_a$	$A_f$ dB	No. of iterations
1.	Kaiser	93	$0.059\pi$	$4.056 \times 10^{-3}$	$2.92 \times 10^{-4}$	70.33	78
2.	Cosh	97	$0.059\pi$	$3.97 \times 10^{-3}$	$2.38 \times 10^{-4}$	79.69	77

TABLE IV PERFORMANCE COMPARISON AT  $N = 97$  FOR 16-CHANNEL FILTERBANK

S. No.	Window function	$A_s$ dB	$\omega_s$	$E_{pp}$	$E_a$	$A_f$ dB	No. of iterations
1.	Kaiser	47.2	$0.059\pi$	$5.503 \times 10^{-3}$	$4.94 \times 10^{-4}$	76.50	83
2.	Cosh	45.0	$0.059\pi$	$3.79 \times 10^{-3}$	$2.38 \times 10^{-4}$	79.69	77

TABLE V PERFORMANCE COMPARISONS WITH REPORTED WORK OF KHA ET AL. [9]

S.No.	M	$A_s$ dB	$\omega_s$	N	$E_{pp}$	$E_a$	optimization
Kha et al [8]	8	35.8	$0.12\pi$	41	$5.50 \times 10^{-3}$	$2.47 \times 10^{-3}$	Nonlinear
<b>Proposed</b>	8	35.8	$0.12\pi$	45	$2.00 \times 10^{-3}$	$2.01 \times 10^{-3}$	Linear
Kha et al [8]	16	45	$0.059\pi$	103	$5.95 \times 10^{-3}$	$3.89 \times 10^{-4}$	Nonlinear
<b>Proposed</b>	16	45	$0.059\pi$	97	$3.79 \times 10^{-3}$	$2.38 \times 10^{-4}$	Linear

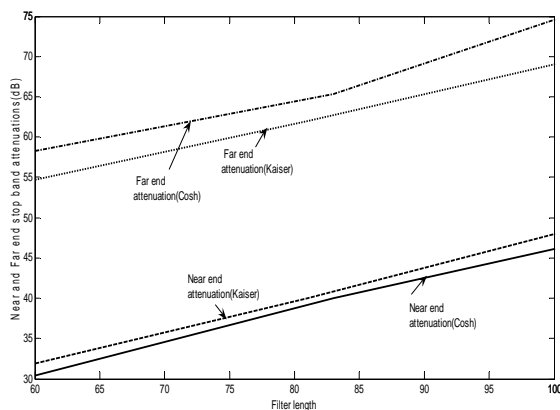


Fig. 12. Plot of Filter length vs. near and far end stopband attenuation

## VI. DISCUSSION

Experimental result shows that the performance of Cosh window based filterbank is better than the Kaiser window based filterbank. In case of 8-channel filterbank at fixed stopband attenuation of 35.8dB, the amplitude and aliasing distortions for Cosh window function are  $2.00 \times 10^{-3}$  and  $2.01 \times 10^{-3}$ , which is much less than the distortion parameters obtained for Kaiser window function, *i.e.*,  $5.503 \times 10^{-3}$ ,  $2.47 \times 10^{-3}$ . Similarly, at fixed filter length the performance of Cosh window function is better than the Kaiser window function.

Also in case of 16-channel filterbank, Cosh window based filterbank provides better results than Kaiser window based filterbank.

Apart from the distortion parameters, the other performance parameters like far-end attenuation and required number of iteration in optimization are also showing the superiority of Cosh window function. It is clear from the Fig. 6 and Fig. 11, that the proposed Cosh window function provides lower value of amplitude distortion than the Kaiser window for the wide range of filter length. It is clear from Fig. 12, the far-end attenuation of cosh window is better than the Kaiser window function.

In Table-5, the performance of proposed work is compared with recent work of Kha *et al.* [9]. For the same input parameters like stopband attenuation, stopband frequency the proposed design provides lower value of amplitude and aliasing distortions.

## VII. CONCLUSION

The design of  $M$ -channel filterbank using newly reported Cosh window function is presented. The results are compared with Kaiser window function and better performance is achieved. The obtained results are also superior than recently reported publication. The higher attenuation at the far-end can be utilized in application like beam forming and for better suppression of cross-talk and echo cancellation in audio and video fields.

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## REFERENCES

- [1] A.N.Akansu and R.A. Haddad, *Multiresolution Signal Decomposition: Transforms, Sub bands, Wavelets*. New York: Academic, 2001.
- [2] P.N. Heller, T. Karp, and T.Q. Nguyen, "A general formulation of modulated filter banks," *IEEE Trans. Signal Process.* vol.47, no. 4, pp. 986-1002, April 1999.
- [3] P. P. Vaidyanathan, *Multirate systems and filter banks*, Englewood Cliffs, NJ: Prentice-Hall, 1993.
- [4] T.Q.Nguyen, "Near-perfect-reconstruction pseudo-QMF banks," *IEEE Trans. Signal Process.*, vol.42, no.1, pp. 65-76, Jan. 1994.
- [5] T.Q.Nguyen, "Digital filter bank design quadratic-constrained formulation," *IEEE Trans. Signal Process.*, vol.43, no. 9, pp. 2103-2108, Sep.1995.
- [6] S.K. Mitra, *Digital signal processing: A computer Based Approach*. New York, NY: McGraw Hill, 1998.
- [7] C.D.Creusere and S.K. Mitra, "A simple method for designing high-quality prototype filters for  $M$ -band pseudo QMF banks," *IEEE Trans. on Signal Processing*, vol. 43, no. 4, pp. 1005-1007, April 1995.
- [8] Kemal Avci and Arif Nacaroglu, "Cosh window family and its application to FIR filter design," *AEU-International Journal of Electronics and Communications*, vol.63, pp .907-916, November 2009.
- [9] H.H.Kha, H.D.Tuan and T.Q.Nguyen, "Efficient design of cosine-modulated filter banks via convex optimization," *IEEE Trans. on Signal Processing*, vol. 57, no. 3, pp. 966-976, March 2009.
- [10] A.Jain, R.Saxena and S.C.Saxena, "An improved and simplified design of cosine modulated pseudo-QMF filter banks," *Digital Signal Processing*, vol. 16, no. 3, pp. 225-232, May, 2006.
- [11] Y.P. Lin and P. P.Vaidyanathan, "A Kaiser window approach for the design of prototype filters of cosine-modulated filter banks," *IEEE Signal Processing Letter*, vol. 5, no. 6, pp.132- 134, June1998.

APPENDIX

