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Some $(v+1, b+r+\lambda+1, r+\lambda+1, k, \lambda+1)$ Balanced Incomplete Block Designs (BIBDs) from Lotto Designs (LDs)

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Abstract—The paper considered the construction of BIBDs using potential Lotto Designs (LDs) earlier derived from qualifying parent BIBDs. The study utilized Li's condition $\lfloor \frac{pr}{t-1} \rfloor \binom{t-1}{2} + \binom{pr - \lfloor \frac{pr}{t-1} \rfloor (t-1)}{2} < \binom{p}{2} \lambda$, to determine the qualification of a parent BIBD (v, b, r, k, λ) as LD (n, k, p, t) constrained on $v \geq k$, $v \geq p$, $t \leq min\{k, p\}$ and then considered the case $k = t$ since t is the smallest number of tickets that can guarantee a win in a lottery. The $(15, 140, 28, 3, 4)$ and $(7, 7, 3, 3, 1)$ BIBDs were selected as parent BIBDs to illustrate the procedure. These BIBDs yielded three potential LDs each. Each of the LDs was completely generated and their properties studied. The three LDs from the $(15, 140, 28, 3, 4)$ produced $(9, 84, 28, 3, 7)$, $(10, 120, 36, 3, 8)$ and $(11, 165, 45, 3, 9)$ BIBDs while those from the $(7, 7, 3, 3, 1)$ produced the $(5, 10, 6, 3, 3)$, $(6, 20, 10, 3, 4)$ and $(7, 35, 15, 3, 5)$ BIBDs. The produced BIBDs follow the generalization $(v+1, b+r+\lambda+1, r+\lambda+1, k, \lambda+1)$ where (v, b, r, k, λ) are the parameters of the $(9, 84, 28, 3, 7)$ and $(5, 10, 6, 3, 3)$ BIBDs. All the BIBDs produced are unreduced designs.

Keywords—Balanced Incomplete Block Designs, Lotto Designs, Unreduced Designs, Lottery games.

I. INTRODUCTION

Balanced Incomplete Block Designs (BIBDs) are used in the design and analysis of experiments to improve efficiency and meet experimental necessities in randomized block designs. They are used to design experiments where the subjects must be divided into blocks (subsets) of the same size to receive different treatments, such that each subject is tested the same number of times and every pair of subjects appears in the same number of subsets. Instances of biological and physical conditions that can lead to a predetermined, fixed block size where the researcher is compelled to use BIBDs are given in [1]. Some other applications of BIBDs include their use in secrecy and authentication codes, tournament scheduling, group testing and so on.

A Balanced Incomplete Block Design (BIBD) [2] with parameters (v, b, r, k, λ) is a pair (X, A) , where X is a set, A is a collection of subsets of X, and the five parameters are nonnegative integers defined as follows:

- v (order) is the size of X (elements of X are points, varieties or treatments),
- b (block number) is the number of elements of A (elements of A are blocks),
- r (replication number) is the number of blocks to which every point belongs,
- k (block size) is the common size of each block, and
- λ (index) is the number of blocks to which every pair of distinct points belongs.

Several construction methods for BIBDs exist in literature. These include [3], [4], [5] and [6]. New BIBDs can also be constructed from old BIBDs. For instance, the sum and block complementation techniques for constructing BIBDs were provided in [7]. Block section and intersection techniques for constructing new designs from parent symmetric designs were described in [8]. In this study, we present the construction of some BIBDs from parent BIBDs using potential Lotto Designs.

II. LOTTO DESIGNS

In a typical lottery game, a person chooses k numbers from v numbers with a small amount of money. This constitutes the ticket. The sale of tickets is stopped at a certain point and the organizers pick p numbers from the v numbers randomly. These p numbers are called the winning numbers. If any of the tickets sold match t or more of the winning numbers, a prize is given to the holder of the matching ticket. The larger the value of t, the larger the prize. The historical background of lottery and some types of lottery are given in [9].

A formal definition of Lotto Designs is given in [10]:

> "Suppose v, k, p and t are integers and B is a collection of k-subsets of a set X of v elements (usually X is $X(v)$). Then, B is an (v, k, p, t) Lotto Design (LD) if an arbitrary p-subset of $X(v)$ intersects relevant k-set of B in at least t elements. The k-sets in B are known as the blocks of the Lotto design B. The elements of X are known as the varieties of the design."

The author also defined potential lotto designs as collections of k-sets formed during the construction which may or may not be lotto designs.

The primary aim of most researchers and players is to know the minimum number of tickets required to obtain a match of at least t numbers. This minimum is usually denoted by $L(v, k, p, t)$. Several researches have been carried out in this regard. In [11], a computer program that can be used to construct minimal (v, k, p, t) lottery design was presented while [12] determined the values for $L(v, 6, 6, 2)$ for $v \leq 54$. Several upper bound construction methods for LDs, one of

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which is the use of BIBDs was presented in [10] while the notion of a lottery graph and closed-form bound formulations for the lottery number were introduced in [13].

III. Relationship between LDs and BIBDs

BIBDs can be used in constructing upper bounds for lotto designs. However, not all BIBDs can produce Lotto Designs. To recognize the qualifying BIBDs, [10] gave the general condition they must satisfy in the following theorem:

Theorem[10]:
If B is the set of blocks of a (v, b, r, k, λ) BIBD and p, t are positive integers where $\lfloor \frac{pr}{t-1} \rfloor \binom{t-1}{2} + \binom{pr - \lfloor \frac{pr}{t-1} \rfloor (t-1)}{2} < \binom{p}{2} \lambda$, then B is the set of blocks of an (v, k, p, t) Lotto design. Hence $L(v, k, p, t) \leq b$.

IV. Unreduced BIBDs

A (v, b, r, k, λ) BIBD is said to be an unreduced design if $b = \binom{v}{k}$, $r = \binom{v-1}{k-1}$, and $\lambda = \binom{v-2}{k-2}$. They are designs that contain all possible combinations of k out of v varieties. Unreduced designs are the simplest type of BIBDs. They have the advantage of being simple and easy to construct. They have been found to be useful in the construction of Partially Balanced Incomplete Block Designs (PBIBD)[14] and in the theory of resistant designs [15].

V. Methodology

Two BIBDs, the $(15, 140, 28, 3, 4)$ and the $(7, 7, 3, 3, 1)$ BIBDs were selected from a list of BIBDs presented in [8] for the purpose of illustration. The parameters p, r, t and λ specified in Li's condition were integers defined such that $3 \leq p \leq 11$, $3 \leq t \leq 8$ and $t \leq min\{k, p\}$ and $k = t$; r and λ were obtained from the selected BIBD (v, b, r, k, λ) with $v \geq p$, $v \geq k$. The ranges of p and t were selected to fit the range of most lottery formats available. A FORTRAN language program was written to implement the expression for Li's condition so as to determine the (v, k, p, t) lotto designs that can be derived from the selected BIBDs. A complete generation of these LDs was made using a Microsoft Office Access database computer program which was adapted from [16] and their properties were then investigated for compliance with properties of BIBDs.

VI. Results and Discussion

Tables I and II show the results of the implementation of Li's condition for the $(15, 140, 28, 3, 4)$ and $(7, 7, 3, 3, 1)$ BIBDs. COMBT, COMBP, COMTOT and COMBL represent computations of the different components in Li's condition. YES indicates Li's condition is satisfied and NO indicates that the condition is not satisfied. Hence the $(15, 140, 28, 3, 4)$ BIBD was found to qualify as $LD(15, 3, 9, 3)$, $LD(15, 3, 10, 3)$ and $LD(15, 3, 11, 3)$ while the $(7, 7, 3, 3, 1)$ BIBD qualified as $LD(7, 3, 5, 3)$, $LD(7, 3, 6, 3)$, $LD(7, 3, 7, 3)$.

Tables III-VIII show the complete generation of the LDs. Any of the p values can be used for the generation. This gives the same set of parameters although the elements used for the arrangement would be different. This implies that, with any p, a BIBD with the same set of parameters will still be obtained. All the LDs produced satisfied the necessary conditions for BIBDs, hence BIBDs can be obtained from them. The $(15, 140, 28, 3, 4)$ parent BIBD led to the production of $(9, 84, 28, 3, 7)$, $(10, 120, 36, 3, 8)$ and $(11, 165, 45, 3, 9)$ BIBDs while $(5, 10, 6, 3, 3)$, $(6, 20, 10, 3, 4)$ and $(7, 35, 15, 3, 5)$ are the BIBDs obtained from the $(7, 7, 3, 3, 1)$ BIBD. All the BIBDs produced are unreduced designs and can be generalized as $(v+1, b+r+\lambda+1, r+\lambda+1, k, \lambda+1)$ where (v, b, r, k, λ) are the parameters of the first BIBD produced. This is shown in Table IX.

TABLE I
Implementation of Li's Condition for $(15, 140, 28, 3, 4)$ BIBD

P	T	R	L	COMBT	COMBP	COMTOT	COMBL	ANSW
3	3	28	4	42.00	0.00	42.00	12.00	NO
3	4	28	4	84.00	0.00	84.00	12.00	NO
3	5	28	4	126.00	0.00	126.00	12.00	NO
3	6	28	4	160.00	6.00	166.00	12.00	NO
3	7	28	4	210.00	0.00	210.00	12.00	NO
3	8	28	4	252.00	0.00	252.00	12.00	NO
4	3	28	4	56.00	0.00	56.00	24.00	NO
4	4	28	4	111.00	0.00	111.00	24.00	NO
4	5	28	4	168.00	0.00	168.00	24.00	NO
4	6	28	4	220.00	1.00	221.00	24.00	NO
4	7	28	4	270.00	6.00	276.00	24.00	NO
4	8	28	4	336.00	0.00	336.00	24.00	NO
5	3	28	4	70.00	0.00	70.00	40.00	NO
5	4	28	4	138.00	1.00	139.00	40.00	NO
5	5	28	4	210.00	0.00	210.00	40.00	NO
5	6	28	4	280.00	0.00	280.00	40.00	NO
5	7	28	4	345.00	1.00	346.00	40.00	NO
5	8	28	4	420.00	0.00	420.00	40.00	NO
6	3	28	4	84.00	0.00	84.00	60.00	NO
6	4	28	4	168.00	0.00	168.00	60.00	NO
6	5	28	4	252.00	0.00	252.00	60.00	NO
6	6	28	4	330.00	3.00	333.00	60.00	NO
6	7	28	4	420.00	0.00	420.00	60.00	NO
6	8	28	4	504.00	0.00	504.00	60.00	NO
7	3	28	4	98.00	0.00	98.00	84.00	NO
7	4	28	4	195.00	0.00	195.00	84.00	NO
7	5	28	4	294.00	0.00	294.00	84.00	NO
7	6	28	4	390.00	0.00	390.00	84.00	NO
7	7	28	4	480.00	6.00	486.00	84.00	NO
7	8	28	4	588.00	0.00	588.00	84.00	NO
8	3	28	4	112.00	0.00	112.00	112.00	NO
8	4	28	4	222.00	1.00	223.00	112.00	NO
8	5	28	4	336.00	0.00	336.00	112.00	NO
8	6	28	4	440.00	6.00	446.00	112.00	NO
8	7	28	4	555.00	1.00	556.00	112.00	NO
8	8	28	4	672.00	0.00	672.00	112.00	NO
9	3	28	4	126.00	0.00	126.00	144.00	YES
9	4	28	4	252.00	0.00	252.00	144.00	NO
9	5	28	4	378.00	0.00	378.00	144.00	NO
9	6	28	4	500.00	1.00	501.00	144.00	NO
9	7	28	4	630.00	0.00	630.00	144.00	NO
9	8	28	4	756.00	0.00	756.00	144.00	NO
10	3	28	4	140.00	0.00	140.00	180.00	YES
10	4	28	4	279.00	0.00	279.00	180.00	NO
10	5	28	4	420.00	0.00	420.00	180.00	NO
10	6	28	4	560.00	0.00	560.00	180.00	NO
10	7	28	4	690.00	6.00	696.00	180.00	NO
10	8	28	4	840.00	0.00	840.00	180.00	NO
11	3	28	4	154.00	0.00	154.00	220.00	YES
11	4	28	4	306.00	1.00	307.00	220.00	NO
11	5	28	4	462.00	0.00	462.00	220.00	NO
11	6	28	4	610.00	3.00	613.00	220.00	NO
11	7	28	4	765.00	1.00	766.00	220.00	NO
11	8	28	4	924.00	0.00	924.00	220.00	NO

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TABLE IX
GENERALIZATION OF BIBDS PRODUCED

Parent BIBD	LDs	BIBD Produced	$(v+1, b+r+\lambda+1, r+\lambda+1, k, \lambda+1)$
(15,140,28,3,4)	$LD(15,3,9,3)$,	(9,84,28,3,7),	(9,84,28,3,7),
	$LD(15,3,10,3)$,	(10,120,36,3,8),	(9+1,84+28+7+1,28+7+1,3,7+1),
	$LD(15,3,11,3)$	(11,165,45,3,9)	(10+1,120+36+8+1,36+8+1,3,8+1)
(7,7,3,3,1)	$LD(7,3,5,3)$,	(5,10,6,3,3),	(5,10,6,3,3)
	$LD(7,3,6,3)$,	(6,20,10,3,4),	(5+1,10+6+3+1,6+3+1,3,3+1)
	$LD(7,3,7,3)$	(7,35,15,3,5)	(6+1,20+10+4+1,10+4+1,3,4+1)

TABLE II
IMPLEMENTATION OF LI'S CONDITION FOR $(7,7,3,3,1)$ BIBD

P	T	R	L	COMBT	COMBP	COMTOT	COMBL	ANSW
3	3	3	1	4.00	0.00	4.00	3.00	NO
3	4	3	1	9.00	0.00	9.00	3.00	NO
3	5	3	1	12.00	0.00	12.00	3.00	NO
3	6	3	1	10.00	6.00	16.00	3.00	NO
3	7	3	1	15.00	3.00	18.00	3.00	NO
3	8	3	1	21.00	1.00	22.00	3.00	NO
4	3	3	1	6.00	0.00	6.00	6.00	NO
4	4	3	1	12.00	0.00	12.00	6.00	NO
4	5	3	1	18.00	0.00	18.00	6.00	NO
4	6	3	1	20.00	1.00	21.00	6.00	NO
4	7	3	1	30.00	0.00	30.00	6.00	NO
4	8	3	1	21.00	10.00	31.00	6.00	NO
5	3	3	1	7.00	0.00	7.00	10.00	YES
5	4	3	1	15.00	0.00	15.00	10.00	NO
5	5	3	1	18.00	3.00	21.00	10.00	NO
5	6	3	1	30.00	0.00	30.00	10.00	NO
5	7	3	1	30.00	3.00	33.00	10.00	NO
5	8	3	1	42.00	0.00	42.00	10.00	NO
6	3	3	1	9.00	0.00	9.00	15.00	YES
6	4	3	1	18.00	0.00	18.00	15.00	NO
6	5	3	1	24.00	1.00	25.00	15.00	NO
6	6	3	1	30.00	3.00	33.00	15.00	NO
6	7	3	1	45.00	0.00	45.00	15.00	NO
6	8	3	1	42.00	6.00	48.00	15.00	NO
7	3	3	1	10.00	0.00	10.00	21.00	YES
7	4	3	1	21.00	0.00	21.00	21.00	NO
7	5	3	1	30.00	0.00	30.00	21.00	NO
7	6	3	1	40.00	0.00	40.00	21.00	NO
7	7	3	1	45.00	3.00	48.00	21.00	NO
7	8	3	1	63.00	0.00	63.00	21.00	NO
8	3	3	1	12.00	0.00	12.00	28.00	YES
8	4	3	1	24.00	0.00	24.00	28.00	YES
8	5	3	1	36.00	0.00	36.00	28.00	NO
8	6	3	1	40.00	6.00	46.00	28.00	NO
8	7	3	1	60.00	0.00	60.00	28.00	NO
8	8	3	1	63.00	3.00	66.00	28.00	NO
9	3	3	1	13.00	0.00	13.00	36.00	YES
9	4	3	1	27.00	0.00	27.00	36.00	YES
9	5	3	1	36.00	3.00	39.00	36.00	NO
9	6	3	1	50.00	1.00	51.00	36.00	NO
9	7	3	1	60.00	3.00	63.00	36.00	NO
9	8	3	1	63.00	15.00	78.00	36.00	NO
10	3	3	1	15.00	0.00	15.00	45.00	YES
10	4	3	1	30.00	0.00	30.00	45.00	YES
10	5	3	1	42.00	1.00	43.00	45.00	YES
10	6	3	1	60.00	0.00	60.00	45.00	NO
10	7	3	1	75.00	0.00	75.00	45.00	NO
10	8	3	1	84.00	1.00	85.00	45.00	NO
11	3	3	1	16.00	0.00	16.00	55.00	YES
11	4	3	1	33.00	0.00	33.00	55.00	YES
11	5	3	1	48.00	0.00	48.00	55.00	YES
11	6	3	1	60.00	3.00	63.00	55.00	NO
11	7	3	1	75.00	3.00	78.00	55.00	NO
11	8	3	1	84.00	10.00	94.00	55.00	NO

TABLE III
COMBINATIONS GENERATED FROM $LD(15,3,9,3)$

Report on Intersections
$Comparison\ value\ p = 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ ,\ \ t = 3$

1 2 3	1 4 7	2 3 6	2 6 9	3 6 9	5 6 8
1 2 4	1 4 8	2 3 7	2 7 8	3 7 8	5 6 9
1 2 5	1 4 9	2 3 8	2 7 9	3 7 9	5 7 8
1 2 6	1 5 6	2 3 9	2 8 9	3 8 9	5 7 9
1 2 7	1 5 7	2 4 5	3 4 5	4 5 6	5 8 9
1 2 8	1 5 8	2 4 6	3 4 6	4 5 7	6 7 8
1 2 9	1 5 9	2 4 7	3 4 7	4 5 8	6 7 9
1 3 4	1 6 7	2 4 8	3 4 8	4 5 9	6 8 9
1 3 5	1 6 8	2 4 9	3 4 9	4 6 7	7 8 9
1 3 6	1 6 9	2 5 6	3 5 6	4 6 8	
1 3 7	1 7 8	2 5 7	3 5 7	4 6 9	
1 3 8	1 7 9	2 5 8	3 5 8	4 7 8	
1 3 9	1 8 9	2 5 9	3 5 9	4 7 9	
1 4 5	2 3 4	2 6 7	3 6 7	4 8 9	
1 4 6	2 3 5	2 6 8	3 6 8	5 6 7	

TABLE IV
COMBINATIONS GENERATED FROM $LD(15,3,10,3)$

Report on Intersections
$Comparison\ value\ p = 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ ,\ \ t = 3$

1 2 3	1 4 9	2 3 6	2 6 9	3 6 7	4 7 8	6 8 9
1 2 4	1 4 10	2 3 6	2 6 10	3 6 8	4 7 9	6 8 10
1 2 5	1 5 6	2 3 7	2 7 8	3 6 9	4 7 10	6 9 10
1 2 6	1 5 7	2 3 8	2 7 9	3 6 10	4 8 9	7 8 9
1 2 7	1 5 8	2 3 9	2 7 10	3 7 8	4 8 10	7 8 10
1 2 8	1 5 9	2 3 10	2 8 9	3 7 9	4 9 10	7 9 10
1 2 9	1 5 10	2 4 5	2 8 10	3 7 10	5 6 7	8 9 10
1 2 10	1 6 7	2 4 6	2 9 10	3 8 9	5 6 8	
1 3 4	1 6 8	2 4 7	3 4 5	3 8 10	5 6 9	
1 3 5	1 6 9	2 4 8	3 4 6	3 9 10	5 6 10	
1 3 6	1 6 10	2 4 9	3 4 7	4 5 6	5 7 8	
1 3 7	1 7 8	2 4 10	3 4 8	4 5 7	5 7 9	
1 3 8	1 7 9	2 5 6	3 4 9	4 5 8	5 7 10	
1 3 9	1 7 10	2 5 7	3 4 10	4 5 9	5 8 9	
1 3 10	1 8 9	2 5 8	3 5 6	4 5 10	5 8 10	
1 4 5	1 8 10	2 5 9	3 5 7	4 6 7	5 9 10	
1 4 6	1 9 10	2 5 10	3 5 8	4 6 8	6 7 8	
1 4 7	2 3 4	2 6 7	3 5 9	4 6 9	6 7 9	
1 4 8	2 3 5	2 6 8	3 5 10	4 6 10	6 7 10	

VII. CONCLUSION

In this study, we constructed some BIBDs from their parent BIBDs using Lotto Designs which were constrained on $v \geq k$, $v \geq p$, $t \leq min\{k, p\}$ and $k = t$. The BIBDs produced from these LDs follow the generalization $(v+1, b+r+\lambda+1, r+\lambda+1, k, \lambda+1)$ where (v, b, r, k, λ) are the parameters of the first BIBD produced. All the BIBDs produced are unreduced

TABLE V
COMBINATIONS GENERATED FROM $LD(15, 3, 11, 3)$

Report on Intersections
$Comparison\ value\ p = 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ ,\ \ t = 3$

1 2 3	1 5 9	2 4 6	3 4 5	3 10 11	5 7 8	8 9 11
1 2 4	1 5 10	2 4 7	3 4 6	4 5 6	5 7 9	8 10 11
1 2 5	1 5 11	2 4 8	3 4 7	4 5 7	5 7 10	9 10 11
1 2 6	1 6 7	2 4 9	3 4 8	4 5 8	5 7 11	
1 2 7	1 6 8	2 4 10	3 4 9	4 5 9	5 8 9	
1 2 8	1 6 9	2 4 11	3 4 10	4 5 10	5 8 10	
1 2 9	1 6 10	2 5 6	3 4 11	4 5 11	5 8 11	
1 2 10	1 6 11	2 5 7	3 5 6	4 6 7	5 9 10	
1 2 11	1 7 8	2 5 8	3 5 7	4 6 8	5 9 11	
1 3 4	1 7 9	2 5 9	3 5 8	4 6 9	5 10 11	
1 3 5	1 7 10	2 5 10	3 5 9	4 6 10	6 7 8	
1 3 6	1 7 11	2 5 11	3 5 10	4 6 11	6 7 9	
1 3 7	1 8 9	2 6 7	3 5 11	4 7 8	6 7 10	
1 3 8	1 8 10	2 6 8	3 6 7	4 7 9	6 7 11	
1 3 9	1 8 11	2 6 9	3 6 8	4 7 10	6 8 9	
1 3 10	1 9 10	2 6 10	3 6 9	4 7 11	6 8 10	
1 3 11	1 9 11	2 6 11	3 6 10	4 8 9	6 8 11	
1 4 5	1 10 11	2 7 8	3 6 11	4 8 10	6 9 10	
1 4 6	2 3 4	2 7 9	3 7 8	4 8 11	6 9 11	
1 4 7	2 3 5	2 7 10	3 7 9	4 9 10	6 10 11	
1 4 8	2 3 6	2 7 11	3 7 10	4 9 11	7 8 9	
1 4 9	2 3 7	2 8 9	3 7 11	4 10 11	7 8 10	
1 4 10	2 3 8	2 8 10	3 8 9	5 6 7	7 8 11	
1 4 11	2 3 9	2 8 11	3 8 10	5 6 8	7 9 10	
1 5 6	2 3 10	2 9 10	3 8 11	5 6 9	7 9 11	
1 5 7	2 3 11	2 9 11	3 9 10	5 6 10	7 10 11	
1 5 8	2 4 5	2 10 11	3 9 11	5 6 11	8 9 10	

TABLE VI
COMBINATIONS GENERATED FROM $LD(7, 3, 5, 3)$

Report on Intersections
$Comparison\ value\ p = 1\ 2\ 3\ 4\ 5\ ,\ \ t = 3$

1	2	3
1	2	4
1	2	5
1	3	4
1	3	5
1	4	5
2	3	4
2	3	5
2	4	5
3	4	5

TABLE VII
COMBINATIONS GENERATED FROM $LD(7, 3, 6, 3)$

Report on Intersections
$Comparison\ value\ p = 1\ 2\ 3\ 4\ 5\ 6\ ,\ \ t = 3$

1	2	3	2	3	4
1	2	4	2	3	5
1	2	5	2	3	6
1	2	6	2	4	5
1	3	4	2	4	6
1	3	5	2	5	6
1	3	6	3	4	5
1	4	5	3	4	6
1	4	6	3	5	6
1	5	6	4	5	6

TABLE VIII
COMBINATIONS GENERATED FROM $LD(7, 3, 7, 3)$

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$Comparison\ value\ p = 1\ 2\ 3\ 4\ 5\ 6\ 7\ ,\ \ t = 3$

1	2	3	2	3	7
1	2	4	2	4	5
1	2	5	2	4	6
1	2	6	2	4	7
1	2	7	2	5	6
1	3	4	2	5	7
1	3	5	2	6	7
1	3	6	3	4	5
1	3	7	3	4	6
1	4	5	3	4	7
1	4	6	3	5	6
1	4	7	3	5	7
1	5	6	3	6	7
1	5	7	4	5	6
1	6	7	4	5	7
2	3	4	4	6	7
2	3	5	5	6	7
2	3	6			

designs.

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