

# ML Detection with Symbol Estimation for Nonlinear Distortion of OFDM Signal

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**Abstract**— In this paper, a new technique of signal detection has been proposed for detecting the orthogonal frequency-division multiplexing (OFDM) signal in the presence of nonlinear distortion. There are several advantages of OFDM communications system. However, one of the existing problems is remain considered as the nonlinear distortion generated by high-power-amplifier at the transmitter end due to the large dynamic range of an OFDM signal. The proposed method is the maximum likelihood detection with the symbol estimation. When the training data are available, the neural network has been used to learn the characteristic of received signal and to estimate the new positions of the transmitted symbol which are provided to the maximum likelihood detector. Resulting in the system performance, the nonlinear distortions of a traveling wave tube amplifier with OFDM signal are considered in this paper. Simulation results of the bit-error-rate performance are obtained with 16-QAM OFDM systems

**Keywords**— OFDM, TWTA, nonlinear distortion, detection.

## I. INTRODUCTION

An OFDM signal is the sum of sinusoidal wave and transmits as multicarrier so that the peak power of OFDM signal increases in proportion to the number of subcarriers. As a result, multicarrier systems are more sensitive than the single-carrier systems to the presence of nonlinearities when such a signal is input to a nonlinear amplifier. Due to the large dynamic range of modulated signal, the nonlinear distortion at the power amplifier of transmitter causes interference both inside (intermodulation between subcarrier) and outside (spectral-spreading of the OFDM signal) the signal bandwidth. The out-of-band component affects adjacent frequency band whereas the in-band component determines a degradation of the system bit-error rate (BER). This paper focuses on the effects of in-band interference at the receiver in which a new algorithm of signal detection has been proposed.

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All most the radio system employ high power amplifier (HPA) such as a traveling wave tube amplifier (TWTA) in the transmitter. To achieve the maximum output power efficiency, the HPA is usually operated near the saturation region and this introduces nonlinear distortion into over all system. Unfortunately, the OFDM signal is characterized by high peak-to-average power ratio (PARR) and a large dynamic variation of signal amplitude. A highly linear power amplifier is required otherwise the nonlinear distortion arises and results in the spectral spreading and higher BER in the receiver. An alternative is that the high-output back off (OBO) is required, but resulting in low-power efficiency.

From above reason, there are several methods have been proposed. The first method is to decrease the PARR of input signal, the simplest one is clipping of the signal that is input of HPA [1]. Second method is the HPA linearization by analog techniques [2]. Another methods are predistortion methods [3], the linearized constant peak-power code OFDM (LCP-OFDM) and the partial transmit sequence (PTS) method.

In this paper, a new technique of the maximum likelihood detection with symbol estimation at the receiver has been proposed for detecting the received OFDM signal in the presence of nonlinear distortion. The major task is to estimate the new symbol position  $\tilde{\mathbf{s}}_m$  from received signal constellation which cause by nonlinearity of power amplifier. For the package data transmission, there are training data and information period. The  $\tilde{\mathbf{s}}_m$  will be estimated by neural network and provided to ML detector when the training data are available and communications channel are stationary during information period.

The paper is organized as follow. The principles of OFDM system and nonlinear distortion of traveling wave tube amplifier model are given in the next section. The Maximum likelihood detection and unsupervised clustering are briefly reviewed in the section III. Simulation results of 16-QAM OFDM system performances are obtained in the section IV. Finally, some conclusions are given in the section V.

## II. PRELIMINARIES

## A. System Model

Let consider an OFDM system employing  $N$  subcarriers. A stream of data  $\{a_i\}$ , which are belonging to  $M$ -QAM modulation, is serial to parallel converted. And  $a_n(i) = a_{In}(i) + ja_{Qn}(i)$  is the transmitted symbol in the  $i$ th time slot on the  $n$ th subchannel with subcarrier frequency  $f_n = n\Delta f$ ,  $\Delta f = \frac{1}{T}$  is the subcarrier interval. In the case of a  $M$ -QAM constellation, one of  $M$  possible signal waveform is transmitted to the receiver can be written as

$$a_{In} \in A, a_{Qn} \in A, \quad (1)$$

$$A = \{2m-1 - \sqrt{M}, m = 1, 2, \dots, \sqrt{M}\}$$

In each frame interval, a vector of  $N$  complex symbols  $a_n(i), n = 1, 2, \dots, N$  is transformed by means of an IFFT to a vector of  $N$  complex channel symbols  $A_n(t)$  of time domain samples. In order to eliminate any interference between adjacent OFDM symbols, the guard interval  $T_g$  (cyclic prefix) which assume longer than the overall channel impulse response is appended to  $A_n(t)$ . The resulting extended vector or the complex envelope of the OFDM signal can be written as

$$x(t) = \sum_{n=0}^{N-1} \sum_{i=-\infty}^{+\infty} a_n(i) \Phi_n(t - iT_s) \quad (2)$$

Where  $T_s = T + T_g$  is the period of the time domain OFDM signal,  $N$  is the number of subcarrier,  $T_g$  is guard interval

$$\Phi_n = \begin{cases} \sqrt{2\varepsilon/T_s} e^{j2\pi f_n t}, & \text{for } -T_g \leq t \leq T \\ 0, & \text{otherwise} \end{cases} \quad (3)$$

Where  $\varepsilon$  is the transmitted pulse energy. A baseband scheme of IFFT/FFT-based OFDM system is illustrated in Figure 1. In this figure, at the output of the parallel to serial (P/S) converter both transmitter and receiver, the samples of the OFDM signal are generated at rate  $N/T$ , then processed by a low pass filter  $H_t(f)$  to generate the analog complex signal  $x(t)$ . The OFDM signal is amplified and corrupted by additive white Gaussian noise  $N(t)$  with double-sided spectral power density  $N_0/2$ . Since attention is focused on

the effects of nonlinearity, a nonfrequency selective channel is assumed. The received signal is first processed by a linear filter having a low-pass transfer function  $H_r(f)$ . In the following analysis, the subsystems for carrier recovery and symbol synchronization are considered ideal and are disappeared in the Figure 1.

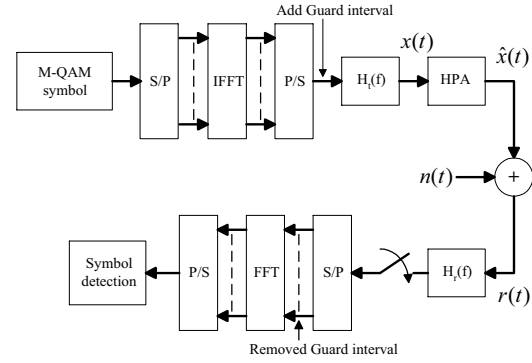


Figure 1. Baseband OFDM system.

## B. HPA model

A widely accepted HPA model is a nonlinear memoryless transformation between the complex envelope of the input and output signal. The complex envelope of the input signal to the HPA is  $x(t) = \varphi(t) \cdot e^{j\phi(t)}$ , And the complex envelope is  $\hat{x}(t) = A[\varphi(t)] \cdot e^{j\{\phi(t) + \Psi[\varphi(t)]\}}$  where  $A[\varphi]$  and  $\Psi[\varphi]$  represent the AM/AM and AM/PM conversion characteristic of the nonlinear amplifier, respectively. According to the memoryless Saleh travelling-wave tube (TWT) model [4]. The AM/AM and AM/PM conversion characteristics can be expressed as

$$A[\varphi(t)] = \frac{\alpha_a \varphi(t)}{1 + \beta_a \varphi^2(t)} \quad (4)$$

$$\Psi[\varphi(t)] = \frac{\alpha_p \varphi^2(t)}{1 + \beta_p \varphi^2(t)} \quad (5)$$

Where  $\varphi(t)$  is the input amplitude. In order to describe the different output power levels, the conventional method use the output power backoff (OBO) of the HPA which defined as

$$OBO_{dB} = 10 \log_{10} \frac{P_{\max}}{\hat{P}} \quad (6)$$

Where the  $P_{\max}$  represents the maximum output power of the HPA at the saturation point and  $\hat{P}$  denotes the mean power of

the signal at the HPA output. At the receiver, the output of the receiver filter which has impulse response  $h_r(t) = h_t(-t)$  is sampled at rate  $1/T$ . Figure 2 shows the received signal constellation for a 16-QAM OFDM scheme using travelling-wave tube amplifier (TWTA) with OBO=7 dB and  $N=128$  subcarriers.

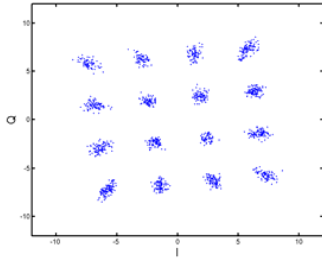


Figure 2. The received signal constellation for a 16-QAM OFDM using TWTA with OBO=7 dB and  $N=128$  subcarriers.

### III. SIGNAL DETECTION

#### A. Maximum Likelihood Detection

When a signal is transmitted over AWGN channel, a correlation demodulation or a matched filter demodulator produces a received vector in the form of

$$\mathbf{r}(t) = \mathbf{s}_m(t) + \mathbf{n}(t) \quad m = 1, 2, \dots, M \quad (7)$$

where  $\mathbf{r}(t)$  and  $\mathbf{s}_m(t)$  are received and transmitted signal with  $s$  dimensional vector, respectively.  $M$  is the possible transmitted signals and  $\mathbf{n}(t)$  is an additive noise vector with independent and identically distributed random variables. In this study, we assume that there is no memory in signals transmitted in successive signal intervals. From Bayes decision theory, minimum error-rate classification can be archived by finding the maximum of a posteriori,

$$p(\mathbf{s}_m | \mathbf{r}) = \frac{p(\mathbf{r} | \mathbf{s}_m)p(\mathbf{s}_m)}{p(\mathbf{r})} \quad (8)$$

This decision criterion is well known as the maximum a posterior probability criterion, where  $p(\mathbf{r} | \mathbf{s}_m)$  is the condition probability density function of the received vector given  $\mathbf{s}_m$ .  $p(\mathbf{s}_m)$  is the a priori probability of the  $m$ th signal

being transmitted and  $p(\mathbf{r}) = \sum_{m=1}^M p(\mathbf{r} | \mathbf{s}_m)p(\mathbf{s}_m)$ . When

the  $M$  possible transmitted signals are equally probability, the decision criterion is based on the maximum of  $p(\mathbf{r} | \mathbf{s}_m)$  over  $M$  signals. In this case, the MAP criterion is equivalent to the maximum likelihood (ML) criterion. For the maximum of ML detector the system need to find the  $\hat{m}_{ML}$  that minimized the Euclidean distance to the received signal.

$$\hat{m}_{ML} = \arg \min_m \|\mathbf{r} - \mathbf{s}_m\|, m = 1, 2, \dots, M \quad (9)$$

From the received OFDM signal with interference of nonlinear distortion, the system will estimate the new center of received symbol  $\tilde{\mathbf{s}}_m$  from the training data. Figure 3 shows a block diagram of the proposed method. The  $\tilde{\mathbf{s}}_m$  will be estimated by neural network to which the received signal and priori training data at the receiver are provided.

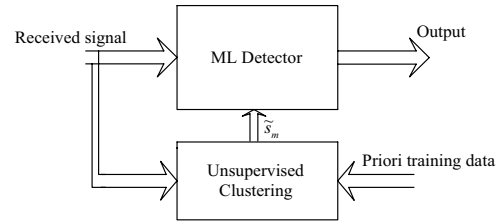


Figure 3. Block diagram of the proposed system.

#### B. Unsupervised Clustering Neural Network

$k$  mean clustering is a technique by which a data set is partitioned into disjoint subgroups [5], [6]. It means that each data point  $\mathbf{r}_j$  ( $j=1, 2, \dots, k$ ) is assigned to only one data cluster. On this sense, these clusters are called partitions. At the beginning process, the number of  $c$  is given and the initial cluster center  $\mathbf{z}_1, \mathbf{z}_2, \dots, \mathbf{z}_c$  are chosen where  $\mathbf{z}_i = \{z_{i1}, z_{i2}, \dots, z_{im}\}$  ( $i=1, 2, \dots, c$ ). After the input vector is fed into all clusters, the cluster is selected based on the similarity or the distance measurement. The selected cluster is expressed with the selection index

$$\delta_{ij} = \begin{cases} 1 & \mathbf{r}_j \in A_i \\ 0 & \mathbf{r}_j \notin A_i \end{cases} \quad (10)$$

For the hard partition,  $\sum_{i=1}^c \delta_{ij} = 1$  and  $\delta_{ij} \wedge \delta_{kj} = 0$  should be satisfied for all  $j$ . Defined a matrix  $\mathbf{U}$  which comprises of  $\delta_{ij}$  ( $i=1, 2, \dots, c; j=1, 2, \dots, k$ ). We then define a hard  $c$ -partition space as the following matrix set:

$$\mathbf{M}_C = \left\{ \mathbf{U} | \delta_{ij} \in \{0, 1\}, \sum_{i=1}^c \delta_{ij} = 1, \right. \\ \left. 0 < \sum_{j=1}^k \delta_{ij} < k \right\} \quad (11)$$

The HCM objective function is given by

$$\mathbf{J}(\mathbf{U}, \mathbf{z}) = \sum_{i=1}^c \sum_{j=1}^k \delta_{ij} d_{ij}^2, \quad (12)$$

where  $\mathbf{U}$  is partition matrix, the parameter  $\mathbf{z}$  is a vector of cluster centers,  $d_{ij}$  is a Euclidean distance between the  $j^{\text{th}}$  data point and  $i^{\text{th}}$  cluster center  $z_i$ . The optimum partition matrix based on the minimum least square error, i.e.,  $\mathbf{U}^*$ , is obtained by using the following condition,

$$J(\mathbf{U}^*, \mathbf{z}^*) = \min_{\mathbf{U} \in M_c} J(\mathbf{U}, \mathbf{z}). \quad (13)$$

Then the estimated  $\tilde{\mathbf{s}}_m$  define as

$$\tilde{\mathbf{s}}_m = \frac{\sum_{j=1}^c z(j) l_m(j)}{\sum_{j=1}^c l_m(j)} \quad m = 1, 2, \dots, M \quad (14)$$

where  $l_m(j)$  is the index of label

$$l_m(j) = \begin{cases} 1 & \text{for } \|\mathbf{z}(i) - \mathbf{s}_{m(\text{training data})}(k)\| > \\ & \|\mathbf{z}(j) - \mathbf{s}_{m(\text{training data})}(k)\|, \text{ all } i \neq j \\ 0 & \text{otherwise} \end{cases} \quad (15)$$

where  $\mathbf{s}_{m(\text{training data})}$  is the priori training data at receiver.

#### IV. EXPERIMENTAL RESULTS

In this section, we provide some results to illustrate the performance of the symbol estimation based maximum likelihood detection of OFDM signal with the effect of TWTA nonlinear distortion. An example at OBO=9.1 dB, 256 symbols of training data are provided into the system in which the network is initial 50 random clusters. After neural network has adapted with training data, the priori training data sequence at the receiver have been fed to the network for labeling of adapted clusters. The result of 16-QAM labeling is shown in Figure 4.

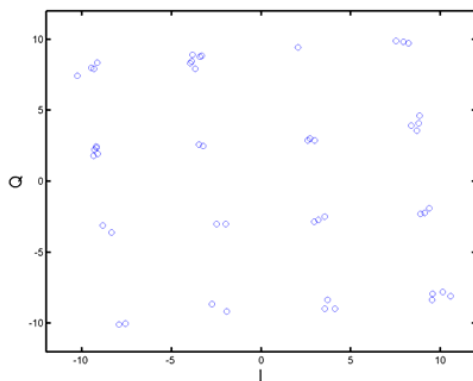


Figure 4. Clusters of 16-QAM OFDM symbol estimation obtained from the network.

The simulation confirmed that the proposed method has improved the system performance. Figure 5 reports BER versus SNR (dB) of 16-QAM OFDM system performance in the presence of TWTA nonlinear distortion. It can observe that, at BER =  $10^{-4}$ , the ML detection with symbol estimation required the SNR below 21.5 dB for OBO more than 8.7 dB.

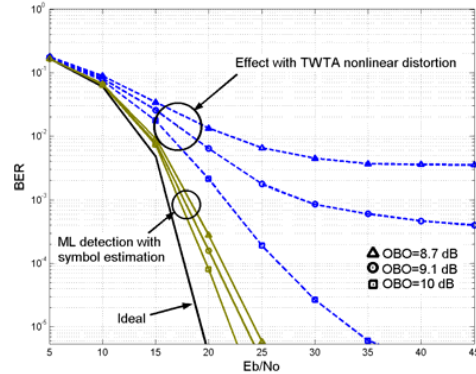


Figure 5. The BER versus signal-to-noise ratio of 16-QAM OFDM system.

#### V. CONCLUSIONS

In this paper, we considered the problem of signal detection in the OFDM communication system. The study investigates a new technique of maximum likelihood detector with symbol estimation for detecting the OFDM signal in the presence of nonlinear distortion generated by TWTA. The estimated symbols are obtained by the unsupervised clustering to which the received signal constellation and the priori training data at the receiver are provided. The simulation results of ML detector with symbol estimation show that the proposed method has enhanced the BER performance 16-QAM OFDM systems.

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