Accelerating GLA with an M-Tree

Olli Luoma, Johannes Tuikkala, and Olli Nevalainen

Abstract—In this paper, we propose a novel improvement for the generalized Lloyd Algorithm (GLA). Our algorithm makes use of an M-tree index built on the codebook which makes it possible to reduce the number of distance computations when the nearest code words are searched. Our method does not impose the use of any specific distance function, but works with any metric distance, making it more general than many other fast GLA variants. Finally, we present the positive results of our performance experiments.

Keywords—Clustering, GLA, M-Tree, Vector Quantization .

I. Introduction

Clustering - a process of classifying objects into groups according to their similarity [1] - has been extensively studied and put to use in many different application areas, such as data mining [2] and pattern recognition [3]. The importance of clustering has also passed over to the area of data compression where clustering is used as a means for codebook generation in vector quantization [4]. In this setting, clustering is used to find a given number of *code vectors*, in other words, a *codebook*, for a given set of *training vectors* by minimizing the average pairwise distance between the training vectors and their representative code vectors.

In this paper, we concentrate on the widely known *generalized Lloyd algorithm* (GLA). The algorithm starts with an initial, e.g., random, codebook which is iteratively improved until some convergence condition is met. Each iteration of GLA consists of two steps, the *partition step* in which each training vector is assigned to its closest code vector, and the *codebook step* in which the code vectors are updated based on the partitioning found in the partition step. If GLA is straightforwardly implemented, every partition step requires $n \cdot m$ distance computations, where n denotes the size of the codebook and m the number of training vectors.

Our idea is to reduce the number of distance calculations by building an M-tree [5] over the codebook and using this tree to find the nearest code vector. Since the nearest code vector for any given training vector can now be found with a logarithmic number of distance computations, only $O(\log(n)) \cdot m$ distance computations are needed in the partition step. Obviously, this method yields better results as the size of the codebook increases, and thus our method is at its best in clustering tasks involving relatively high number of clusters.

Our method also has the additional advantage of being more general than many other GLA variants. Many other methods which force the use of Euclidean or otherwise restricted distance function, but here, the distance function is only assumed to be *metric*, i.e., to satisfy the non-negativity, symmetry, and triangle inequality postulates.

The remainder of this paper is organized as follows. In section II, we briefly review the related work, and in section III, we present some basic concepts related to M-trees. Our clustering method is presented in section IV and the results of our performance evaluation in section V. Section VI concludes this article and discusses our future work.

II. Related Work

Index structures have previously been used to speed up GLA by building an index on training vectors and using simple geometric reasoning to reduce the number of distance calculations [6, 7, 8]. The use of an kd-tree or other spatial access methods, however, imposes the distance metric to be an L_P norm ruling out the use of more sophisticated distance functions, such as Levenshtein distance or Mahalanobis distance. Furthermore, as observed in [8], the performance of these methods is seriously degraded as the dimensionality of the data increases.

The *partial distortion search* (PDS) [9] aims at reducing the number of distance calculations by computing the distance between a code vector candidate and a training vector cumulatively by summing up the squared differences in each dimension. If the cumulative distance exceeds the distance between the training vector and the closest code vector found thus far, the code vector candidate is rejected. The *mean-distance-ordered partial search* (MPS) [10] utilizes a less expensive distance function to find a lower bound for the distance between a code vector candidate and a training vector. If this value is greater than the current minimum distance the candidate is rejected. Since most of the information needed to calculate the lower bound can be precalculated, this method can reduce the runnig time substantially. However, MPS can only be applied if Euclidean distance metric is used.

The *triangular inequality elimination* technique (TIE) [11], on the contrary, does not force the use of any specific distance

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O. Luoma is with TUCS (Turku Center for Computer Science) and Department of Information Technology, University of Turku, Lemminkäisenkatu 14 A, FIN-20520 Turku, FINLAND (phone: 358-(0)2-3338666; fax: 358-(0)2-3338600; e- mail: olli.luoma@it.utu.fi).

J. Tuikkala is with TUCS and Department of Information Technology, University of Turku, FINLAND (e-mail: johannes.tuikkala@it.utu.fi).

O. Nevalainen is with TUCS and Department of Information Technology, University of Turku, FINLAND (e-mail: olli.nevalainen@it.utu.fi).

function, and thus we regard TIE as the most relevant GLA variant to our paper. In TIE, it is assumed that the distance function is metric, and hence the distance calculation between a code vector C_i and a training vector T_j can be avoided if

$$d(C_i, C_a) > 4 \cdot d(T_j, C_a) \qquad (1)$$

where $d(O_x, O_y)$ denotes the distance between O_x and O_y and C_a denotes the nearest code vector found thus far. A practical implementation of TIE utilizes a matrix of the distances between all code vectors which is updated at the beginning of each partition step. Since updating the matrix requires $n(n-1)/2$ distance calculations, where n denotes the size of the codebook, the performance of TIE significantly degrades when the number of clusters increases.

The *code vector activity detection* proposed by Kaukoranta *et al.* [12] is based on the concepts of *active* and *static code vectors*. If a training vector is assigned to a static code vector, i.e., a code vector which was not changed in the last codebook step, only the distances to active code vectors, i.e., code vectors which were changed in the last codebook step, have to be computed. This method can be applied to a wide range of GLA variants, including the method proposed in this paper.

III. METRIC SPACES AND M-TREES

A *metric space* is defined as a pair $M = (D, d)$, where D is a domain of feature values and $d : D \times D \to R^+$ is a distance function such that for all $O_x, O_y, O_z \in D$:

$$d(O_x, O_y) = 0 \Leftrightarrow O_x = O_y \qquad (2)$$
$$d(O_x, O_y) = d(O_y, O_x) \qquad (3)$$
$$d(O_x, O_y) \leq d(O_x, O_z) + d(O_z, O_y) \qquad (4)$$

Metric spaces can be indexed using so called metric trees [13] which only consider the relative distances between objects. One metric tree structure is the M-tree proposed by Ciaccia *et al.* [5]. In an M-tree, all indexed objects reside on the leaf level. Each inner node stores a *routing object* and its *covering radius*, i.e., the maximum distance between the routing object and the objects residing in the subtrees corresponding to the routing object.

For accessing the indexed objects, M-tree provides two search methods. In the *range query*, the query object and the maximum distance are specified, and in the *k nearest neighbors query*, the query object and the cardinality of the result set are the input parameters. Both types of queries start from the root and recursively traverse all the paths which cannot be excluded from the search. For our purposes, it is sufficient to say that in most cases, the number distance computations involved in both types of queries grows logarithmically with respect to the size of the tree. This, of course, is typical to tree structures, since the heigth of the tree also grows logarithmically with respect to tree size. For a detailed description of the M-tree, we refer the reader to [5].

IV. OUR ALGORITHM

The main intuition behind our algorithm is very simple. By building an M-tree over the codebook at the start of each partition step, we can expect to find the nearest code vector for each of the training vectors with a logarithmic number of distance computations. Of course, the building of the M-tree introduces some overhead which in typical clustering tasks, however, is negligible since the number of clusters compared to the number of training vectors is typically relatively small. This is evident in the results of our experimental evaluation presented in section VI.

Our algorithm is presented in Fig. 1. Operation clear empties the M-tree and operation kNN(O,k) returns k nearest neighbors of O. Each partition step is preceded by building of a new M-tree over the codebook which takes approximately $n \cdot \log(n)$ distance computations. We then use the M-tree to find the nearest code vector for each of the training vectors by issuing a nearest neigbor query. This can be done using $O(\log(n)) \cdot m$ distance computations. After this, the codebook is updated and the improvement of the partition checked.

M-TREE-GLA(T)
IN: Training set T .
  Generate codebook C by any algorithm;
  REPEAT
    tree.clear;
    FOR EACH $C_i \in C$ DO
      tree.insert(C_i);
    FOR EACH $T_i \in T$ DO
      T_i .cluster =tree.kNN(T_i ,1);
    FOR EACH $C_i \in C$ DO
      C_i .update;
  UNTIL no improvement achieved

Figure 1. Pseudo-code of our algorithm.

V. EXPERIMENTAL RESULTS

We evaluated the performance of our method by performing codebook generation tasks on three standard CCIT test images presented in Fig. 2 - Fig. 4, with varying number of clusters. The size of all images was 256x256 pixels and the training vectors were 4x4 pixel blocks from the images. Thus, the number of training vectors in all cases is 4096. We also implemented the simple GLA and TIE as proposed in [11]. All algorithms were implemented using Java.

Fig. 3 illustrates the average number of distance computations per training vector per iteration for these three images using six different codebook sizes. The $n \cdot (n-1)/2$ distance computations needed to update the distance matrix in TIE and the $n \cdot \log(n)$ distance computations needed to build the tree in our M-tree variant are included in the results.

Fig. 5 clearly shows that the performance of TIE degrades significantly as the number of clusters inceases. In the case of 1024 clusters, for example, updating the distance matrix requires 524288 distance computations at the beginning of

each partition step, whereas building the M-tree requires approximately 10000 distance computations.

Fig. 6 illustrates the average running time per training vector per iteration. These results include not only the time needed for the distance computations, but also the time needed to sort the rows of the distance matrix in TIE, and thus TIE performs even worse. Overall, our M-tree variant clearly outperforms both TIE and simple GLA in clustering tasks involving a large number of clusters.

Figure 2. Bridge (256x256).

Figure 3. Camera (256x256).

Figure 4. Couple (256x256).

VI. CONCLUSION AND FUTURE WORK

We introduced an improvement to GLA which utilizes an M-tree built on the codebook. Unlike many other methods, our variant does not force the use of any specific distance function, which makes it more general than many other methods. We also presented the results of our performance evaluation which suggested that the M-tree variant can outperform the TIE method proposed by Chen and Tsieh [11].
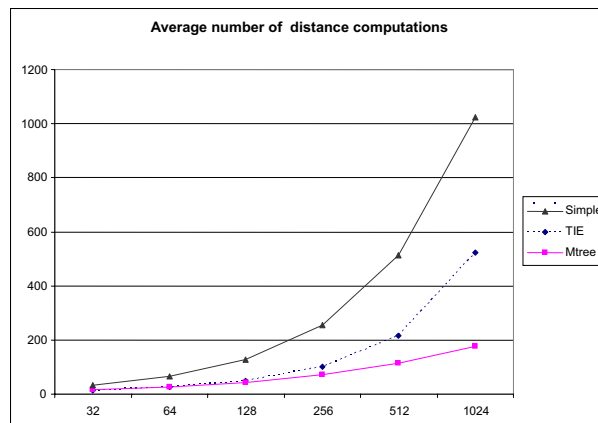

Figure 5. The average number of distance calculations per training vector per iteration. The results are averages for Bridge, Camera, and Couple, 10 runs for each image.
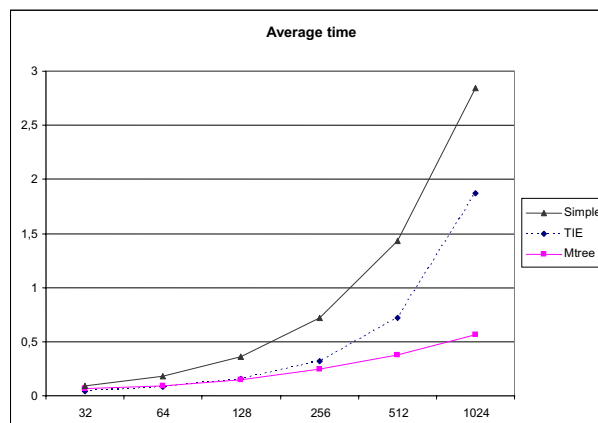

Figure 6. The average running time (in milliseconds) per training vector per iteration. The results are averages for Bridge, Camera, and Couple, 10 runs for each image.

Our method provides good performance especially when the number of clusters is relatively high, and thus it suits well to codebook generation in vector quantization, for example. However, our algorithm still has to visit every training vector. We plan to tackle this problem by building an index also on the training vectors, which makes it possible to assign a large number of training vector to a cluster at the same time [6, 7, 8]. We are also going to efficiently incorporate the code vector activity detection [12] in our method, since the most

obvious solution, building separate M-trees for active code vectors and for all code vectors, introduces some unnecessary overhead.

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