A Family of Affine Projection Adaptive Filtering Algorithms With Selective Regressors

Mohammad Shams Esfand Abadi, Nader Hadizadeh Kashani, and Vahid Mehrdad

Abstract—In this paper we present a general formalism for the establishment of the family of selective regressor affine projection algorithms (SR-APA). The SR-APA, the SR regularized APA (SR-R-APA), the SR partial rank algorithm (SR-PRA), the SR binormalized data reusing least mean squares (SR-BNDR-LMS), and the SR normalized LMS with orthogonal correction factors (SR-NLMS-OCF) algorithms are established by this general formalism. We demonstrate the performance of the presented algorithms through simulations in acoustic echo cancellation scenario.

Keywords—Adaptive filter, affine projection, selective regressor.

I. INTRODUCTION

ADAPTIVE filtering is an important subfield of digital signal processing having been actively researched for more than four decades and having important applications such as noise cancellation, system identification, telecommunications channel equalization, and telephony acoustic and network echo cancellation [1], [2], [3]. In some of these applications, a large number of filter coefficients are needed to achieve an acceptable performance. Therefore the computational complexity is the main problem in these applications.

Several adaptive filter algorithms such as the adaptive filter algorithms with selective partial updates (SPU) have been proposed to solve this problem. The Max-NLMS [4], the MMax-NLMS [5], [6], the variants of the selective partial update normalized least mean square algorithms (SPU-NLMS) [7], [8], [9], and the SPU affine projection algorithm (SPU-APA) [8] are important examples of this family of adaptive filter algorithms.

Recently, an affine projection adaptive filtering algorithm with selective regressors (SR) was proposed in [10]. This paper presents a novel affine projection algorithm which reduces computational complexity by optimally selecting a subset of input regressors at every iteration.

In this paper we extend the approach in [10] to present the family of SR-AP algorithms. The SR regularized APA (SR-R-APA), the SR partial rank algorithm (SR-PRA), the SR binormalized data reusing LMS (SR-BNDR-LMS), and the SR NLMS with orthogonal correction factors (SR-NLMS-OCF) are established through the general formalism.

What we propose in this paper can be summarized as follows:

Mohammad Shams Esfand Abadi is with the Shahid Rajaee Teacher Training University, Department of Electrical Engineering, Tehran, Iran (corresponding author to provide phone: +98-21-22970003; e-mail: mshams@srttu.edu).

Nader Hadizadeh Kashani is B.S. student in Islamic Azad University (South Teharn Branch), Young Researchers Club.

Vahid Mehrdad is the master student in Shahid Rajaee Teacher Training University, Department of Electrical Engineering, Tehran, Iran (e-mail: vmehrdad@srttu.edu.)

- The establishment of the family AP algorithms.
- Extension of the selective regressor approach, and the establishment of the family of SR-AP algorithms.
- Demonstrating of the presented algorithms in acoustic echo cancellation scenario.

This paper is organized as follows. In Section II, we presents the data model and general update equation for the family of AP algorithms. In the next section, the family of SR-AP algorithms is introduced. Section VI presents the computational complexity of the SR-AP algorithm. We conclude the paper by showing a comprehensive set of simulation results in system identification and acoustic echo cancellation scenarios.

Throughout the paper, the following notations are used:

$\|.\|$	Euclidean norm of a vector.
$\mathrm{Tr}(.)$	Trace of a matrix.
$(.)^T$	Transpose of a vector or a matrix.

II. BACKGROUND ON NLMS ALGORITHM

Figure 1 shows a typical adaptive filter setup, where $x(n)$, $d(n)$ and $e(n)$ are the input, the desired and output error signals, respectively. Here, $\mathbf{h}(n)$ is the $M \times 1$ column vector of filter coefficients at iteration n. The desired signal assumed to conform to the following linear data model

$$d(n) = \mathbf{x}^T(n)\mathbf{h}_t + v(n), \qquad (1)$$

where $\mathbf{x}(n) = [x(n), x(n-1), \ldots, x(n-M+1)]^T$ is the input signal regressors, $v(n)$ is the measurement noise, assumed to be zero mean, white, Gaussian, and independent of $\mathbf{x}(n)$, and $\mathbf{h}_t$ is the unknown filter vector.
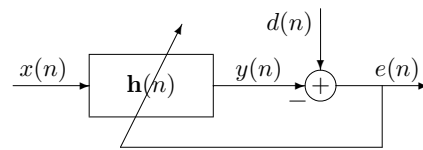

Fig. 1. A typical adaptive filter setup.

It is well known that the NLMS algorithm can be derived from the solution of the following optimization problem:

$$\min_{\mathbf{h}(n+1)} \|\mathbf{h}(n+1) - \mathbf{h}(n)\|^2 \qquad (2)$$

subject to

$$d(n) = \mathbf{x}^T(n)\mathbf{h}(n+1). \qquad (3)$$

Using the method of Lagrange multipliers to solve this optimization problem leads to the following recursion

$$\mathbf{h}(n+1) = \mathbf{h}(n) + \frac{\mu}{\|\mathbf{x}(n)\|^2}\mathbf{x}(n)e(n), \qquad (4)$$

where $e(n) = d(n) - \mathbf{x}^T(n)\mathbf{h}(n)$, and μ is the step-size that determines the convergence speed and excess MSE (EMSE).

III. FAMILY OF AFFINE PROJECTION ALGORITHMS (APA)

Now define the $M \times K$ matrix of the input signal as

$$\mathbf{X}(n) = [\mathbf{x}(n), \mathbf{x}(n-D), \dots, \mathbf{x}(n-(K-1)D)], \qquad (5)$$

and the $K \times 1$ vector of desired signal as

$$\mathbf{d}(n) = [d(n), d(n-D), \dots, d(n-(K-1)D)]^T, \qquad (6)$$

where K is positive integer (usually, but not necessarily $K \leq M$), and D is the positive integer parameter ($D \geq 1$) that can increase the separation and consequently reduce the correlation among the regressors in $\mathbf{X}(n)$.

The family of APA can be established by minimizing (2) but subject to $\mathbf{d}(n) = \mathbf{X}^T(n)\mathbf{h}(n)$. Again by using the method of Lagrange multipliers, the filter vector update equation for the family of APA is given by

$$\mathbf{h}(n+1) = \mathbf{h}(n) + \mu\mathbf{X}(n)\mathbf{W}(n)\mathbf{e}(n), \qquad (7)$$

where $\mathbf{e}(n)$ is the output error vector which is defined as

$$\mathbf{e}(n) = \mathbf{d}(n) - \mathbf{X}^T(n)\mathbf{h}(n), \qquad (8)$$

and the matrix $\mathbf{W}(n)$ is obtained from Table I[1]. The NLMS, ϵ-NLMS, standard version of the APA, the binormalized data-reusing LMS (BNDR-LMS) [11], the regularized APA (R-APA) [12], the NLMS with orthogonal correction factors (NLMS-OCF) [13] are established form (11). From (11), the partial rank algorithm (PRA) [14] can also be established when the adaptation of the filter coefficients is performed only once every K iterations.

TABLE I
FAMILY OF AFFINE PROJECTION ADAPTIVE FILTER ALGORITHMS

Algorithm	K	D	$\mathbf{W}(n)$
NLMS	$K = 1$	$D = 1$	$\frac{1}{\|\mathbf{x}(n)\|^2}$
ϵ-NLMS	$K = 1$	$D = 1$	$\frac{1}{\epsilon + \|\mathbf{x}(n)\|^2}$
APA	$K \leq M$	$D = 1$	$(\mathbf{X}^T(n)\mathbf{X}(n))^{-1}$
BNDR-LMS	$K = 2$	$D = 1$	$(\mathbf{X}^T(n)\mathbf{X}(n))^{-1}$
R-APA	$K \leq M$	$D = 1$	$(\epsilon\mathbf{I} + \mathbf{X}^T(n)\mathbf{X}(n))^{-1}$
NLMS-OCF	$K \leq M$	$D \geq 1$	$(\mathbf{X}^T(n)\mathbf{X}(n))^{-1}$

[1]In Table I, ϵ is the regularization parameter, and $\mathbf{I}$ is the identity matrix.

IV. THE FAMILY OF SELECTIVE REGRESSOR APA (SR-APA)

In [10], another novel affine projection algorithm with selective regressors (SR) which was called (SR-APA) was presented. In this section we extend this approach to present the family of SR-APA. The SR-APA, minimizes (4) subject to

$$\mathbf{d}_G(n) = \mathbf{X}_G^T(n)\mathbf{h}(n), \qquad (9)$$

where $G = \{i_1, i_2, \dots, i_P\}$ denote the P subset (subset with P member) of the set $\{0, 1, \dots, K-1\}$,

$$\mathbf{X}_G(n) = [\mathbf{x}(n - i_1 D), \mathbf{x}(n - i_2 D), \dots, \mathbf{x}(n - i_P D)], \qquad (10)$$

is the $M \times P$ matrix of the input signal and

$$\mathbf{d}_G(n) = [d(n - i_1 D), d(n - i_2 D), \dots, d(n - i_P D)]^T, \qquad (11)$$

is the $P \times 1$ vector of the desired signal. Using the method of Lagrange multipliers to solve this optimization problem leads to the following update equation

$$\mathbf{h}(n+1) = \mathbf{h}(n) + \mu\mathbf{X}_G(n)(\mathbf{X}_G^T(n)\mathbf{X}_G(n))^{-1}\mathbf{e}_G(n), \qquad (12)$$

where

$$\mathbf{e}_G(n) = \mathbf{d}_G(n) - \mathbf{X}_G^T(n)\mathbf{h}(n). \qquad (13)$$

The indices of G are obtained by the following procedure:
1) Compute the following values for $0 \leq i \leq K - 1$

$$\frac{e^2(n - iD)}{\|\mathbf{x}(n - iD)\|^2}, \qquad (14)$$

where $\mathbf{e}(n) = [e(n), e(n-D), \dots, e(n-(K-1)D)]^T$.
2) The indices of G are correspond to P largest values of (12).

Setting $D = 1$ leads to SR-APA presented in [10]. Furthermore, from (10), the family of SR-APA such as SR-BNDR-LMS, SR-NLMS-OCF adaptive algorithms will be established.

Equation (12) can also be represented as

$$\mathbf{h}(n+1) = \mathbf{h}(n) + \mu\mathbf{X}(n)\mathbf{B}(n)$$
$$\times(\mathbf{B}^T(n)\mathbf{X}^T(n)\mathbf{X}(n)\mathbf{B}(n))^{-1}\mathbf{B}^T(n)\mathbf{e}(n), \qquad (15)$$

where $\mathbf{B}(n) = [\mathbf{1}_{i_1}, \mathbf{1}_{i_2}, \dots, \mathbf{1}_{i_P}]$ is the $K \times P$ matrix and $\mathbf{1}_{i_p} = [0, \dots, 0, 1, 0, \dots, 0]^T$ is the $K \times 1$ vector with the element 1 in the position i_p.

Based on (15), the general filter update equation for the family of AP with SR is introduces as

$$\mathbf{h}(n+1) = \mathbf{h}(n) + \mu\mathbf{X}(n)\mathbf{Z}(n)\mathbf{e}(n). \qquad (16)$$

where $\mathbf{Z}(n)$ matrix is obtained from Table II.

V. COMPUTATIONAL COMPLEXITY

The computational complexity of the APA, and SR-APA has been presented in Table III. The computational complexity of the APA is from [15]. The computational complexity of the SR-APA is from [10]. Also, the computational complexity of SR-PRA is reduced by the factor of K, because the adaptation of the filter coefficients is performed only once every K iterations.

TABLE II
FAMILY OF SR AFFINE PROJECTION ALGORITHMS

Algorithm	K	D	$\mathbf{Z}(n)$
SR-APA	$K \leq M$	$D = 1$	$\mathbf{B}(n)(\mathbf{B}^T(n)\mathbf{X}^T(n)\mathbf{X}(n)\mathbf{B}(n))^{-1}\mathbf{B}^T(n)$
SR-BNDR-LMS	$K = 2$	$D = 1$	$\mathbf{B}(n)(\mathbf{B}^T(n)\mathbf{X}^T(n)\mathbf{X}(n)\mathbf{B}(n))^{-1}\mathbf{B}^T(n)$
SR-R-APA	$K \leq M$	$D = 1$	$\mathbf{B}(n)(\epsilon\mathbf{I} + \mathbf{B}^T(n)\mathbf{X}^T(n)\mathbf{X}(n)\mathbf{B}(n))^{-1}\mathbf{B}^T(n)$
SR-NLMS-OCF	$K \leq M$	$D \geq 1$	$\mathbf{B}(n)(\mathbf{B}^T(n)\mathbf{X}^T(n)\mathbf{X}(n)\mathbf{B}(n))^{-1}\mathbf{B}^T(n)$

TABLE III
THE COMPUTATIONAL COMPLEXITY OF THE APA, AND SR-APA

Algorithm	Multiplications	Divisions	Additional Multiplications	Comparisons
APA	$(K^2 + 2K)M + K^3 + K^2$	$-$	-	-
SR-APA	$(P^2 + 2P)M + P^3 + P^2$	K	$(K - P)M + K + 1$	$K log_2 P + O(K)$

VI. SIMULATION RESULTS

We demonstrate the performance of the proposed algorithms by several computer simulations in an acoustic echo cancellation scenario. The unknown system is from the exact exact impulse responses of the car echo path with 256 taps[2]. The input signal $x(n)$ is a first order autoregressive (AR) signal generated by

$$x(n) = \rho x(n - 1) + w(n) \qquad (17)$$

where $w(n)$ is either a zero mean white Gaussian signal. The value of ρ is set to 0.9, generating a highly colored Gaussian signal. The measurement noise $v(n)$ with $\sigma_v^2 = 10^{-3}$ is added to the noise-free desired signal $d(n) = \mathbf{h}_t^T\mathbf{x}(n)$, where $\mathbf{h}_t^T$ is the unknown filter vector. The adaptive filter and the unknown channel are assumed to have the same number of taps. In all simulations, the simulated learning curves are obtained by ensemble averaging over 200 independent trials.

To make the comparison fair, the step-sizes of the family of SR-APA were chosen to get approximately the same steady-state MSE. Fig. 7 shows the impulse responses of the car echo path that should be identified. The input signal is the same as in previous simulations and the order of the system shown in Fig. 7 was 256. Fig. 8 shows the simulated learning curves of SR-APA with $K = 4$ and different values for $P = 2, 3, 4$. As we can see, by increasing the parameter P, the convergence speed increases. Fig. 9 shows the results for SR-PRA. The same performance as Fig. 8 with lower computational complexity compare with SR-APA can be seen. The learning curves of SR-NLMS-OCF with $K = 4$, $D = 4$, and different values for P have been presented in Fig. 10. Fig. 11 compares the performance of the SR-APA and SR-PRA. As we can see the convergence speed of SR-PRA will be close to SR-APA with $P = 3$. Furthermore, the computational complexity of SR-PRA is less than SR-APA, especially in this application.

VII. SUMMARY AND CONCLUSIONS

In this paper we presented the family of SR affine projection algorithms. The SR-APA, the SR regularized APA (SR-R-

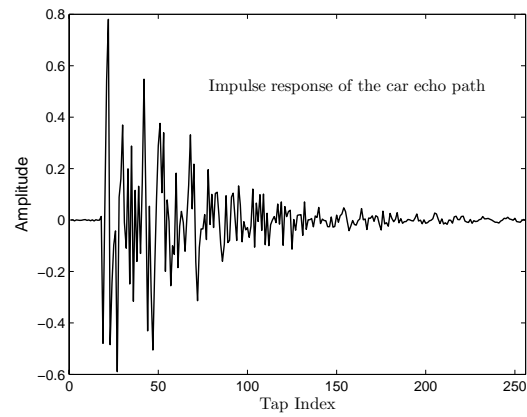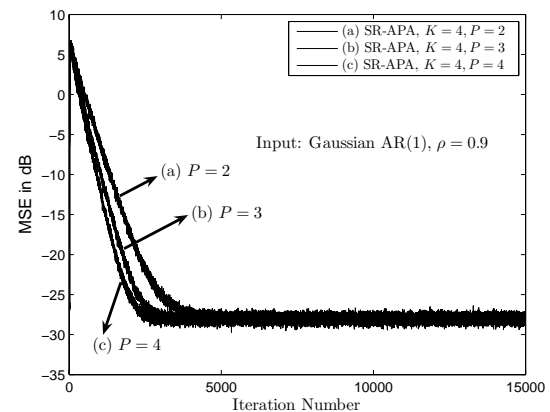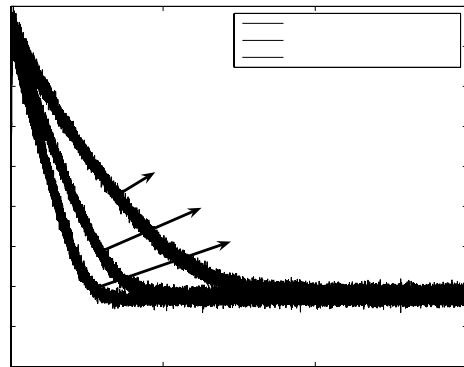[2]The impulse response of the car echo path is from [8].

Fig. 2. Impulse response of the car echo path

Fig. 3. Simulated learning Curves of SR-APA with $K = 4$, and $P = 2, 3, 4$ (input: Gaussian AR(1), $\rho = 0.9$).

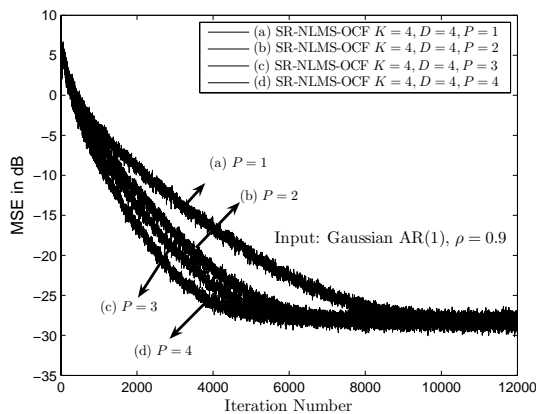APA), the SR partial rank algorithm (SR-PRA), the SR bi-normalized data reusing least mean squares (SR-BNDR-LMS), and the SR normalized LMS with orthogonal correction factors (SR-NLMS-OCF) algorithms were established. We demonstrated the performance of the presented algorithms through several simulations in acoustic echo cancellation application.

Fig. 4. Simulated learning Curves of SR-PRA with $K = 4$, and $P = 2, 3, 4$ (input: Gaussian AR(1), $\rho = 0.9$).

Fig. 5. Simulated learning Curves of SR-NLMS-OCF with $K = 4$, $D = 4$, and $P = 1, 2, 3, 4$ (input: Gaussian AR(1), $\rho = 0.9$).
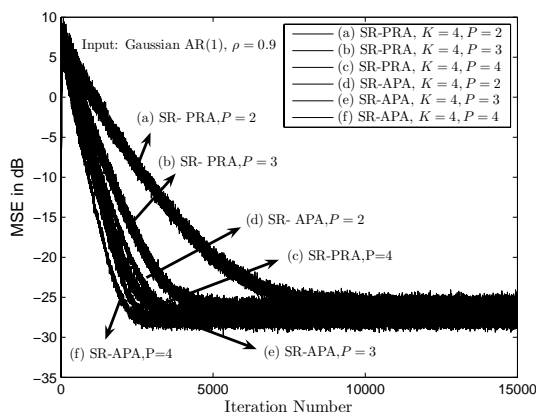

Fig. 6. Simulated learning Curves of SR-APA and SR-PRA with $K = 4$, and $P = 2, 3, 4$ (input: Gaussian AR(1), $\rho = 0.9$).

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Mohammad Shams Esfand Abadi was born in Tehran, Iran, on September 18, 1978. He received the B.S. degree in Electrical Engineering from Mazandaran University, Mazandaran, Iran and the M.S. degree in Electrical Engineering from Tarbiat Modarres University, Tehran, Iran in 2000 and 2002, respectively, and the Ph.D. degree in Biomedical Engineering from Tarbiat Modarres University, Tehran, Iran in 2007. Since 2004 he has been with the Department of Electrical Engineering, Shahid Rajaee University, Tehran, Iran. During the fall of 2003, spring of 2005 and again in the spring of 2007, he has a visiting scholar with the Signal Processing Group at the University of Stavanger, Norway. His research interests include digital filter theory and adaptive signal processing algorithms.