

# Mathematical Modelling of Partially Filled Fluid Coupling Behaviour

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**Abstract**—Modelling techniques for a fluid coupling taken from published literature have been extended to include the effects of the filling and emptying of the coupling with oil and the variation in losses when the coupling is partially full. In the model, the fluid flow inside the coupling is considered to have two principal velocity components; one circumferentially about the coupling axis (centrifugal head) and the other representing the secondary vortex within the coupling itself (vortex head). The calculation of liquid mass flow rate circulating between the two halves of the coupling is based on: the assumption of a linear velocity variation in the circulating vortex flow; the head differential in the fluid due to the speed difference between the two shafts; and the losses in the circulating vortex flow as a result of the impingement of the flow with the blades in the coupling and friction within the passages between the blades.

**Keywords**—Fluid Coupling, Mathematical Modelling, partially filled.

## I. INTRODUCTION

FLUID couplings are widely used in industry to provide a soft drive between driving and driven rotating machines. In most applications the coupling operates under steady conditions with a fixed quantity of liquid within the coupling. The characteristics of partially filled fluid couplings are complex and depend on the behaviour of the two-phase, unsteady, three dimensional flows. Little work has been found in open literature describing modelling of fluid couplings and the methods available are for fully filled couplings. In the application considered in this paper, the fluid coupling is used to provide an intermittent drive between two shafts where the coupling is engaged and disengaged by filling and then draining oil from the coupling. It thus operates partially filled for a portion of the drive cycle.

A number of studies have been reported concerning the analysis, design and performance prediction of a fluid coupling [1-5]. Rolfe [1] developed a mathematical analysis of hydraulic couplings using conventional design formulae and general rotor-dynamic theory, based on the use of non-dimensional parameters. The theory derived showed close agreement with experimental data taken from a test rig developed for this purpose. Qualman & Egbert [2] presented different methods to predict the performance of fluid couplings. In their analysis, they considered a single effective flow path along which the fundamental equations of a one dimensional flow were applied. This model, for a fully filled fluid coupling, was then extended by Wallace et al [3] to

model the fluid coupling performance using two approaches; the mean flow path approach (constant velocity) and the linear velocity approach where the velocity is assumed to increase linearly with distance from the mean radius. The performance prediction for a fully filled coupling was compared with experimental data and it was found that the linear velocity approach gave an improvement over the constant velocity approach. This model was then used by Whitfield et al [4] to study the effect of a baffle plate on the torque characteristics of a fully filled coupling and the linear velocity approach again showed better agreement with experimental data. In a further paper, Whitfield et al [5] used the simpler constant velocity approach [2 & 3] to develop a general performance prediction model for more complex multi-element torque converters. The model was verified by comparison with comparison with three and five element torque converters.

The fluid coupling considered in this study is illustrated in Fig. 1. It comprises two almost identical halves each connected to one of the shafts. The only difference between each half of the coupling is that there one more blade in one half to eliminate resonance at the blade passing frequency. A shroud is then fitted over the driven half of the coupling and is fixed to the driving half of the coupling. The purpose of the shroud is to retain the oil in the coupling. The oil is fed to the coupling down a hole along the centre of the shaft of the driven half of the coupling and then through the gap in between each coupling half.

## II. THEORY AND MODELLING:

The analytical model described in this paper is based on an extension of the model described by Wallace et al [3] to account for a partially filled fluid coupling with filling and emptying. This model uses an empirical calculation for the energy losses used to determining the mass flow rate circulating between each half of the coupling. This circulating mass flow rate is directly proportional to the transmitted torque.

The fluid flow inside the coupling is considered to have two principal components; one is the circumferential velocity about the coupling axis (this gives rise to the centrifugal head) and the other is the circulation of the vortex passing fluid between the two halves of the coupling in the plane of coupling axis (this gives rise to the vortex head).

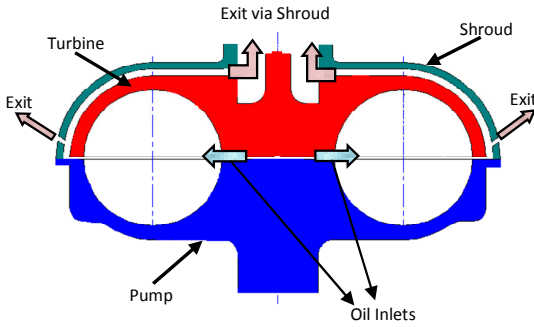


Fig. 1 Schematic diagram of fluid coupling with filling and emptying

Oil fed into the coupling is retained by the shroud. Fig. 2 shows a cross section of a fully filled fluid coupling indicating the main flow features. A pressure balance is made at the outer radius of the coupling where the fluid pressure circulating inside the coupling balances that held in the shroud.

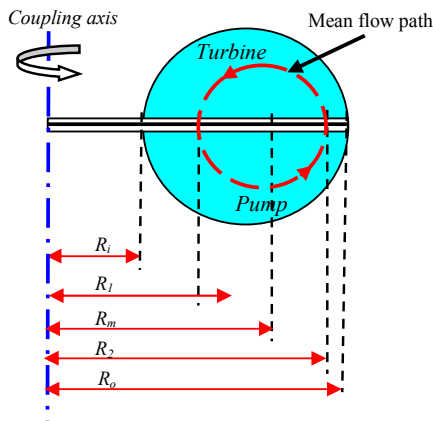


Fig. 2 Cross section of a fully filled fluid coupling showing main flow features

### A. Flow Regimes

For a partially filled coupling, two types of flow regime occur; the *annular flow* regime and the *outer centred flow* regime. If the vortex head, which is generated by the angular velocity of the fluid circulating between the two halves of the coupling, is greater than the centrifugal head, which is generated by the rotational speed of the pump and turbine, the flow is considered to be *annular flow*. Alternatively, if the centrifugal head dominates, the flow moves to the outer edge of the coupling and is considered to be *outer centred flow*. Figure 3 shows the annular and the outer centred flow regimes in a partially filled coupling. The outer centred flow is shown to have an idealised circular vortex path and this is used to simplify the modelling. In reality the liquid would be flung to the outer part of the coupling and the free surface would be at constant radius from the coupling axis.

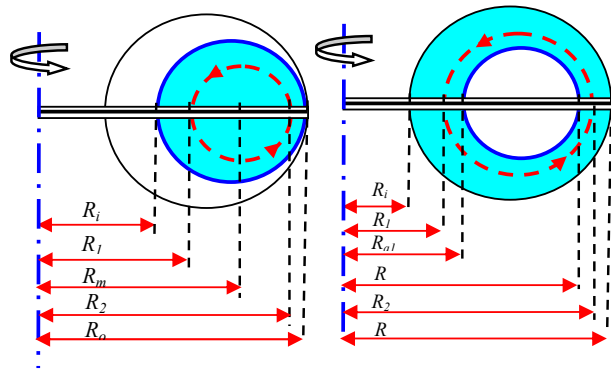


Fig. 3 Idealized Outer flow regime and Annular flow Centred flow regime

Applying the continuity equation to the lower and upper parts of the coupling, the centre of the mean flow path,  $R_m$ , and the inner and outer radial location of the mean flow path,  $R_l$  and  $R_2$ , can be determined as function of the coupling size, the oil volume fraction inside the coupling and the coupling flow regime (whether the flow is annular or outer centred).

#### 1) Outer centred flow regime:

By using the continuity condition of equal flow rates in the lower and upper portions of the coupling, the centre of the mean flow path is at radius  $R_m$  is given by:

$$R_m = \sqrt{(R_o^2 + R_i^2) / 2} . \quad (1a)$$

Similarly, the inner and outer radii of the mean flow path are given by:

$$R_l = \sqrt{(R_m^2 + R_i^2) / 2} , \quad (1b)$$

$$R_2 = \sqrt{(R_m^2 + R_o^2) / 2} , \quad (1c)$$

where the inner and outer radii of the oil inside the coupling,  $R_i$  and  $R_o$ , can be written as function of the centrifugal flow,  $D'$ , diameter of and the vortex flow diameter,  $d'$ , as:

$$R_i = (D' - d') / 2 , \quad (2a)$$

$$R_o = (D' + d') / 2 . \quad (2b)$$

The diameter of the main flow  $D'$  and the diameter of the vortex flow  $d'$  can be linked to the pitch circle diameter,  $D$ , and the minor diameter,  $d$ , by the oil volume fraction as:

$$f_{oil} = V_{oil} / V_{tot} = \frac{d'^2 D'}{d^2 D} . \quad (3a)$$

Also, from geometrical configurations:

$$D' + d' = D + d . \quad (3b)$$

Substituting (3b) into (3a), gives

$$d'^3 + (D + d)d'^2 + fDd = 0 . \quad (4)$$

Equation (4) is a third order algebraic equation which can be solved for  $d'$  at any given volume fraction and this gives values for all the flow radii using (1-3).

## 2) Annular flow regime:

Similarly, from the continuity condition and the volume fraction definition, it can be found that:

$$R_{a1} = \sqrt{(2R_i^2 + f(R_o^2 - R_i^2))/2}, \quad (5a)$$

$$R_{a2} = \sqrt{(2R_o^2 - f(R_o^2 - R_i^2))/2}, \quad (5b)$$

$$R_l = \sqrt{(4R_i^2 + f(R_o^2 - R_i^2))/4}, \quad (5c)$$

$$R_2 = \sqrt{(4R_o^2 - f(R_o^2 - R_i^2))/4}, \quad (5d)$$

where,

$$R_i = (D - d)/2, \quad (6a)$$

$$R_o = (D + d)/2. \quad (6b)$$

### B. Constant and linear velocity approaches:

The torque developed in any turbo machine is given by the rate of change of angular momentum as the fluid passes through the passages in between the blades. The fluid coupling under consideration has straight blades, (blade angle equal to zero), and if the circumferential component of fluid velocity is considered to be equal to the blade speed, i.e. that the secondary vortex flow is parallel to the blades, or no slip, the torque can be written as:

$$T = \dot{m}(\omega_p R_2^2 - \omega_t R_1^2) = \dot{m} \omega_p (R_2^2 - S R_1^2), \quad (7)$$

where,  $S$  is the speed ratio given as:  $S = \frac{\omega_t}{\omega_p}$ .

To calculate the transmitted torque, the vortex mass flow rate,  $\dot{m}$ , needs to be determined. In modelling the vortex mass flowrate,  $\dot{m}$ , of the fluid between the halves of the coupling (i.e. the circulation shown in Fig. 2) two velocity profiles are considered: a *constant velocity* in the vortex, in which it is assumed that the all the fluid moves at one speed (equal to that at the mean flow path) circulating around the vortex, and a *linear velocity* in which it is assumed that the fluid moves as a solid body rotating about the vortex with zero velocity at the mean radius and a maximum at the periphery. Figures 4 show a schematic diagram of these constant velocity and the linear velocity approaches.

#### Constant velocity approach:

The power input to the pump and the power output from the turbine are given as:

$$P_p = \dot{m} \omega_p^2 (R_2^2 - S R_1^2), \quad (8a)$$

$$P_T = \dot{m} \omega_T \omega_p (R_2^2 - S R_1^2), \quad (8b)$$

The power dissipated through the coupling is given as the difference between the power input to the pump and the power output from the turbine:

$$P_L = P_p - P_T. \quad (9)$$

This can be expressed as a specific energy loss as:

$$p_L = (\omega_p^2 - \omega_T \omega_p) (R_2^2 - S R_1^2). \quad (10)$$

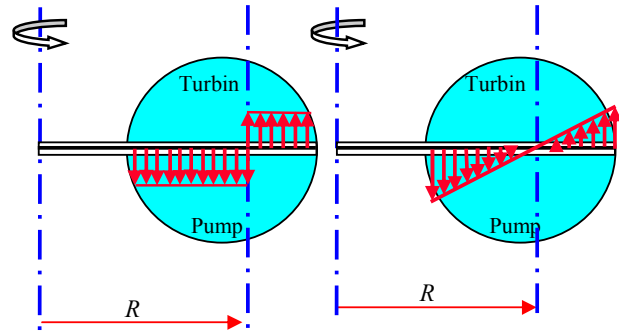


Fig. 4 Constant and linear velocity profiles

The losses are usually considered into two parts [1 & 2]; the incidence loss, due to the loss as the flow strikes the blades as the liquid moves from one half of the coupling to the other, and the flow path circulation loss or passage loss, due to friction within the passages in the coupling. Qualman and Egbert [2] gave the total incidence power losses as:

$$p_i = \frac{1}{2} (\omega_p - \omega_T) (R_2^2 + R_1^2). \quad (11)$$

The passage power loss was considered to be proportional to the square of the flow velocity in similar manner to that of pipe friction [3] as:

$$p_f = \frac{1}{2} K C^2. \quad (12)$$

By equating the difference between power input to the pump and the power output from the turbine with the summation of the incidence loss and the flow path circulation loss, the mean flow velocity can be written as:

$$C_{vor} = \frac{\omega_p}{\sqrt{K}} \left[ (1 - S^2) (R_2^2 - R_1^2) \right]^{0.5}. \quad (13)$$

Thus, the mass flow rate through the coupling can be calculated using the vortex velocity as:

$$\dot{m} = \rho \pi C_{vor} (R_m^2 - R_i^2) \quad (\text{Outer centred Flow}), \quad (14a)$$

$$\dot{m} = \rho \pi C_{vor} (R_o^2 - R_g^2) \quad (\text{Annular Flow}). \quad (14b)$$

#### 1) Linear velocity approach:

Applying the methodology used by Wallace et al [3] for a fully filled coupling to a partially filled coupling, the transmitted torque and the absorbed power can be written as:

$$T = K_I \omega_{vor} \omega_p, \quad (15)$$

$$P_L = K_I \omega_{vor} \omega_p^2 (1 - S), \quad (16)$$

where  $K_I$  depends on the oil volume fraction, the coupling geometry and type of flow regime - annular or outer centred flow.

$K_I$  for outer centred flow is given as:

$$K_i = 2\pi\rho \left[ \begin{array}{l} \frac{R_o^5}{5} + \frac{R_o^4 R_m}{4} + \frac{R_m^5}{20} \\ -S \left( \frac{R_i^5}{5} + \frac{R_i^4 R_m}{4} + \frac{R_m^5}{20} \right) \end{array} \right], \quad (17a)$$

and for annular flow given as:

$$K_i = 2\pi\rho \left[ \begin{array}{l} \frac{R_o^5}{5} + \frac{R_o^4 R_{a2}}{4} + \frac{R_{a2}^5}{20} \\ -S \left( \frac{R_i^5}{5} + \frac{R_i^4 R_{a1}}{4} + \frac{R_{a1}^5}{20} \right) \end{array} \right]. \quad (17b)$$

In order to calculate the transmitted torque, the vortex rotational speed is required. This can be found by calculating the power losses following the same procedure used by Wallace et al [3] where the power loss was divided into three components; (i) the *incidence loss* which is generally taken to be given by the square of the change in the tangential component of velocity relative to the rotation of the component which the fluid is entering, (ii) the *friction loss* which is obtained by applying pipe friction formulae to the flow within the passages of each half of the fluid coupling, (iii) *Circulation loss* which represents other losses and in this case accounts for the secondary losses in the vortex as the flow passes between the blades. It is represented as equivalent to the loss due to four 90° bends..

The total power loss ( $P_L$ ) which is experienced by the coupling is given as:

$$P_L = (K_f + K_c) \omega_{vor}^3 + K_i \omega_p^2 (1-S)^2 \omega_{vor} \quad (18)$$

$K_i$ ,  $K_f$  and  $K_c$  are incident loss coefficient, friction loss coefficient and circulation loss coefficient respectively. Their values depend on the oil volume fraction, the coupling geometry and whether the flow is annular or outer centred flow. Following Wallace et al [3] these coefficients are:

For outer centred flow:

$$K_i = \pi\rho \left[ \frac{R_m^5}{10} - \frac{R_m(R_i^4 + R_o^4)}{4} + \frac{(R_i^5 + R_o^5)}{5} \right], \quad (19a)$$

$$K_f = \frac{\pi\rho f}{4} \left[ \frac{(ZD_L + \pi R_i)(R_m - R_i)^4}{(ZD_U + \pi R_o)(R_o - R_m)^4} + \frac{\pi\rho f Z}{20} \left[ (R_m - R_i - D_L)^5 + (R_o - R_m - D_U)^5 \right] \right], \quad (19b)$$

$$K_c = 2\pi\rho K \left[ \frac{R_m^5}{10} - \frac{R_m^3(R_i^2 + R_o^2)}{2} + R_m^2(R_i^3 + R_o^3) - \frac{3R_m(R_i^4 + R_o^4)}{4} + \frac{(R_i^5 + R_o^5)}{5} \right], \quad (19c)$$

and for annular flow:

$$K_i = \pi\rho \left[ \frac{(R_{a1}^5 + R_{a2}^5)}{20} - \frac{(R_{a1}R_i^4 + R_{a2}R_o^4)}{4} + \frac{(R_i^5 + R_o^5)}{5} \right], \quad (20a)$$

$$K_f = \frac{\pi\rho f}{4} \left[ \frac{(ZD_L + \pi R_i)(R_{a1} - R_i)^4}{(ZD_U + \pi R_o)(R_o - R_{a2})^4} + \frac{\pi\rho f Z}{20} \left[ (R_{a1} - R_i - D_L)^5 + (R_o - R_{a2} - D_U)^5 \right] \right], \quad (20b)$$

$$K_c = 2\pi\rho K \left[ \frac{(R_{a1}^5 + R_{a2}^5)}{20} - \frac{(R_{a1}^3 R_i^2 + R_{a2}^3 R_o^2)}{2} + \frac{(R_i^5 + R_o^5)}{5} - \frac{3(R_{a1}R_i^4 + R_{a2}R_o^4)}{4} \right]. \quad (20c)$$

In the above equations, the depth of the lower and upper stream lines thickness,  $D_U$  and  $D_L$ , are specified following Wallace et al [3] as:

$$\begin{array}{ll} D_L = (R_m - R_i)/10 & \text{Outer centered flow} \\ D_L = R_{a1} - R_i & \text{Annular flow} \end{array}, \quad (21)$$

$$D_U = \left( \frac{R_m - R_i}{R_o - R_m} \right) \frac{R_i}{R_o} D_L \quad \text{Outer centered flow}, \quad (22)$$

$$D_U = \left( \frac{R_{a1} - R_i}{R_o - R_{a2}} \right) \frac{R_i}{R_o} D_L \quad \text{Annular flow}$$

Equating the empirically derived power loss equation, (18), with the power absorbed in the coupling, (16), yields:

$$(K_f + K_c) \omega^3 + \omega_p^2 (1-S) [K_i (1-S) + K_j] \omega_{vor} = 0. \quad (23)$$

The above equation is a third order algebraic equation which can be solved for the fluid vortex angular velocity. With the vortex angular velocity known, the torque can be calculated from (15). The value taken for the empirical loss coefficient  $K$  in (12) (19c) and (20c) was 1. This is typical of values given in the literature.

### C. Head Calculations

The total coupling head at the outermost edge of the coupling where the flow passes into the shroud may be written as the summation of the centrifugal head and the vortex head,

$$H_{coup} = H_{cen} + H_{vor}. \quad (24)$$

Following Rolfe [1], the centrifugal head and the vortex head can be written as:

$$H_{cen} = \left( \frac{\omega_p + \omega_t}{2} \right)^2 (R_o^2 - R_i^2) / 2g, \quad (25)$$

$$H_{vor} = \frac{\omega_{vor}^2}{2g} (R_o - R_m)^2 \quad \text{Outer centred Flow}, \quad (26a)$$

$$H_{vor} = \frac{\omega_{vor}^2}{2g} (R_o - R_{a2})^2 \quad \text{Annular Flow}, \quad (26b)$$

At equilibrium, the coupling head balances the head of liquid in the shroud which is given by:

$$H_{shr} = \left( \frac{\omega_p + \omega_t}{2} \right)^2 (R_o^2 - L_{shr}^2) / 2g. \quad (27)$$

The shroud length,  $L_{shr}$ , is the distance from the axis of the coupling to the free surface in the shroud. As the shroud head balances the total coupling head, the shroud length can be calculated by equating (24) with (27).

### D. Volume Balance:

To calculate the oil volume fraction inside the coupling at any given time the volume of the oil inside the coupling must be known and this can be found by the volume balance:

$$V_{coup} + V_{shr} = V_{in} - V_{ex\_hole} - V_{ex\_shr} \quad (28)$$

In (28) the volumes represent the cumulative flow from the time at which the coupling starts. The inlet volume,  $V_{in}$ , is given by the volume flow rate at the inlet.

The shroud volume,  $V_{shr}$ , can be calculated from the shroud geometry and the shroud length  $L_{shr}$ , can be calculated from (24) and (27).

Since the location and size of the shroud holes is known, the head at the shroud exit holes can be written as:

$$H_{ex\_hole} = \left( \frac{\omega_p + \omega_t}{2} \right)^2 (L_{ex\_hole}^2 - L_{shr}^2) / 2g \quad (29)$$

where  $L_{ex\_hole}$  is the distance between the coupling centre to the exit holes. The velocity of the oil leaving the shroud through the shroud holes can be calculated as:

$$C_{ex\_hole} = \sqrt{2g \cdot H_{ex\_hole}} \quad (30)$$

From the exit velocity, the discharge volume through the holes,  $V_{ex\_hole}$ , can be calculated by assuming a discharge coefficient.

If the calculated shroud length is greater than the minimum shroud length (determined by the diameter of the hole in the middle of the shroud) this means that the shroud is not full and thus the discharge volume through the shroud,  $V_{ex\_shr}$ , is equal to zero. But if the calculated shroud length is less than the minimum shroud length this means that the shroud is full (and overflowing) and the shroud length is then equal to the minimum shroud length. When the shroud is full the coupling head must balance the shroud head and in this case the volume fraction inside the coupling can be calculated by equating (24) with (27).

Since all the volumes at any time can be calculated, the oil volume fraction inside the coupling can be specified and thus the flow regimes inside the coupling can be found from the head balance. Once the flow regime (annular or outer centred) and the volume fraction inside the coupling are known all the coupling characteristics can be found using the appropriate equations.

### III. RESULTS AND DISCUSSIONS

In this study two different geometrical cases are considered; (i) Case A where the pitch circle diameter,  $D$ , is 133 mm, the minor diameter,  $d$ , is 50 mm and the oil filling rate is 230 litres per hour, (ii) Case B where the pitch circle diameter is 184 mm, the minor diameter 70 mm, and the oil filling rate is also 230 litres per hour. In these cases the pump is driven by an electric motor and the turbine is connected to a large mass to give a high moment of inertia but low frictional losses.

Figure 5 shows the transmitted torque calculated using both the constant velocity approach and the linear velocity approach. It can be seen that both approaches produce reasonable results. This is because it includes terms to calculate the increased flow path circulation losses that occur when the coupling is only partially full and also it has a more realistic representation of the vortex circulation which is used to calculate the torque. During the second acceleration and deceleration cycle, this lag is not evident. It is suggested that

during the initial fill, either air becomes trapped within the coupling preventing the oil volume fraction building up, or the oil inflow interacts with the liquid vortex circulating within the coupling reducing the circulation rate and hence the torque. Clearly further work is needed to develop the modelling for this case.

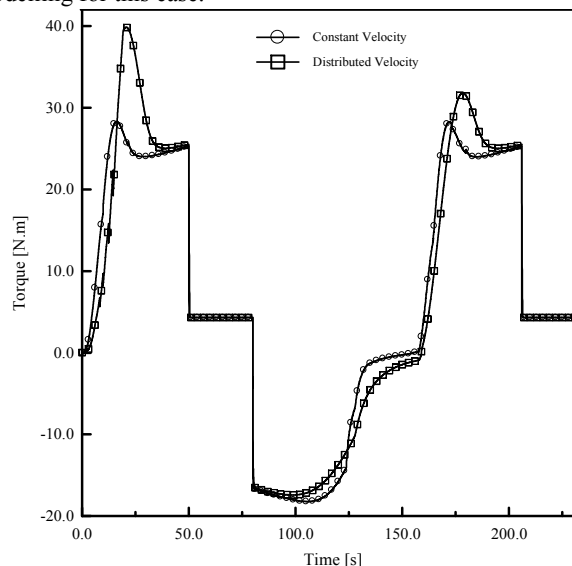


Fig. 5 Calculated torque using two different flow approaches for case A

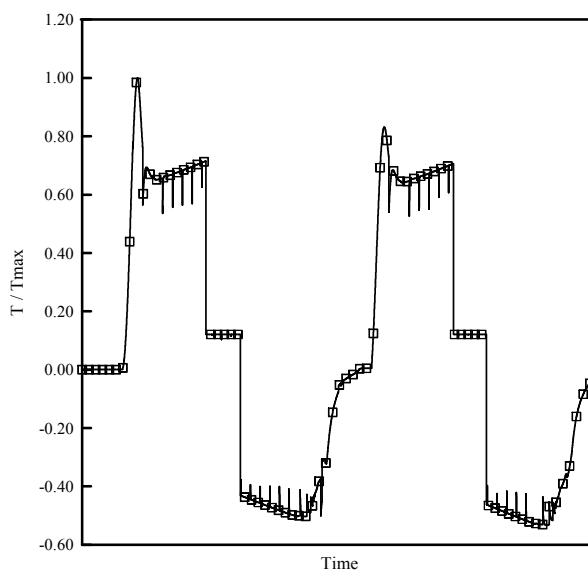


Fig. 6 Calculated torque using the linear velocity approach for case B

The linear velocity approach is used to study and understand the flow within fluid coupling for Case B. Figure 6 shows a calculated torque for Case B. It can also be seen that this approach is able to predict unsteadiness in the torque occurring during acceleration and deceleration and possible reasons for this are commented on later in the discussion.

Calculated turbine rotational speed for Case A together with resulted torque is shown in Fig. 7. It is clear that the torque reaches its maximum positive value at the speed up and its

negative value at the speed down regime. A convenient way of characterizing the torque generated by a fluid coupling is to express it in non-dimensional terms as a moment coefficient. The moment coefficient is defined as:

$$C_m = \frac{T}{\rho \omega_p^2 D^5} \quad (31)$$

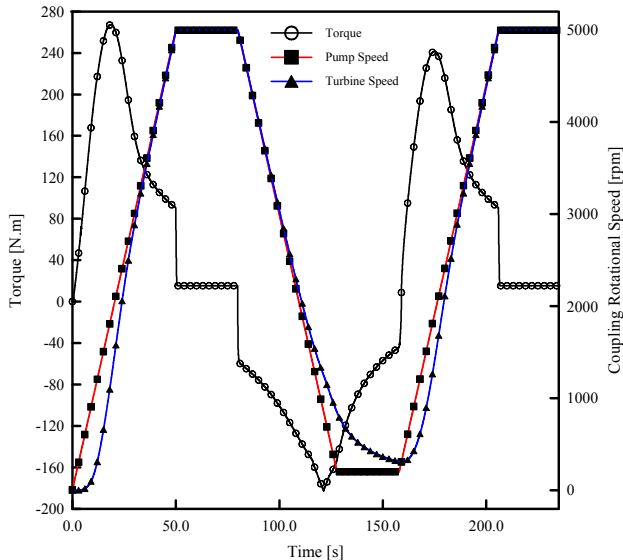


Fig. 7 Calculated turbine and pump rotational speed and the resulted torque for case A

An interesting result is seen in Fig. 8 which shows the variation of the oil volume fraction with time for Case A and Case B. It can be seen from that the coupling in Case A is filling faster than Case B. This is because the oil flow rate is the same but the volume of Case A is less than that for Case B. More interestingly it can be also be noticed that in Case A, when the coupling is full it stays full, while in Case B, after the oil volume fraction in the coupling reaches 1 it then starts oscillating between 0.78 and 0.96. This occurs after an initial peak in the predicted torque

#### IV. CONCLUSION

Existing analytical models for the prediction of the performance of a fully filled fluid coupling have been extended to model a coupling which is partially filled and in which two distinct flow regimes occur: *annular flow* and *outer centred flow*. It is found that a one dimensional flow within the circulating vortex in the coupling does not give as good a prediction of performance as a linear velocity distribution within the circulating vortex, as has been reported by earlier workers.

Although for the smaller coupling tested it is apparent that the model over-predicts the torque produced whilst the coupling is filling. It is suggested that a more detailed model is required to give a better representation of the way the inflow of oil into the coupling interacts with the vortex flow within the coupling during filling.

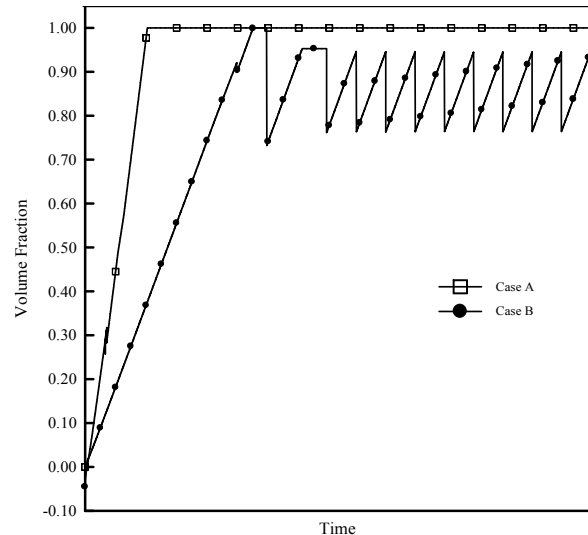


Fig. 8 Variation of oil volume fraction during initial start up for case A and case B

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