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Certain Subordination Results For A Class Of Analytic Functions Defined By The Generalized Integral Operator

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Abstract**—We obtain several interesting subordination results for a class of analytic functions defined by using a generalized integral operator.**

Keywords**—Analytic functions, Hadamard product, Subordinating factor sequence**

I. Introduction

Let $\mathcal{A}$ be the class of analytic functions

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \tag{1}$$

defined on the open unit disc $\mathcal{U} = \{z : | z | < 1\}$. Let S denote the subclass of $\mathcal{A}$ consisting of functions that are univalent in $\mathcal{U}$. The Hadamard product of two functions $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ and $g(z) = z + \sum_{n=2}^{\infty} b_n z^n$ in $\mathcal{A}$ is given by

$$(f * g)(z) = z + \sum_{n=2}^{\infty} a_n\, b_n\, z^n \tag{2}$$

For complex parameters $\alpha_1, \ldots, \alpha_q$ and $\beta_1, \ldots, \beta_s$ $(\beta_j \in \mathbb{C} \setminus \mathbb{Z}_0^-; \mathbb{Z}_0^- = 0, -1, -2, \ldots; j = 1, \ldots, s)$, we define the generalized hypergeometric function ${}_qF_s(\alpha_1, \ldots, \alpha_q; \beta_1, \ldots, \beta_s; z)$ by

$$\begin{aligned} &{}_qF_s(\alpha_1, \alpha_2, \ldots, \alpha_q; \beta_1, \beta_2, \ldots, \beta_s; z) \\ &= \sum_{n=0}^{\infty} \frac{(\alpha_1)_n \ldots (\alpha_q)_n}{(\beta_1)_n \ldots (\beta_s)_n} \frac{z^n}{n!} \end{aligned}$$

$$(q \leq s+1;\ q, s \in \mathcal{N}_0 = \mathcal{N} \cup \{0\};\ z \in \mathcal{U}),$$

where $\mathcal{N}$ denotes the set of positive integers and $(x)_k$ is the Pochhammer symbol defined,

$$(x)_k = \begin{cases} 1 & if\, k = 0 \\ x(x+1)\ \ldots\ (x+k-1) & if\, k \in N = \{1, 2, , \ldots\}. \end{cases}$$

Corresponding to a function $h(\alpha_1, \ldots, \alpha_q; \beta_1, \ldots, \beta_s; z)$ defined by

$$\begin{aligned} &h(\alpha_1, \ldots, \alpha_q; \beta_1, \ldots, \beta_s; z) \\ &= z\, {}_qF_s(\alpha_1, \alpha_2, \ldots, \alpha_q; \beta_1, \beta_2, \ldots, \beta_s; z) \end{aligned} \tag{3}$$

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Dziok and Srivastava considered a linear operator

$$H(\alpha_1, \ldots, \alpha_q; \beta_1, \ldots, \beta_s; z) \quad : \quad \mathcal{A} \longrightarrow \mathcal{A}$$

defined by Hadamard product (or convolution):

$$\begin{aligned} &H(\alpha_1, \ldots, \alpha_q; \beta_1, \ldots, \beta_s; z) f(z) \\ &= h(\alpha_1, \ldots, \alpha_q; \beta_1, \ldots, \beta_s; z) * f(z) \end{aligned}$$

The linear operator $H(\alpha_1, \ldots, \alpha_q; \beta_1, \ldots, \beta_s; z) f(z)$ includes (as its special cases) various other linear operators which were introduced and studied by Hohlov [9], Carlson and Shaffer [2], Ruscheweyh [13] and so on.

Corresponding to the function $h(\alpha_1, \ldots, \alpha_q; \beta_1, \ldots, \beta_s; z)$ defined by (3), we define a function $h_{\mu,\,p}(\alpha_1, \ldots, \alpha_q; \beta_1, \ldots, \beta_s; z)$ given by

$$\begin{aligned} &z + \sum_{n=2}^{\infty} \frac{(n+\mu)^p}{(\mu+1)^p} z^n * h_{\mu,\,p}(\alpha_1, \ldots, \alpha_q; \beta_1, \ldots, \beta_s; z) \\ &= h(\alpha_1, \ldots, \alpha_q; \beta_1, \ldots, \beta_s; z) \end{aligned} \tag{4}$$

Analogous to $H(\alpha_1, \ldots, \alpha_q; \beta_1, \ldots, \beta_s; z)$, Selvaraj et al. [14] considered a linear operator
$\mathcal{G}_\mu^p(\alpha_1, \ldots, \alpha_q; \beta_1, \ldots, \beta_s; z)$ on $\mathcal{A}$ as follows:

$$\begin{aligned} &\mathcal{G}_\mu^p(\alpha_1, \ldots, \alpha_q; \beta_1, \ldots, \beta_s; z) f(z) \\ &= h_{\mu,\,p}(\alpha_1, \ldots, \alpha_q; \beta_1, \ldots, \beta_s; z) * f(z) \end{aligned} \tag{5}$$

$(\alpha_i, \beta_j \in \mathbb{C} \setminus \mathbb{Z}_0^-; i = 1, \ldots, q; j = 1, \ldots, s; \mu \neq -1; p \in \mathcal{N}_0 = \{0, 1, 3, \ldots\})$

For convenience, we write

$$\mathcal{G}_{\mu,\,q,\,s}^p(\alpha_1) = \mathcal{G}_\mu^p(\alpha_1, \ldots, \alpha_q; \beta_1, \ldots, \beta_s; z) \tag{6}$$

It can be easily verified from the definition of (5) that

$$\begin{aligned} &z(\mathcal{G}_{\mu,\,q,\,s}^{p+1}(\alpha_1) f(z))' \\ &= (\mu+1)\mathcal{G}_{\mu,\,q,\,s}^p(\alpha_1) f(z) - \mu\, \mathcal{G}_{\mu,\,q,\,s}^{p+1}(\alpha_1) f(z) \end{aligned} \tag{7}$$

and

$$\begin{aligned} &z(\mathcal{G}_{\mu,\,q,\,s}^p(\alpha_1) f(z))' \\ &= \alpha_1 \mathcal{G}_{\mu,\,q,\,s}^p(\alpha_1+1) f(z) - (\alpha_1 - 1)\mathcal{G}_{\mu,\,q,\,s}^p(\alpha_1) f(z) \end{aligned} \tag{8}$$

we now define the most generalized subclass of $\mathcal{A}$ by using the operator (5).

Definition 1.1. For any non-zero complex number λ, $0 \le \gamma < 1$ and $k \ge 0$, a function $f \in \mathcal{A}$ is said to be in $\mathcal{I}(\alpha_1, q, s: k, \gamma, \lambda)$ if and only if it satisfies the condition

$$\mathfrak{Re}\left\{\frac{\lambda \mathcal{G}^p_{\mu,q,s}(\alpha_1+1)f(z)}{\mathcal{G}^p_{\mu,q,s}(\alpha_1)f(z)} - (\lambda-1)\right\} > k\left|\frac{\lambda \mathcal{G}^p_{\mu,q,s}(\alpha_1+1)f(z)}{\mathcal{G}^p_{\mu,q,s}(\alpha_1)f(z)} - \lambda\right| + \gamma \qquad (9)$$

The family $\mathcal{I}(\alpha_1, q, s: k, \gamma, \lambda)$ is of special interest for it contains many well-known as well as many new classes of analytic univalent functions. For $\lambda = \alpha_1$ and for appropriate choices of the parameters $\mathcal{I}(\alpha_1, q, s: k, \gamma, \lambda)$ reduces to $\mathcal{L}(a, c; \alpha, \beta)$[5]. We note that the family $\mathcal{S}^*(\alpha)$ of starlike function of order $\alpha\,(0 \le \alpha < 1)$[3], [6], the family $\mathcal{C}(\alpha)$ of convex function of order $\alpha\,(0 \le \alpha < 1)$[3], [6], $k - UCV(\alpha)$[1], $k - UST(\alpha)$ and many other well known subclasses of $\mathcal{S}$ (see also the work of Kanas and Srivastava [10], Goodman [7], [8] and Rønning [11], [12]) are the special cases of $\mathcal{I}(\alpha_1, q, s: k, \gamma, \lambda)$.

We now state the definitions and lemmas which we need in the sequel.

Definition 1.2. An analytic function g is said to be subordinate to an analytic function f if $g(z) = f(w(z))$, $z \in \mathcal{U}$ for some analytic function w with $|\, w(z) \,| \le |\, z \,|$ and we write $f(z) \prec g(z)$

Definition 1.3. A sequence $\{b_n\}_{n=1}^{\infty}$ of complex numbers is called a subordinating factor sequence if, whenever $f(z)$ is analytic, univalent and convex in $\mathcal{U}$, we have the subordination given by

$$\sum_{n=1}^{\infty} b_n\, a_n z^n \prec f(z) \qquad (z \in \mathcal{U},\, a_1 = 1). \qquad (10)$$

Lemma 1.1[16]The sequence $\{b_n\}_{n=1}^{\infty}$ is a subordinating factor sequence if and only if

$$Re\left\{1 + 2\sum_{n=1}^{\infty} b_n z^n\right\} > 0 \qquad (z \in (U). \qquad (11)$$

Lemma 1.2 If

$$\sum_{n=2}^{\infty}\Big(\lambda(1+k)(\alpha_1+1)_{n-1} + \big[(1-\gamma) - \lambda(1+k)\big](\alpha_1)_{n-1}\Big)\Psi(n)\ |\, a_n \,| \le (1-\gamma) \qquad (12)$$

where $\Psi(n) = \dfrac{(\mu+1)^p(\alpha_2)_{n-1}\ldots(\alpha_q)_{n-1}}{(n+\mu)^p(\beta_1)_n\ \ldots\ (\beta_s)_n}$,
then $f(z) \in \mathcal{I}(\alpha_1, q, s: k, \gamma, \lambda)$

Proof. It suffices to show that

$$k\left|\frac{\lambda \mathcal{G}^p_{\mu,q,s}(\alpha_1+1)f(z)}{\mathcal{G}^p_{\mu,q,s}(\alpha_1)f(z)} - \lambda\right| - \mathfrak{Re}\left\{\frac{\lambda \mathcal{G}^p_{\mu,q,s}(\alpha_1+1)f(z)}{\mathcal{G}^p_{\mu,q,s}(\alpha_1)f(z)} - \lambda\right\} \le 1-\gamma.$$

We have

$$k\left|\frac{\lambda \mathcal{G}^p_{\mu,q,s}(\alpha_1+1)f(z)}{\mathcal{G}^p_{\mu,q,s}(\alpha_1)f(z)} - \lambda\right| - \mathfrak{Re}\left\{\frac{\lambda \mathcal{G}^p_{\mu,q,s}(\alpha_1+1)f(z)}{\mathcal{G}^p_{\mu,q,s}(\alpha_1)f(z)} - \lambda\right\}$$

$$\le\ (1+k)\left|\frac{\lambda \mathcal{G}^p_{\mu,q,s}(\alpha_1+1)f(z)}{\mathcal{G}^p_{\mu,q,s}(\alpha_1)f(z)} - \lambda\right|$$

$$\le \frac{(1+k)\sum_{n=2}^{\infty}\lambda(\alpha_1+1)_{(n-1)}\Psi(n)\ |\, a_n \,||\, z \,|^{n-1}}{1-\sum_{n=2}^{\infty}(\alpha_1)_{n-1}\Psi(n)\ |\, a_n \,||\, z \,|^{n-1}} - \frac{\lambda(\alpha_1)_{(n-1)}\,\Psi(n)\ |\, a_n \,||\, z \,|^{n-1}}{1-\sum_{n=2}^{\infty}(\alpha_1)_{n-1}\Psi(n)\ |\, a_n \,||\, z \,|^{n-1}}$$

$$\le \frac{(1+k)\sum_{n=2}^{\infty}\lambda(\alpha_1+1)_{(n-1)}\Psi(n)\ |\, a_n \,||\, z \,|^{n-1}}{1-\sum_{n=2}^{\infty}(\alpha_1)_{n-1}\Psi(n)\ |\, a_n \,||\, z \,|^{n-1}} - \frac{\lambda(\alpha_1)_{(n-1)}\,\Psi(n)\ |\, a_n \,||\, z \,|^{n-1}}{1-\sum_{n=2}^{\infty}(\alpha_1)_{n-1}\Psi(n)\ |\, a_n \,||\, z \,|^{n-1}}$$

The last expression is bounded by $1-\gamma$ if

$$\sum_{n=2}^{\infty}\Big(\lambda(1+k)(\alpha_1+1)_{n-1} + \big[(1-\gamma) - \lambda(1+k)\big](\alpha_1)_{n-1}\Big)\Psi(n)\ |\, a_n \,| \le (1-\gamma)$$

which completes the proof of the Lemma.

For convenience we shall henceforth denote

$$\xi_n(\alpha_1, q, s: k, \gamma, \lambda) = \Big(\lambda(1+k)(\alpha_1+1)_{n-1} + \big[(1-\gamma) - \lambda(1+k)\big](\alpha_1)_{n-1}\Big)\Psi(n) \qquad (13)$$

Let $\mathcal{I}^*(\alpha_1, q, s: k, \gamma, \lambda)$ denote the class of functions $f(z) \in \mathcal{A}$ whose coefficients satisfy the conditions (12). We note that $\mathcal{I}^*(\alpha_1, q, s: k, \gamma, \lambda) \subseteq \mathcal{I}(\alpha_1, q, s: k, \gamma, \lambda)$.

II. Main Results

we begin with the following:

Theorem 2.1 Let the function $f(z)$ defined by (1) be in the class $\mathcal{I}^*(\alpha_1, q, s: k, \gamma, \lambda)$ where $0 \le \gamma < 1$; $k \ge 0$. Also let $\mathcal{C}$ denote the familiar class of functions $f(z) \in \mathcal{A}$ which are univalent and convex in $\mathcal{U}$. Then

$$\frac{\xi_2(\alpha_1, q, s: k, \gamma, \lambda)}{2\big[1-\gamma+\xi_2(\alpha_1, q, s: k, \gamma, \lambda)\big]}(f * g)(z) \prec g(z) \qquad (14)$$

and $(z \in \mathcal{U};\, g \in \mathcal{C})$

$$\mathfrak{Re}(f(z)) > -\frac{1-\gamma+\xi_2(\alpha_1, q, s: k, \gamma, \lambda)}{\xi_2(\alpha_1, q, s: k, \gamma, \lambda)} \qquad (15)$$

The constant $\dfrac{\xi_2(\alpha_1, q, s: k, \gamma, \lambda)}{2\big[1-\gamma+\xi_2(\alpha_1, q, s: k, \gamma, \lambda)\big]}$ is the best estimate.

Proof. Let $f(z) \in \mathcal{I}^*(\alpha_1, q, s: k, \gamma, \lambda)$ and let $g(z) = z + \sum_{n=2}^{\infty} b_n z^n \in \mathcal{C}$. Then

$$\frac{\xi_2(\alpha_1, q, s: k, \gamma, \lambda)}{2\big[1-\gamma+\xi_2(\alpha_1, q, s: k, \gamma, \lambda)\big]}(f * g)(z) = \frac{\xi_2(\alpha_1, q, s: k, \gamma, \lambda)}{2\big[1-\gamma+\xi_2(\alpha_1, q, s: k, \gamma, \lambda)\big]}\left(z + \sum_{n=2}^{\infty} a_n b_n z^n\right).$$

Thus, by Definition (1.3), the assertion of the theorem will hold if the sequence

$$\left\{\frac{\xi_2(\alpha_1,q,s:\,k,\gamma,\lambda)}{2\big[1-\gamma+\xi_2(\alpha_1,q,s:\,k,\gamma,\lambda)\big]}a_n\right\}_{n=1}^{\infty}$$

is a subordinating factor sequence, with $a_1=1$. In view of Lemma (1.1), this will be true if and only if ($z\in\mathcal{U}$)

$$\mathfrak{Re}\left\{1+2\sum_{n=1}^{\infty}\frac{\xi_2(\alpha_1,q,s:\,k,\gamma,\lambda)}{2\big[1-\gamma+\xi_2(\alpha_1,q,s:\,k,\gamma,\lambda)\big]}a_nz^n\right\}>0. \tag{16}$$

Now

$$\mathfrak{Re}\left\{1+\frac{\xi_2(\alpha_1,q,s:\,k,\gamma,\lambda)}{1-\gamma+\xi_2(\alpha_1,q,s:\,k,\gamma,\lambda)}\sum_{n=1}^{\infty}a_n\,z^n\right\}$$

$$=\mathfrak{Re}\left\{1+\frac{\xi_2(\alpha_1,q,s:\,k,\gamma,\lambda)}{1-\gamma+\xi_2(\alpha_1,q,s:\,k,\gamma,\lambda)}z+\right.$$

$$\left.\frac{1}{1-\gamma+\xi_2(\alpha_1,q,s:\,k,\gamma,\lambda)}\sum_{n=2}^{\infty}\xi_2(\alpha_1,q,s:\,k,\gamma,\lambda)a_nz^n\right\}$$

$$\geq 1-\left\{\frac{\xi_2(\alpha_1,q,s:\,k,\gamma,\lambda)}{1-\gamma+\xi_2(\alpha_1,q,s:\,k,\gamma,\lambda)}r-\right.$$

$$\left.\frac{1}{1-\gamma+\xi_2(\alpha_1,q,s:\,k,\gamma,\lambda)}\sum_{n=2}^{\infty}\xi_2(\alpha_1,q,s:\,k,\gamma,\lambda)a_nr^n\right\}.$$

Since $\xi_n(\alpha_1,q,s:\,k,\gamma,\lambda)$ is an increasing function of n ($n\geq 2$)

$$1-\left\{\frac{\xi_2(\alpha_1,q,s:\,k,\gamma,\lambda)}{1-\gamma+\xi_2(\alpha_1,q,s:\,k,\gamma,\lambda)}r-\right.$$

$$\left.\frac{1}{1-\gamma+\xi_2(\alpha_1,q,s:\,k,\gamma,\lambda)}\sum_{n=2}^{\infty}\xi_2(\alpha_1,q,s:\,k,\gamma,\lambda)a_nr^n\right\}$$

$$>1-\frac{\xi_2(\alpha_1,q,s:\,k,\gamma,\lambda)}{1-\gamma+\xi_2(\alpha_1,q,s:\,k,\gamma,\lambda)}r$$

$$-\frac{1-\gamma}{1-\gamma+\xi_2(\alpha_1,q,s:\,k,\gamma,\lambda)}r>0.$$

Thus (16) holds true in $\mathcal{U}$. This proves the inequality (14). The inequality (15) follows by taking the convex function $g(z)=\frac{z}{1-z}=z+\sum_{n=2}^{\infty}z^n$ in (14). To prove the sharpness of the constant $\frac{\xi_2(\alpha_1,q,s:\,k,\gamma,\lambda)}{2\big[1-\gamma+\xi_2(\alpha_1,q,s:\,k,\gamma,\lambda)\big]}$, we consider $f_0(z)\in\mathcal{I}^*(\alpha_1,q,s:\,k,\gamma,\lambda)$ given by

$$f_0(z)=z-\frac{1-\gamma}{\xi_2(\alpha_1,q,s:\,k,\gamma,\lambda)}z^2 \qquad (-1\leq\gamma<1).$$

Thus from (14), we have

$$\frac{\xi_2(\alpha_1,q,s:\,k,\gamma,\lambda)}{2\big[1-\gamma+\xi_2(\alpha_1,q,s:\,k,\gamma,\lambda)\big]}f_0(z)\prec\frac{z}{1-z}. \tag{17}$$

It can be easily verified that

$$\min\left\{Re\left(\frac{\xi_2(\alpha_1,q,s:\,k,\gamma,\lambda)}{2\big[1-\gamma+\xi_2(\alpha_1,q,s:\,k,\gamma,\lambda)\big]}f_0(z)\right)\right\}=-\frac{1}{2}$$

This shows that the constant $\frac{\xi_2(\alpha_1,q,s:\,k,\gamma,\lambda)}{2\big[1-\gamma+\xi_2(\alpha_1,q,s:\,k,\gamma,\lambda)\big]}$ is best possible. This completes the proof of the Theorem.

Let $\lambda=\alpha_1$ and for the appropriate choice of μ,p,q,s, we have the following:

Corollary 2.1.[4] Let $f(z)$ be defined by (1) be in the class $\mathcal{L}^*(a,c,\,k,\gamma)$ and satisfy the condition

$$\sum_{n=2}^{\infty}\left(\frac{(1+k)(\alpha_1)_n}{(\beta_1)_{n-1}}\right.$$

$$\left.+\frac{\big[(1-\gamma)-\alpha_1(1+k)\big](\alpha_1)_{n-1}}{(\beta_1)_{n-1}}\right)\mid a_n\mid\leq(1-\gamma),$$

then for $z\in\mathcal{U}$; $g\in\mathcal{C}$,

$$\frac{\sigma_2(\alpha_1,\beta_1\,k,\gamma)}{2\big[1-\gamma+\sigma_2(\alpha_1,\beta_1\,k,\gamma)\big]}(f*g)(z)\prec g(z), \tag{18}$$

and

$$\mathfrak{Re}(f(z))>-\frac{1-\gamma+\sigma_2(\alpha_1,\beta_1\,k,\gamma)}{\sigma_2(\alpha_1,\beta_1\,k,\gamma)} \tag{19}$$

where

$\sigma_2(\alpha_1,\beta_1\,k,\gamma)=\frac{(1+k)(\alpha_1)_n+\big[(1-\gamma)-\alpha_1(1+k)\big](\alpha_1)_{n-1}}{(\beta_1)_{n-1}}$.

The constant $\frac{\sigma_2(\alpha_1,\beta_1\,k,\gamma)}{2\big[1-\gamma+\sigma_2(\alpha_1,\beta_1\,k,\gamma)\big]}$ is the best estimate.

Corollary 2.2. Let the function $f(z)$ be defined by (1) be starlike of order γ in $\mathcal{U}$ and satisfy the condition $\sum_{n=2}^{\infty}(n-\alpha)\mid a_n\mid\leq 1-\alpha$, then

$$\frac{2-\alpha}{6-4\alpha}(f*g)(z)\prec g(z) \qquad (z\in\mathcal{U};\,g\in\mathcal{C}), \tag{20}$$

and

$$Re(f(z))>-\frac{3-2\alpha}{2-\alpha}, \qquad (z\in\mathcal{U}). \tag{21}$$

Corollary 2.3.[15] Let the function $f(z)$ be defined by (1) be starlike in $\mathcal{U}$ and satisfy the condition $\sum_{n=2}^{\infty}n\mid a_n\mid\leq 1$, then

$$\frac{1}{3}(f*g)(z)\prec g(z) \qquad (z\in\mathcal{U};\,g\in\mathcal{C}) \tag{22}$$

and

$$Re(f(z))>-\frac{3}{2}, \qquad (z\in\mathcal{U}). \tag{23}$$

Corollary 2.4. Let the function $f(z)$ be defined by (1) be convex in $\mathcal{U}$ and satisfy the condition $\sum_{n=2}^{\infty}n^2\mid a_n\mid\leq 1$, then

$$\frac{2}{5}(f*g)(z)\prec g(z) \qquad (z\in\mathcal{U};\,g\in\mathcal{C}) \tag{24}$$

and

$$Re(f(z))>-\frac{5}{4}, \qquad (z\in\mathcal{U}). \tag{25}$$

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