

# Multi-stage Directional Median Filter

Zong Chen and Li Zhang

**Abstract**—Median filter is widely used to remove impulse noise without blurring sharp edges. However, when noise level increased, or with thin edges, median filter may work poorly. This paper proposes a new filter, which will detect edges along four possible directions, and then replace noise corrupted pixel with estimated noise-free edge median value. Simulations show that the proposed multi-stage directional median filter can provide excellent performance of suppressing impulse noise in all situations.

**Keywords**—Impulse noise, Median filter, Multi-stage, Edge-preserving

## I. INTRODUCTION

IMPULSE noise is often introduced into digital images during image acquisition and/or transmission. It is important to remove noise in the images before subsequent processing, such as edge detection, image segmentation, and object recognition. Many impulse noise removal techniques have been developed. Among them, median filter has attracted much attention because of its effective noise suppression capability and high computational efficiency [1].

Conventional median filtering approaches apply the median operation to each pixel unconditionally, that is, without considering whether it is uncorrupted or corrupted. As a result, the image details contributed from the uncorrupted pixels are still subject to be filtered, and this causes image quality degradation. An intuitive solution to overcome this problem is to implement an impulse-noise detection mechanism prior to filtering; hence, only those pixels identified as “corrupted” would undergo the filtering process, while those identified as “uncorrupted” would remain intact. By incorporating such noise detection mechanism into the median filtering framework, the so-called “switching” median filters such as [2]–[6] had shown significant performance improvement. The drawback of these algorithms is that the noisy pixels are replaced by some median value in their vicinity without taking into account local features such as the possible presence of edges. Hence details and edges are not recovered satisfactorily.

For images corrupted by Gaussian noise, least-squares methods based algorithms [7]–[10] have been used successfully to preserve the edges and details in images. These

methods fail in the presence of impulse noise because the noise is heavily tailed. Moreover, the restoration will alter all pixels in the image, including those noise free pixels [11].

In this paper, we propose a multi-stage directional median filter. The noise pixels are identified in the first stage. Then edges are detected in the second stage. In the third stage, noise pixels are restored along detected edges. The algorithm is expressed in Section II. Simulation is expressed in Section III. Experiment shows that the multi-stage directional median filter can remove most noise while preserving detail features, even single pixel width lines.

## II. MULTI-STAGE DIRECTIONAL MEDIAN FILTER

Impulse noise is caused by errors in the data transmission. The corrupted pixels are either set to the maximum value by positive impulse (which looks like white dot or snow in the image) or set to zero value by negative impulse (which looks like black dot or pepper in the image), giving the image a “salt and pepper” like appearance. Unaffected pixels always remain unchanged. The noise is usually quantified by the percentage of pixels which are corrupted. For example, if an image is corrupted by  $R=30\%$  impulse noise, then 15% of the pixels in the image are corrupted by positive impulses and 15% of the pixels corrupted by negative impulses.

Let  $x(i,j)$ , where  $i,j \in I \equiv \{1, 2, \dots, M\} \times \{1, 2, \dots, N\}$ , be the gray level of a  $M$ -by- $N$  image at location  $(i,j)$ , and  $[Z_{\min}, Z_{\max}]$  be the dynamic range of  $x$ , i.e.,  $Z_{\min} \leq x(i,j) \leq Z_{\max}$  for all  $i,j \in I$ . Denote  $y$  as a noisy image. Then,

$$y(i,j) = \begin{cases} Z_{\min}, & \text{with probability } p \\ Z_{\max}, & \text{with probability } q \\ x(i,j), & \text{with probability } 1-p-q \end{cases}$$

Since salt-and-pepper noise only set corrupted pixels to extreme values, pixels from noisy image equal to  $Z_{\min}$  or  $Z_{\max}$  can be considered as noise candidates. A binary flag map is generated during the impulse detection procedure. Let  $y(i,j)$  represents the pixel value on position of  $(i,j)$  in the input image, and  $f(i,j)$  represents the flag value on position of  $(i,j)$  in the resulted binary flag map. For each pixel  $y(i,j)$ , if  $y(i,j) = Z_{\max}$  or  $y(i,j) = Z_{\min}$ , we set  $f(i,j) = 1$ . Otherwise,  $y(i,j)$  is considered as a noise free pixel and we set  $f(i,j) = 0$ .

If the input pixel is classified as an impulse noise according to the binary noise flag map  $f(i,j)$ , the pixel value then should be replaced by an estimated value. Otherwise, its original intensity is the output. The estimated value should be calculated only from noise-free pixels within the filter window.

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Most median based filters use median value or weighted median value from the whole window as the estimated value. However, since the noisy pixels are replaced by median value in their vicinity without taking into account local features such as the possible presence of edges, details and edges are not recovered satisfactorily.

As suggest by [12], edges are usually aligned along four directions. Let  $D_k$  ( $k = 1$  to 4) denotes a set of coordinates aligned with the  $k$ th direction centered at  $(0, 0)$ , i.e.,  $D_1 = \{(-2, -2), (-1, -1), (0, 0), (1, 1), (2, 2)\}$ ,  $D_2 = \{(0, -2), (0, -1), (0, 0), (0, 1), (0, 2)\}$ ,  $D_3 = \{(2, -2), (1, -1), (0, 0), (-1, 1), (-2, 2)\}$ ,  $D_4 = \{(-2, 0), (-1, 0), (0, 0), (1, 0), (2, 0)\}$ , as shown in Fig. 1.

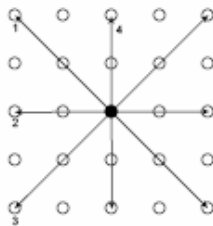


Fig. 1. Four directions for edges

If there is one edge along  $D_1$ , then all pixels along  $D_1$  should be similar, while pixels along other directions should be different. Since standard deviation describes how tightly all the value are clustered around the mean in the set of pixels, it can be used to calculate the similarity along directions. Standard deviations ( $S_k(i, j)$ ,  $k = 1$  to 4) and median values ( $M_k(i, j)$ ,  $k = 1$  to 4) along 4 directions need to be calculated. Smallest standard deviation along  $D_1$  suggests a possible diagonal edge existed in window. Therefore, median value of pixels along that direction in filter window,  $M_1(i, j)$ , is used as the estimated value to replace the center noise pixel.

This algorithm can detect most edges but have problem for thin edges especially single pixel width edges. An example is shown in Figure 2. In a  $5 \times 5$  filter window, there is one edge along  $D_1$ , and all other pixels belong to a flat background. The center pixel is corrupted by positive impulse and should be replaced by an estimated value. Since the estimated value should be calculated only from noise-free pixels within the filter window, the center pixel is excluded from calculation. Along  $D_1$ , there are four dark gray pixels, thus  $M_1 =$  dark gray, and  $S_1 = 0$ . Along  $D_2$ , there are four light gray pixels, thus,  $M_2 =$  light gray, and  $S_2 = 0$ . And along  $D_3$  and  $D_4$ ,  $M_3 = M_4 =$  light gray, and  $S_3 = S_4 = 0$ . Since  $S_1 = S_2 = S_3 = S_4 = 0$ , the above directional algorithm will fail to find the edge.



Fig. 2. Noise on single pixel width edge

Background information should be considered in the edge detection procedure. In Figure 1, 17 pixels are used to detect edge along four possible directions. The other 8 pixels,  $\{(-2, 1), (-2, -1), (2, -1), (2, 1), (-1, -2), (-1, 2), (1, -2), (1, 2)\}$  are not used. Median value of these 8 pixels can represent the background information  $B(i, j)$ . Standard deviations ( $S_k(i, j)$ ,  $k = 1$  to 4) and median values ( $M_k(i, j)$ ,  $k = 1$  to 4) along 4 directions still need to be calculated. Smallest standard deviation suggests a possible edge direction. While having more than one directions with same or similar standard deviation, the difference between  $B(i, j)$  and  $M_k(i, j)$  is calculated. Since edge should be different from background, bigger difference suggests a possible edge direction.

With the example in Figure 2, center pixel is noise corrupted and excluded from calculation. Standard deviation along all 4 directions are same,  $S_1 = S_2 = S_3 = S_4 = 0$ . Therefore, background information will be included to detect edges. The differences between  $B(i, j)$  and  $M_1(i, j)$  is bigger than the differences between  $B(i, j)$  and  $M_k(i, j)$ ,  $k = 2, 3, 4$ , thus the edge along  $D_1$  is identified

In the last step, median filter will be applied on noise-free pixels along identified edge.

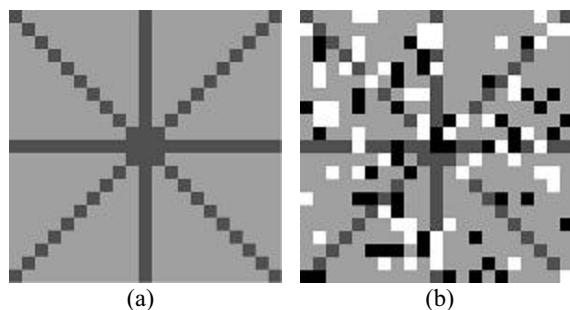
### III. SIMULATION

In our experiment, the original test images are corrupted with salt-and-pepper impulse noise. The Peak signal-to-noise ratio (PSNR) is used to evaluate the restoration performance.

$$PSNR = 10 \log_{10} \frac{\max\{o(i, j), (i, j) \in I\}^2}{\frac{1}{MN} \sum_{(i, j) \in I} [r(i, j) - o(i, j)]^2} \text{ (dB)}$$

where  $r(i, j)$  and  $o(i, j)$  denote the pixel values of the restored image and the original image respectively, and the image size is  $M \times N$ .

Figure 3(a) shows an original image ( $21 \times 21$ ) with single pixel width edges. Figure 3(b) is Figure 3(a) corrupted by 30% impulse noise. Figure 3(c) shows the conventional median filter result. Figure 3(d) shows the directional weighted median filter result. Figure 3(e) shows the adaptive median filter result. Figure 3(f) shows the multi-stage directional median filter result.



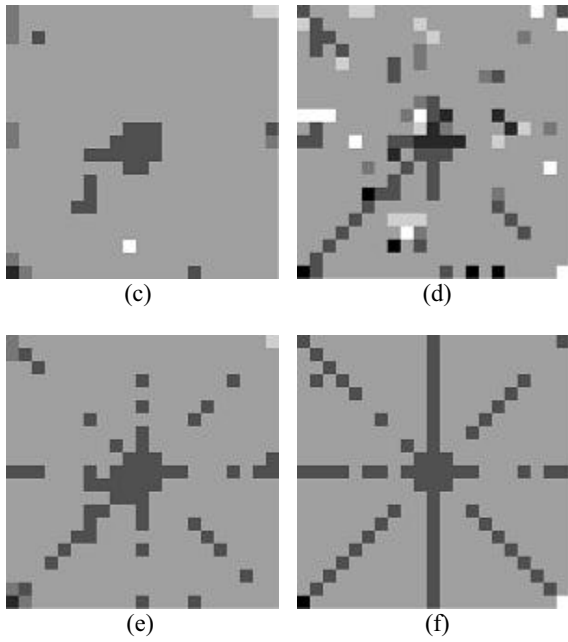


Fig. 3. Test results for image with single pixel width edges

The PSNR for conventional median filter is 16.69 dB. The PSNR for directional weighted median filter is 15.56 dB. The PSNR for adaptive median filter is 19.77 dB. The PSNR for the multi-stage directional median filter is 22.84 dB. The result shows that the multi-stage-directional median filter is the best. Adaptive median filter is the second. Even the directional weighted median filter produces higher PSNR than conventional median filter, Figure 3(d) looks better than Figure 3(c).

Figure 4(a) shows an original image with thin edges. Figure 4(b) is Figure 4(a) corrupted by 30% impulse noise. Figure 4(c) shows the conventional median filter result. Figure 4(d) shows the directional weighted median filter result. Figure 4(e) shows the adaptive median filter result. Figure 4(f) shows the multi-stage directional median filter result.

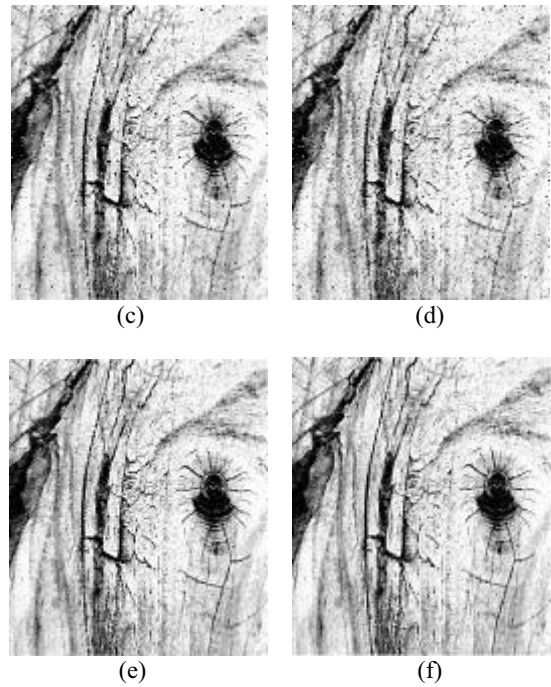
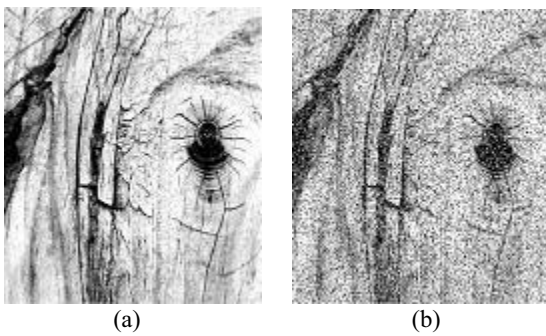


Fig. 4. Test results for wood image with thin edges

Table 1 shows the restoration results when the original image corrupted by different noise levels. The first column is the restoration result for conventional median filter. The second column is the restoration result for directional weighted median filter. The third column is the restoration result for adaptive median filter. The fourth column is the restoration result for multi-stage directional median filter. It is clear when noise level lower than 50%, the multi-stage directional median filter performs better than all other filters. When noise level is higher than 50%, the multi-directional median filter performs closely to adaptive median filter, and much better than the others.

TABLE I RESTORATION RESULTS OF WOOD IMAGE IN PSNR (dB)

Noise Level	CM	DWM	AM	MDM
10%	17.48	20.63	20.66	24.87
20%	16.79	17.00	19.89	22.28
30%	15.73	14.46	18.97	20.50
40%	14.01	12.22	17.90	18.87
50%	12.12	10.40	16.90	17.40
60%	10.14	8.84	15.83	15.91
70%	8.42	7.56	14.74	14.43
80%	6.89	6.40	13.05	12.63
90%	5.53	5.38	9.59	9.95

#### IV. CONCLUSION

A multi-stage directional median filter is proposed in this paper. Impulse noise is detected in the first stage. After noise detection, edges are identified along four directions. Median

filter is applied only along edge directions to preserve the edge information. Simulations results show that the multi-stage directional median filter performs better than other classical median based algorithms in both subject and objective evaluations.

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