Discontinuous Feedback Linearization of an Electrically Driven Fast Robot Manipulator

A. Izadbakhsh, M. M. Fateh, and M. A. Sadrnia

Abstract**—**A multivariable discontinuous feedback linearization approach is proposed to position control of an electrically driven fast robot manipulator. A desired performance is achieved by selecting a useful controller and suitable sampling rate and considering saturation for actuators. There is a high flexibility to apply the proposed control approach on different electrically driven manipulators. The control approach can guarantee the stability and satisfactory tracking performance. A PUMA 560 robot driven by geared permanent magnet dc motors is simulated. The simulation results show a desired performance for control system under technical specifications.

Keywords**—**Fast robot, feedback linearization, multivariable digital control, PUMA560.

I. INTRODUCTION

COMPUTER-assisted direct digital control is being increasingly used in process industries. Many industries use the proportional-integral-derivative control (PID) for direct digital control (DDC). The reasons for its popularity include well-known tuning methods for analog controller design and relaxed requirements for mathematical model of robot manipulators [1]. However, the control problem becomes hypersensitive, when faster trajectories are demanded. For instance, laser cutting of thin films, and arc welding can be mentioned. The main reason for this sensitiveness refers to dynamic problems resulting from high velocities and accelerations. In the other word, the manipulator dynamics is highly nonlinear with strong couplings existing among the joints which complicate the task of a simple PID controller. Therefore, robot performance degrades quickly, as speed increases [2]. To avoid this, using of nonlinear decoupling controllers is the best approach. Recent developments in nonlinear control allow the design of controllers that make full use of existing nonlinear models over a broader operating region. Many of these techniques are special cases of feedback linearization, which has a sound theoretical base in differential geometry [3], [4]. Several researches have been performed for flexible arms based on input-output feedback linearization [5]-[9].

Whenever the number of outputs is larger than the number of inputs, a part of dynamics of flexible manipulator becomes unobservable. These unobservable dynamics which refer to the zero dynamics of the system represent the motion of the flexible subsystem. Feedback linearization was used to linearize and decoupling of incorporated dynamics of manipulator and actuators [10]. A digital implementation of acceleration was used as feedback control without velocity and acceleration measurements [11]. A new scheme was proposed due to coupling. The proposed approach was applied for the three first joint of PUMA 560 Robot driven by flexible geared DC motor [12]. Many researches were performed based on feedback linearization [13]-[14]. However, the aforementioned studies have used continuous-time feedback linearization control while most robot manipulators like as PUMA 560, measure joint position digitally by using shaft encoders. In the other word, in practice, the digital data such as digital measurements are only available at specific instances and control inputs can only be changed at these instances. Hardware configuration of the robot control system is show in Fig. 1. Since a broad spectrum of continuous-time systems is computer controlled, we employ feedback linearization in combination with optimal control technique to design a discontinuous nonlinear feedback controller for trajectory tracking in joint space.

This paper is organized as follows: Section II describes the dynamic model formulation of actuator and robot. Section III describes the general steps of feedback linearization design and dedicate to define the model based control strategy of this study. Section IV is related to sampling rate option. Section V lists general steps for discontinuous feedback linearization and major simulation results in terms of tracking performance in analog and digital space. Section VI is performed for stability proof and conclusions are given in Section VII.

II. ACTUATOR/MANIPULATOR DYNAMIC MODELING

We first notice to familiar differential equations of motion which describe the DC motor dynamics to drive a n degrees of freedom robot. These equations are given by

$$J_m\ddot{\theta}_m + (b_m + \frac{k_m k_b}{R})\dot{\theta}_m = \frac{k_m}{R}V - r\tau_l \tag{1}$$

where $\dot{\theta}_m$ is the load angular velocity, V is the motor voltage, τ_l is the torque load, R is the armature resistance, K_b is the back EMF constant, K_m is the torque constant, J_m is the moment of inertia, b_m is the damping coefficient and r is the gear ratio. It must be noted that, here we are ignored the motor inductance. However, armature resistance is significant in

A. Izadbakhsh (Izadbakhsh_alireza@hotmail.com), M. M. Fateh (mmfateh@shahroodut.ac.ir), and M. A. Sadrnia (Masadrnia@yahoo.com) are with the Department of Electronic and Robotics Engineering, Shahrood University of Technology, Shahrood, Iran (phone: 00982324225697).

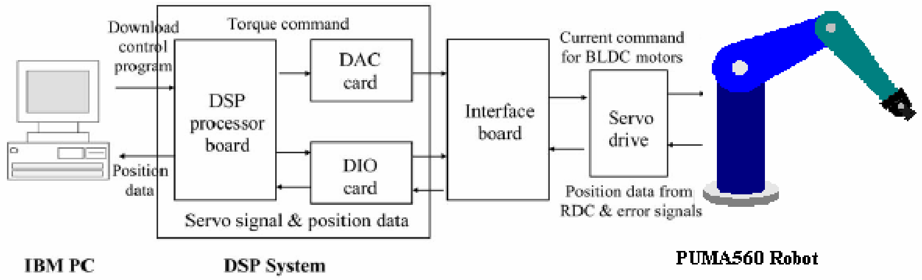

Fig. 1 Hardware configuration of the control robot system

determining the maximum achievable joint velocity [12], [19]. Whereas, feedback linearization is a strongly model-based controller design approach, it is important to obtain an accurate model of the rigid robot manipulator dynamics as follows

$$A(q)\ddot{q}+B(q)\left[\dot{q}\dot{q}\right]+C(q)\left[\dot{q}^{2}\right]+g(q)=\tau \qquad (2)$$

where $\dot{q},\ddot{q}$ are the joint velocity and acceleration vectors, respectively, A(q) denotes the inertia matrix, C(q), B(q) expresses the matrices of coriolis and centripetal forces respectively, and G(q) is the gravity vector.

III. Multivariable Feedback Linearization Design

In this section, we recall briefly the basics of feedback linearization theory. For a detailed treatment, see [4]. Consider a generic nonlinear dynamic system

$$\begin{aligned}\dot{\mathbf{x}}&=f(\mathbf{x})+g(\mathbf{x})u\\ y&=h(\mathbf{x})\end{aligned} \qquad (3)$$

where x is the n-dimensional state and u is the m-dimensional input. The exact state linearization problem via static feedback consists in finding a control law of the form

$$u=\alpha\left(x\right)+\beta\left(x\right)v \qquad (4)$$

with β(x) nonsingular and v an auxiliary input, and a change of coordinates z =ϕ(x) such that, in the new coordinates, the closed-loop system is linear and controllable. Necessary and sufficient conditions exist for the solvability of this problem (Theorem 5.2.3) [4]. For fully actuated robots, i.e., with a number of generalized coordinates equal to the number of input commands, these conditions are trivially satisfied and the control law (2) leads to the well-known computed torque method. If static feedback does not allow one to solve the problem, one can try to obtain exact state linearization by means of a dynamic feedback compensator of the form

$$\begin{aligned}u&=\alpha\left(x,\zeta\right)+\beta\left(x,\zeta\right)v\\ \dot{\zeta}&=\gamma\left(x,\zeta\right)+\delta\left(x,\zeta\right)v\end{aligned} \qquad (5)$$

where ξ is the ν-dimensional compensator state, together with a change of coordinates z =ϕ(x,ζ). Only sufficient conditions are available for the solvability of this problem (see proposition 5.4.4) [4]. In robotics, dynamic feedback has been used for the exact linearization of manipulators with elastic joints [15] and of nonholonomic wheeled mobile robots [16]. In both the static and the dynamic feedback case, if the sufficient conditions are satisfied, the actual construction of the control law requires us to identify an auxiliary m-dimensional output y = h(x), such that the corresponding vector relative degree is well defined and the sum of its elements equals n. In particular, this output vector, together with its derivatives up to a certain order, defines the linearizing coordinate z. As a byproduct, the control laws (4) or (5) also yield input-output decoupling between v and y. In our study, we will restrict our analysis to the multivariable nonlinear systems with the same numbers of inputs and outputs. Under the nonlinear state transformation z =ϕ(x) and the control low (4), the original nonlinear system (3) can be transformed into a simple linear system:

$$\dot{\mathbf{z}}=\mathbf{A}\mathbf{z}+\mathbf{b}\mathbf{v}=\begin{bmatrix}0 & \mathbf{I} & 0 & 0\\ \vdots & & \ddots & 0\\ \vdots & 0 & & \mathbf{I}\\ 0 & \cdots & \cdots & 0\end{bmatrix}\mathbf{z}+\begin{bmatrix}0\\ \vdots\\ \mathbf{I}\end{bmatrix}\mathbf{v} \qquad (6)$$

where v is control input vector and **I** is a diagonal unit matrix [17]. We can achieve any state in finite time using a suitable control low if the given system to be controllable. To control a linearized system, we can use a linear state feedback control law of the form

$$\mathbf{v}=-\mathbf{k}\mathbf{z}+\mathbf{r} \qquad (7)$$

where k is a vector of design parameters for pole placement and r is a desired trajectory. The coefficient vector k is

International Journal of Information, Control and Computer Sciences
ISSN: 2517-9942
Vol:1, No:5, 2007

determined such that the system poles are placed in desired places. Substituting (7) into (6) yields

$$\dot{\mathbf{z}} = \left(\mathbf{A} - \mathbf{bk}\right)\mathbf{z} + \mathbf{br} \tag{8}$$

In this manner, the closed loop poles are eigenvalues of the *A-bk* matrix. The linear state feedback provides possibility of achieving a wide range of closed loop poles. One solution for determination k, is given by the optimum linear control law of the form

$$\mathbf{k} = R^{-1}\mathbf{b}^T P \tag{9}$$

where P and Q are the symmetric positive definite n×n matrixes and R>0 which satisfy the Riccaty equation as follows

$$A^T P + PA - PbR^{-1}b^T P + Q = 0 \tag{10}$$

IV. Sampling Rate Option

A large class of continuous-time systems is in fact computer controlled. In these cases the system data are only available at specific instances and control inputs can only be changed at these instances. Recent advancements in the speed and size of digital technology (DSP, microprocessors, etc.), make possible implementation of digital control design for robot manipulator. However, often it is hard to get around sampling rate for sampling operation. To avoid this, in many of literatures, it does usually assume the sampling frequency is sufficiently high, such that, the closed-loop system considers as a continuous-time system. But how much the sampling frequency is big for actual systems? Examples of systems which are inherently sampled are given by [18]. For calculating a practical sampling frequency, an initial continuous-time design must be performed. To this end, we perform exact feedback linearization on combined dynamics of robot and actuators. After that, the optimal state feedback vector for trajectory tracking is obtained by suitable selection of Q and R weight matrices and using (9). A sampling frequency, ω_s, between of at least ten times and at most twenty times of closed-loop system bandwidth, ω_b, (11) is the best selection based on drawing the frequency response of closed-loop system.

$$10 < \frac{\omega_s}{\omega_b} < 20 \tag{11}$$

Then

$$T_s = \frac{2\pi}{\omega_s} \tag{12}$$

V. Discontinuous Feedback Linearization Design

Continuous-time control method is applied to digital controller design. This design methodology requires a high sampling rate. This method is highly practical. Design of a digital controller usually starts with an analog design, if only to select a suitable sample period. The equivalent diagram of the proposed approach is shown in Fig. 2. So, based on descriptions that mentioned to them in above, the procedure used to design of digital controller entails four steps:

Step1: Initial Continuous-Time Design

1) Combine the dynamic model of robot and actuators as follows:

$$\begin{aligned}(A(q) + r^{-1}J_m r^{-1})\ddot{q} + B(q)\left[\dot{q}\dot{q}\right] + C(q)\dot{q}^2 \\ + G(q) + r^{-1}(b_m + \frac{k_m k_b}{R})r^{-1}\dot{q} = r^{-1}k_m R^{-1}V\end{aligned} \tag{13}$$

where J_m is a diagonal matrix of the moment of inertia. A compact form of (13) is given by

$$D(q)\ddot{q} + h(q,\dot{q}) = u \tag{14}$$

where

$$\begin{aligned}&D(q) = A(q) + r^{-1}J_m r^{-1} \\ &h(q,\dot{q}) = B(q)\left[\dot{q}\dot{q}\right] + C(q)\dot{q}^2 + G(q) + r^{-1}(b_m + \frac{k_m k_b}{R})r^{-1}\dot{q} \\ &u = r^{-1}k_m R^{-1}V\end{aligned} \tag{15}$$

By choosing position and velocity of actuators as state variable (16), the state space form of equation (14) is

$$\begin{cases} X_1 = q \\ X_2 = \dot{q} \end{cases} \tag{16}$$

$$\Rightarrow \begin{pmatrix} \dot{X}_1 \\ \dot{X}_2 \end{pmatrix} = \begin{pmatrix} X_2 \\ -D^{-1}h(X_1, X_2) \end{pmatrix} + \begin{pmatrix} 0 \\ D^{-1} \end{pmatrix} u \tag{17}$$

2) Checking the linearization conditions and forced a change of coordinates z =ϕ(x). As said in Section III, for fully actuated robots, i.e., with a number of generalized coordinates equal to the number of input commands, these conditions are trivially satisfied and so, change of coordinates is formed as

$$\begin{aligned}&z_1 = X_1 \Rightarrow \dot{z}_1 = \dot{X}_1 = X_2 \\ &z_2 = X_2 \Rightarrow \dot{z}_2 = \dot{X}_2 = D^{-1}(u - h(X_1, X_2)) = \upsilon\end{aligned} \tag{18}$$

By comparing (18) and (4), α and β are obtained

$$\alpha = h(X_1, X_2) \quad , \quad \beta = D(q) \tag{19}$$

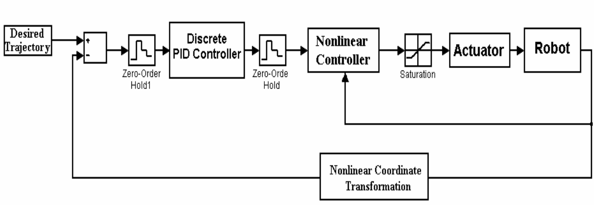

Fig. 2 Discontinuous feedback linearization scheme

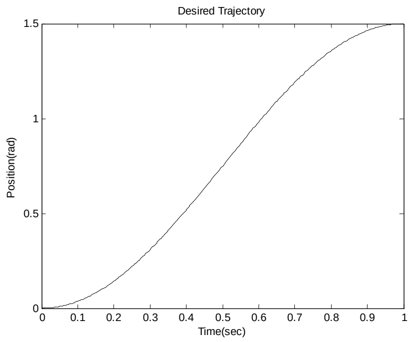

Fig. 3 Desired trajectory

In this manner, the decoupled system is

$$\dot{z} = Az + bv = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix} z + \begin{bmatrix} 0 \\ I \end{bmatrix} v \tag{20}$$

3) By choosing weight matrices Q and R

$$Q = 10000 \times \begin{pmatrix} I_{6\times6} & 0_{6\times6} \\ 0_{6\times6} & 0_{6\times6} \end{pmatrix}, \quad \mathrm{R} = 100 \times \begin{pmatrix} 0_{6\times6} & 0_{6\times6} \\ 0_{6\times6} & I_{6\times6} \end{pmatrix} \tag{21}$$

The optimal state feedback vector K is proposed as

$$K = \left[1000 \times I_{6\times6} \quad 44.7 \times I_{6\times6}\right]_{6\times12} \tag{22}$$

For continuous-time design, desired trajectory, tracking error, and output tracking are shown by Fig. 3, Fig. 4 and Fig. 5, respectively. It must be noted that, the same trajectories are considered for all of joints. Also, a proof of stability for continuous-time design is easily provided.

Step2: Choosing sample period

Bode diagram of closed loop system is shown in Fig. 6. We obtain the closed-loop bandwidth from the closed-loop frequency response. It is approximately 64.9 rad /sec. We can choose a sampling frequency in the following range.

$$\omega_S = (10 \quad to \quad 20) \times 64.9 \ \mathrm{rad/sec} = (649 \quad to \quad 1298)\, rad/\mathrm{sec} \tag{23}$$

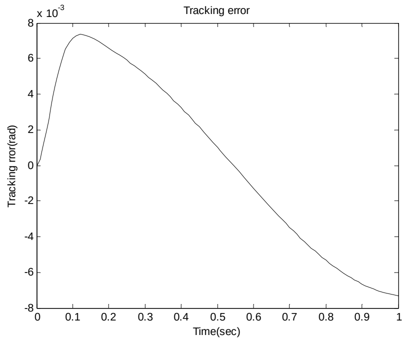

Fig. 4 Tracking error

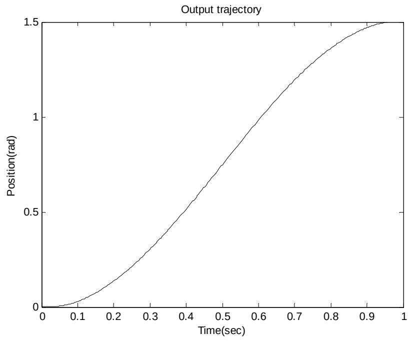

Fig. 5 Output trajectory

Then

$$T_s = \frac{2\Pi}{\omega_S} = 0.0048 \quad to \quad 0.0097\,\mathrm{sec} \tag{24}$$

It is necessary to remind that, we choose the worse of sampling rate for sampling operation (Ts = 0.01 sec).

Step 3: Add sampled-data system elements

A discrete time implementation will introduce a delay of at least one sample period T_s. This delay must be added to decoupled system. In this stage by considering to zero order hold approximation effect, we obtain a controller that establishes design criteria's. The controller is designed as

$$(44.7s + 1000) \times I_{6\times6} \tag{25}$$

Step 4: Digitize control law

After obtaining the compensator (25), which satisfy the design criteria's of continuous-time system, find its discrete time equivalent form, for example, by the Tustin method. The result given by:

$$\left(\frac{9944z - 7944}{z+1}\right) \times I_{6\times6} \tag{26}$$

Output trajectory and tracking error have shown by Fig. 7 and Fig. 8, respectively. It is shown that the tracking errors to be under about 0.008 rad that is acceptable due to

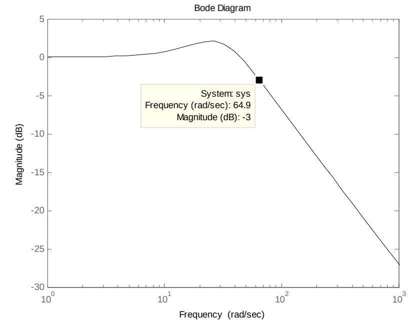

Fig. 6 Closed-loop frequency responses

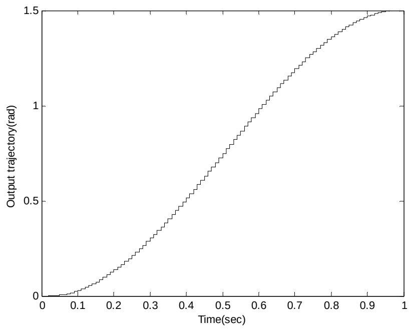

Fig. 7 Output trajectory

mechanical resolution. The technical limits such as voltage limit, and torque limit should be considered. Fig. 9 and Fig.10 show voltages of motors and load torque of robot respectively that designate good operation of actuators. It must be noted that the load torque for the second three joints is approximatly zero because there are no distortion in center wrist. By using Prewrap method, tracking error is given by Fig. 11.

VI. Stability Analysis

In here, we present stability analysis for discontinuous feedback linearization. We had shown that, the combined dynamics of actuators and robots after implementation of feedback linearization is divided to six same independent subsystems as follows

$$\ddot{q}_i = v_i \tag{27}$$

Discrete time equivalent form for i'th subsystem analog controller sees in Fig 12. At first, Z transform of $G_h(s)G_p(s)$ given by

$$\mathcal{Z}\left\{G_h(s)G_p(s)\right\} = \mathcal{Z}\left\{\frac{1-e^{-sT_s}}{s}\frac{1}{s^2}\right\} \tag{28}$$

$$= \left(1-z^{-1}\right)\mathcal{Z}\left\{\frac{1}{s^3}\right\} = \frac{T^2}{2}\frac{Z+1}{\left(Z-1\right)^2}$$

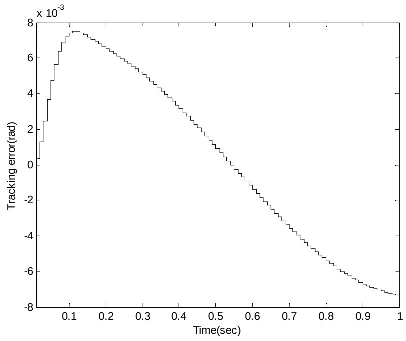

Fig. 8 Digital tracking error (Tustin method)

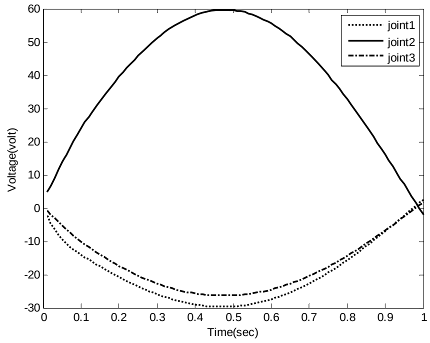

Fig. 9 Motor voltages

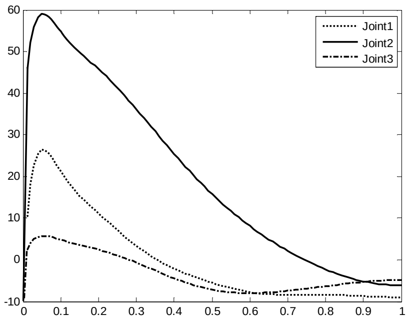

Fig. 10 Load torques

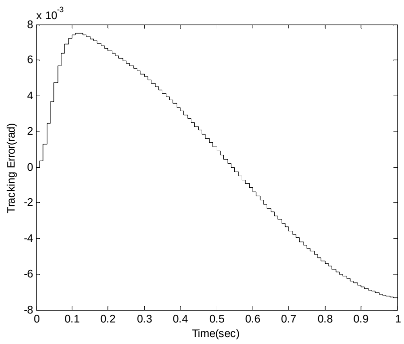

Fig. 11Prewrap method with ω=20rad/sec

International Journal of Information, Control and Computer Sciences
ISSN: 2517-9942
Vol:1, No:5, 2007

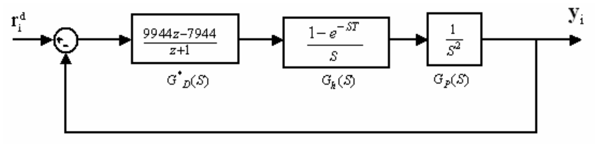

Fig. 12 Discrete time equivalent form for the analog controller

On the other hand, the closed loop equation is

$$1+G(z)=0 \tag{29}$$

where:

$$G(z)=G_D(z)\times \mathcal{Z}\{G_h(s)G_p(s)\}=\frac{T^2(4972z-3972)}{(z-1)^2} \tag{30}$$

Solving (29) leads to

$$z^2-1.5028z+0.6028=0 \tag{31}$$

By using jury stability criteria [23], the closed-loop poles are obtained

$$z_{1,2}= \ 0.7514 \pm 0.1956i \tag{32}$$

where all poles are within circle unit as seen in Fig 13, therefore the closed loop system is stable.

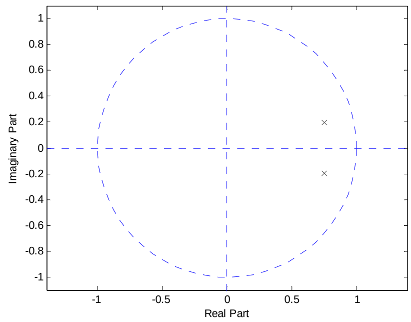

Fig.13 Z plane

VII. Conclusion

Due to the complexity of the nonlinear coupling inertial, coriolis, centripetal and gravitational forces arising from motion of the manipulator, PID control algorithm is not suitable alone for direct digital control in high velocities and accelerations. A multivariable discontinuous feedback linearization scheme was proposed for position control of an electrically driven high speed robot manipulator. A suitable performance was achieved by selecting a suitable controller and desired sampling rate and considering saturation for actuators. It was shown that there is not significant difference between the performance of analog and discontinuous feedback linearization.

References

[1] R. P. Srivastava, Use of genetic algorithms for optimization in digital control of dynamic systems, ACM, 1992, pp. 219-224.
[2] A. Vivas, V. Mosquera, Predictive functional control of a PUMA robot, ACSE 05 Conference, CICC, Cairo, Egypt, 19-21 December 2005.
[3] M. A. Henson, Feedback linearization strategies for nonlinear process Control, PhD thesis, The University of California at Santa Barbara, 1992.
[4] A. Isidori, Nonlinear Control Systems, 3rd Edition, Springer-Verlag, 1995.
[5] A. De Luca, and B. Siciliano, Trajectory control of a nonlinear one-link flexible arm, Int. J. Control 50(5), pp.1699-1715, 1989.
[6] P. Lucibello, Non linear regulation with internal stability of a two link flexible robot arm, in Proceedings of the 28th Conference on Decision and Control, Tempa, FL, 1989.
[7] D. Wang, and M. Vidyasagar, Control of a class of manipulators with a single flexible link: Part II. Observer controller stabilization, Journal of Dynamic Systems, Measurement, and Control 113, pp. 662-668, 1991.
[8] D. Wangand, M. Vidyasagar, Control of a class of manipulators with a single flexible link: Part I. Feedback linearization, Journal of Dynamic Systems, Measurement, and Control 113, pp. 655-661, 1991.
[9] P. Lucibello, and M. D. Di Benedetto, Output tracking for a non linear flexible arm, Journal of Dynamic Systems, Measurement, and Control 115, pp.78-85, 1993.
[10] T. J. Tarn, A. K. Bejczy, X. Yun, and Z. Li, Effects of motor dynamics on nonlinear feedback robot arm control, IEEE Transactions on Robotics and Automation, vol. 7. No. 1, pp. 114-122, February 1991.
[11] J. Studenny, P-R. Belanger, L-K. Daneshmend, A digital implementation of the acceleration feedback control law on a PUMA 560 manipulator, Proceedings of the 30th conference on Decision and control, Brighton, Engeland, December 1991.
[12] P. J. Baines, and J. K. Mills, Feedback Linearized Joint Torque Control of a Geared, DC Motor Driven Industrial Robot, the international journal of robotics research, pp.169-192, 1998.
[13] G. Oriolo, A. D. Luca, and M, Vendittelli, WMR Control Via Dynamic Feedback Linearization: Design, Implementation, and Experimental Validation, IEEE Transactions on control systems technology, Vol. 10, NO. 6, pp. 835-852, November 2002.
[14] T. Sugie, K. Fujimoto, and Y. Kito, Obstacle Avoidance of Manipulators With Rate Constraints, IEEE Transactions on robotics and Automation, Vol. 19, No. 1, pp.168-174, 2003.
[15] A. De Luca, and P. Lucibello, A general algorithm for dynamic feedback linearization of robots with elastic joints, IEEE Int. Conf. on Robotics and Automation, pp. 504–510, 1998.
[16] B. d'Andrea-Novel, G. Bastin, and G. Campion, Control of nonholonomic wheeled mobile robots by state feedback linearization. Int. J. of Robotics Research 14(6):543–559, 1995.
[17] M. W. Spong, M. Vidyasagar, Robot Dynamics And Control, John Wiley And Sons, 1989.
[18] K.J. Astrom, and B. Wittenmark, Computer-Controlled Systems, Theory and Design, Prentice Hall information and System Science Series, 3rd edn, Prentice Hall, 1997.
[19] P. I. Corke, in situ Measurement of Robot Motor Electrical Constants.
[20] B. Armstrong, O. Khatib, J. Burdick, The Explicit Dynamic Model and Inertial Parameters of the PUMA 560, IEEE Transaction, pp.510-518, 1986.
[21] P. I. Corke, B. Armstrong-H´elouvry, A Search for consensus among model parameters reported for the PUMA 560 Robot.
[22] P. I. Corke, The Unimation Puma servo system, MTM-226, July 1994.
[23] K. Ogata, Discrete Time Control Systems, Translated by P. J. Maralani and A. Khakhi sedig, 2000.