

# Project Selection by Using a Fuzzy TOPSIS Technique

M. Salehi, and R. Tavakkoli-Moghaddam

**Abstract**—Selection of a project among a set of possible alternatives is a difficult task that the decision maker (DM) has to face. In this paper, by using a fuzzy TOPSIS technique we propose a new method for a project selection problem. After reviewing four common methods of comparing investment alternatives (net present value, rate of return, benefit cost analysis and payback period) we use them as criteria in a TOPSIS technique. First we calculate the weight of each criterion by a pairwise comparison and then we utilize the improved TOPSIS assessment for the project selection.

**Keywords**—Fuzzy Theory, Pairwise Comparison, Project Selection, TOPSIS Technique.

## I. INTRODUCTION

ENGINEERING economics is the specialized study of financial and economic aspects of the industrial decision making. The role of engineering economics is to assess the appropriateness of a given project, estimate its value, and justify it from an engineering point of view. The main objective is to determine the "best" project(s). The project proposals may be intended for strategic R&D planning (selection of directions, topics, or projects), the development of new commercial products, the management and the implementation of organizational change, the management, the development and the implementation of information technology and the like.

There is a comprehensive literature dedicated to a project selection problem that includes several approaches taking into account various aspects of the given problem. Strategic intent of the project, factors for the project selection, and various qualitative and quantitative project selection models have been thoroughly discussed by Meredith and Mantle [1].

Danila [2] and Shpak and Zaporozhan [3] surveyed a number of the project selection methodologies and discussed several multi-criteria aspects of the problem. Mehrez and Sinuany-Stern [4] formulated a project selection problem as a multi-criteria decision making (MCDM) problem and applied a utility function. Khorramshahgole and Steiner [5] used goal programming associated to a Delphi process for finding the utility map. Chu et al. [6] used a heuristic method based on the

fuzzy logic for ranking projects. The problem for the optimal project funding implies decisions on the new projects and on the projects to be continued. The decision on how to allocate the financial resources between these two types of projects is very important issue studied by Baker and Freeland [7]. Lockett and Stratford [8] presented several 0-1 mathematical programming models which take into account the hierarchical decisions and the fund allocation problem between independent projects. A different approach is based on the reference point and reference level by Lewandowski and Grauer [9] and Wierzbicki [10]. The reference level is represented by a set of performance measures, which are associated to each attribute. The basic idea of the method is to find the nearest feasible non-dominated solution from the point defined by reference levels. Ghasemzadeh, et al. [11] proposed a 0-1 integer linear programming model for selecting and scheduling an optimal project portfolio, based on the organization's objectives and constraints. Gabriel, et al. [12] formulated a multi-objective, integer-constrained optimization model with competing objectives for the project selection by using probability distributions in order to describe costs. Analytical hierarchy process (AHP) has been used by many authors to resolve decision-making issues in the project selection (Dey and Gupta, [13]; Mian and Christine, [14]). Eddie, et al. [15] applied the analytic network process (ANP) to deal with interdependent relationships within an MCDM model. Mohanty, et al. [16] illustrated an application of the fuzzy ANP along with the fuzzy cost analysis in selecting R&D projects. In this approach, triangular fuzzy numbers are used for the preferences of one criterion over another then by using a pairwise comparison with the fuzzy set theory, in which weight of each criterion in the format of triangular fuzzy numbers is calculated. Then the appropriateness of a given project is assessed by using optimistic, likely and pessimistic estimates for each criterion with the fuzzy TOPSIS technique.

The fuzzy TOPSIS is the fuzzy extension of TOPSIS to efficiently handle the fuzziness of the data involved in the decision making. It is easy to understand and it can effectively handle both qualitative and quantitative data in the multi-attribute decision making (MADM) problems.

Other sections of the article are as follows: In Section II, criteria for the project selection have been mentioned. In Section III, the fuzzy set theory explains. In Section IV, we present our methodology. Finally, concluding remarks are provided in Section V.

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## II. THE COMMON METHODS OF COMPARING ALTERNATIVES

The main reasons are given below why \$1000 today is "worth" more than \$1000 one year from today:

1. Inflation
2. Risk
3. Cost of money

Of these, the cost of money is the most predictable, and, hence, it is the essential component of economic analysis. Cost of money is represented by: 1) Money paid for the use of borrowed money; or 2) return on investment. Cost of money is determined by an interest rate. Time value of money is defined as the time-dependent value of money stemming both from changes in the purchasing power of money (inflation or deflation) and from the real earning potential of alternative investments over time [4].

The economic and financial analysis of the project is based on the comparison of the cash flow of all costs and benefits resulting from the project's activities. There are four common methods of comparing alternative investments: 1) Net present value; 2) rate of return; 3) benefit-cost analysis; and 4) pay back period. Each method is dependent on a selected interest rate or discount rate to adjust cash flows at different points in time [8].

### A. Net Present Value

A net present value (NPV) is the present value of future cash inflows minus the cost including cost of investment calculated using an appropriate discounting method. Annual costs, future payments, and gradients should be brought to the present. Converting all cash flows to the present worth is often referred to as discounting. A zero NPV means the project repays original investment plus the required rate of return. A positive or a negative NPV means a better or worse return, respectively, than the return from zero NPV.

The advantages of the NPV method are as follows:

- It gives the correct decision advice assuming a perfect capital market. It also gives correct ranking for mutually exclusive projects.
- It takes into account the time value of money.
- The NPV gives an absolute value.
- Limitations of the NPV.
- It is often difficult to predict future cash flows with certainty.
- It is very difficult to identify the correct discount rate.
- The NPV as a method of investment appraisal requires the decision criteria to be specified before the appraisal can be undertaken.

### B. Rate of Return

A method to analyze investments reflects and accounts for the time value of money. Internal rate of return (IRR) is the discount rate which makes the net present value of revenue flows equal to zero or the investment equal to the present value of revenue flows. If more than one interest factor is

involved, the solution is by trial and error. The calculated interest rate may be compared to a discount rate identified as the "minimum attractive rate of return (MARR)" or to the interest rate yielded by alternatives. Rate-of-return (ROR) analysis is useful when the selection of a number of projects is to be undertaken within a fixed or limited capital budget.

The advantages of the ROR method are as follows:

- Knowing a return is intuitively appealing
- It is a simple way to communicate the value of a project to someone who does not know all the estimation details
- If the IRR is high enough, you may not need to estimate a required return, which is often a difficult task
- Limitations of ROR
- It does not help much in ranking projects of differing sizes or levels of investments.
- Non-conventional cash flows produce multiple RORs

### C. Benefit-Cost Analysis

A benefit-cost analysis is a systematic evaluation of the economic advantages (benefits) and disadvantages (costs) of a set of investment alternatives. Benefit-cost (B/C) analysis is a method of comparison, in which the consequences of an investment are evaluated in monetary terms and divided into the separate categories of benefits and costs. The values are then converted to annual equivalents or present worth for comparison.

### D. Payback Period

The value of time takes to break even on an investment. Since this method ignores the time value of money and cash flows after the payback period, it can provide only a partial picture of whether the investment is worthwhile. The use of the payback period as a capital budgeting decision rule specifies that all independent projects with a payback period less than a specified number of years should be accepted. When choosing among mutually exclusive projects, the project with the quickest payback is preferred.

The advantages of the payback period method are as follows:

- Easy to understand
- Does not accept negative estimated NPV investments when all future cash flows are positive
- Biased towards liquidity
- Limitations of the payback period method
- It ignores the time value of money
- It requires an arbitrary cut-off point
- Ignores cash flows beyond the cutoff point
- Biased against long term projects, such as R&D and new products

Before we describe methodology, we explain the fuzzy set theory.

### III. FUZZY SET THEORY

From long ago, it was believed that the value of a predicate depends on its "true" or "false" states, not on the both. Accordingly, a member is either belongs to a set or it doesn't belong to the set. Since Aristotle, the question has been posed whether there were predicates which valued other than "true" or "false". We see that it reflects some fuzzy logic. So the fuzzy theory has some previous records.

Fuzzy sets theory proposed formally for the first time by Lotfi Asgarzadeh, from University of California in Barkley. He discussed the theory in Control and Information Journal in 1969. The theory has been expanded and deepened a lot since its first appearance and has been applied in many areas.

In the fuzzy sets, the degree of membership is unclear, such as, the set of people who are tall or the set of cardinal numbers. Asgarzadeh analyzed the set by attributing membership degree in range of [0, 1] to the members. For example, who is 170cm or 180cm, so on is considered as a member of tall person with a given degree of membership. Thus the degree of membership in the tall of set the persons who are taller than 180cm is (0.8) for those who are taller than 170cm is (0.7).

If  $U$  is a reference set with some members ( $x$ ), fuzzy set in  $U$  is shown using ordered pairs, such that

$$A = \{(x, \mu_A(x)) | x \in U\}$$

where,  $\mu_A(x)$  is a membership function or degree of membership which offers how much ( $x$ ) belongs to fuzzy set of  $A$ . The real numbers function range is none-negative that has some maximum and in normal state is considered as a close distance [0, 1]. It is worthy noting that there is no certain way to show the membership function and it is mostly experimental and perceptual.

A convex and normal fuzzy set, such as  $A$  with real numbers range of ( $R$ ) is a real fuzzy number if:

- 1- There is only one  $x_0 \in R$  for which we have  $\mu_A(x) = 1$
- 2- Membership function of  $\mu_A(x)$  is a continuous one.

Fuzzy numbers measurement due to their proper structure is time consuming and complex. Proper fuzzy numbers are used to facilitate the calculations. They include bell from, triangular and trapezoid, triangular L-R and trapezoid L-R. We use triangular fuzzy numbers in this study because of their simple from of measurement. A triangular fuzzy number is shown as tree ordered items (l, m, u) (Fig. 1) that l and u are lower and upper bounds, m is the mean and ( $x$ ) is between l and u.

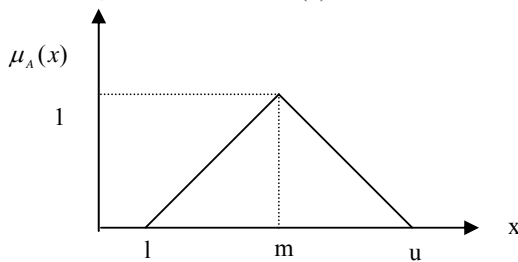


Fig. 1 Triangular numbers

A membership of fuzzy numbers is given in Equation (1).

$$\mu_A(x) = \begin{cases} \frac{x-l}{m-l} & l < x < m \\ 1 & x = m \\ \frac{u-x}{u-m} & m < x < u \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

With the use of the profile concept, we can create a relation as follows between normal and fuzzy numbers. The subset of  $U$  whose degree of membership in fuzzy of  $A$  is at least  $\alpha$  is called "profile- $\alpha$ " and is shown by  $A_\alpha$ :

$$A_\alpha = \{x \in U | \mu_A(x) \geq \alpha\}$$

Robust profile of  $\alpha$  with a robust set of  $\alpha$  is defined as follows:

$$A_\alpha = \{x \in U | \mu_A(x) > \alpha\}$$

To change a fuzzy number to a certain value, there are various methods, such as: gravity point, maximum membership function, giving priority to left and right side of fuzzy number, so on.

We use priority to the left and right side of the fuzzy number in this study so we give its details.

In this method, the total score of a fuzzy number of  $A$  is obtained from adding left and right scores of  $A$ . The left and right scores, in turn, are obtained from two specific sets of (min) and (max) and membership degree of fuzzy number.

(Min) and (max) sets are measured as follows using the given range of fuzzy numbers [0, 1]:

$$\mu_{\min}(x) = \begin{cases} 1-x; & 0 \leq x \leq 1 \\ 0; & \text{otherwise} \end{cases} \quad (2)$$

$$\mu_{\max}(x) = \begin{cases} x; & 0 \leq x \leq 1 \\ 0; & \text{otherwise} \end{cases} \quad (3)$$

where the left scale of  $A$  is obtained by Equation (4).

$$\mu_L(x) = SUP[\mu_{\min}(x) \wedge \mu_x(x)] \quad (4)$$

The right scale of  $A$  is obtained from Equation (5).

$$\mu_R(x) = SUP[\mu_{\max}(x) \wedge \mu_x(x)] \quad (5)$$

After obtaining the scales, we can measure the total scale from Equation (6) which is used later as a proper scale.

$$\mu_T(x) = \frac{\mu_R(x) + 1 - \mu_L(x)}{2} \quad (6)$$

A triangular fuzzy set, such as  $A = (\alpha, \beta, \gamma)$  is given. Fig. 2 shows the right and left scales graphically.

The membership function of fuzzy number of  $A$  is in from of Equation (7):

$$\mu_A(x) = \begin{cases} \frac{x - (m - \alpha)}{\alpha}; & m - \alpha < x < m \\ \frac{(m + \beta) - x}{\beta}; & m < x < m + \beta \end{cases} \quad (7)$$

The right and left scale of a fuzzy number of  $A$  is given by:

$$\begin{aligned} \frac{x_1 - (m - \alpha)}{\alpha} &= 1 - x_1 \Rightarrow x_1 = \frac{m}{1 + \alpha} \Rightarrow \\ \mu_L(A) &= 1 - x_1 = 1 - \frac{m}{1 + \alpha} \\ \frac{(m + \beta) - x_2}{\beta} &= x_2 \Rightarrow x_2 = \frac{m + \beta}{1 + \beta} \Rightarrow \\ \mu_R(A) &= x_2 = \frac{m + \beta}{1 + \beta} \end{aligned} \quad (8)$$

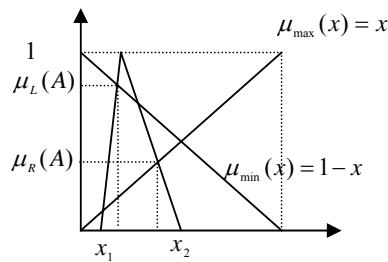


Fig. 2 The graphical form of the left and right scales

IV. METHODOLOGY

Four methods mentioned in Section II consider as criteria to evaluate and select projects. The proposed methodology has two steps: 1) By using a verbal variable pairwise comparison matrix of criteria in the format of triangular fuzzy numbers can be gained (see Fig. 3) then the weight of each criteria is computed using the fuzzy set theory [17]. Also in this step all of projects are also evaluated by using each criterion with optimistic, likely, and pessimistic estimate that those estimation are applied  $l, m, u$  a triangular fuzzy number in order. After this calculation, we can consider Table I.

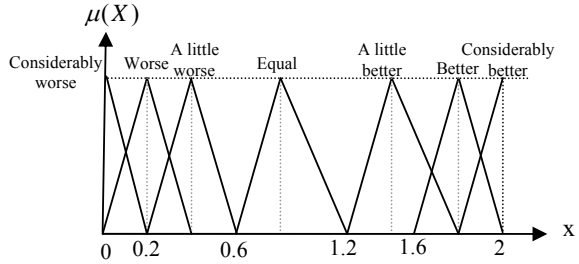


Fig. 3 Verbal variable membership function

In Step 2, the obtained results have been used as input weights in a fuzzy TOPSIS technique. The fuzzy TOPSIS technique by considering an ideal and non ideal solution help decision maker (DM) to evaluate ranking projects and select the best one.

A. The Proposed Method Based on Fuzzy TOPSIS

In this section, a TOPSIS-based model is presented. It is used to obtain appropriateness of given projects in from of the fuzzy logic theory. In this model, desirability of a project in fuzzy from is given, such that in different profiles, the right and left distance of any fuzzy number from ideal value (best) and non-ideal value (worst) are measured. This is the standard measuring of desirability of a project. Following steps are designed to obtain desirability of projects, using a TOPSIS method:

1. Multiplying the fuzzy numbers weight of criteria ( $w_i$ ) by the value of criteria for each project ( $r_{ij}$ ) according to Table II:

$$N_{ij} = R_{ij} \otimes W_i$$

where  $N_{ij}$  is a triangular fuzzy number as follows.

$$\begin{aligned} N_{ij} &= (\alpha_{ij}, r_{ij}, \beta_{ij}) \otimes (\rho_i, \gamma_i, \gamma_i) = \\ &(\alpha_{ij} \cdot \gamma_i + r_{ij} \cdot \rho_i, r_{ij} \cdot \gamma_i, \beta_{ij} \cdot \gamma_i + r_{ij} \cdot \gamma_i) \end{aligned}$$

2. Choosing the profile of  $\alpha_0$ .
3. Calculating the following real numbers for each project and then producing a matrix of  $L_{\alpha_0}$  and  $R_{\alpha_0}$ :

$$\begin{aligned} x_{ij}^- &= \min \{x_{ij} \in R \mid \mu_{N_{ij}}(x_{ij}) \geq \alpha_0\} \\ x_{ij}^+ &= \max \{x_{ij} \in R \mid \mu_{N_{ij}}(x_{ij}) \geq \alpha_0\} \end{aligned}$$

The resulted matrix from  $x_{ij}^-$  is called  $L_{\alpha_0}$  (i.e., meeting point of profile  $\alpha_0$  with left side equation of fuzzy number).

The resulted matrix from  $x_{ij}^+$  is called  $R_{\alpha_0}$  (i.e., meeting point of profile  $\alpha_0$  whit right side equation of fuzzy number).

TABLE I  
FUZZY VALUE OF THE CRITERIA IMPORTANCE AND OPTIMISTIC, LIKELY AND PESSIMISTIC ESTIMATE VALUE FOR PROJECTS USING EACH CRITERIA

Fuzzy weighted criteria	Project n	Project 2	Project 1
$(\rho_1, \gamma_1, \gamma_1)$	$(\alpha_{1n}, r_{1n}, \beta_{1n})$	$(\alpha_{12}, r_{12}, \beta_{12})$	$(\alpha_{11}, r_{11}, \beta_{11})$
$(\rho_2, \gamma_2, \gamma_2)$	$(\alpha_{2n}, r_{2n}, \beta_{2n})$	$(\alpha_{22}, r_{22}, \beta_{22})$	$(\alpha_{21}, r_{21}, \beta_{21})$
$(\rho_m, \gamma_m, \gamma_m)$	$(\alpha_{mn}, r_{mn}, \beta_{mn})$	$(\alpha_{m2}, r_{m2}, \beta_{m2})$	$(\alpha_{m1}, r_{m1}, \beta_{m1})$

The ideal solution and non-ideal one for matrixes of  $L_{\alpha_0}$  and  $R_{\alpha_0}$  due to n project ( $j=1,2,\dots,n$ ), m criterion ( $i=1,2,\dots,m$ ), is defined as follows:

Ideal solution for  $L_{\alpha_0}$  :

$$A_{L_{\alpha_0}}^+ = \{(\max_j x_{ij}^-) | j = 1, 2, \dots, n\} = \{x_1^+, x_2^+, x_1^+, \dots, x_n^+\}$$

Non-ideal solution for  $L_{\alpha_0}$  :

$$A_{L_{\alpha_0}}^- = \{(\min_j x_{ij}^-) | j = 1, 2, \dots, n\} = \{x_1^-, x_2^-, x_1^-, \dots, x_n^-\}$$

Ideal solution for  $R_{\alpha_0}$  :

$$A_{R_{\alpha_0}}^+ = \{(\max_j x_{ij}^+) | j = 1, 2, \dots, n\} = \{x_1^{++}, x_2^{++}, x_1^{++}, \dots, x_n^{++}\}$$

Non ideal solution for  $R_{\alpha_0}$  :

$$A_{R_{\alpha_0}}^- = \{(\min_j x_{ij}^+) | j = 1, 2, \dots, n\} = \{x_1^{+-}, x_2^{+-}, x_1^{+-}, \dots, x_n^{+-}\}$$

4. Calculation the distance of the projects of each matrix from it ideal or non-ideal solution is done using the following equations:

The distance of project (j) of  $L_{\alpha_0}$  from ideal solution

$$dL_j^+ = \left[ \sum_{i=1}^m (x_{ij}^- - x_i^+)^2 \right]^{1/2}, j = 1, 2, \dots, n$$

The distance of project (j) of  $L_{\alpha_0}$  from non ideal solution:

$$dL_j^- = \left[ \sum_{i=1}^m (x_{ij}^- - x_i^-)^2 \right]^{1/2}, j = 1, 2, \dots, n$$

The distance of project (j) of  $R_{\alpha_0}$  from ideal solution:

$$dR_j^+ = \left[ \sum_{i=1}^m (x_{ij}^+ - x_i^{++})^2 \right]^{1/2}, j = 1, 2, \dots, n$$

The distance of project (j) of  $R_{\alpha_0}$  from non ideal solution:

$$dR_j^- = \left[ \sum_{i=1}^m (x_{ij}^+ - x_i^{+-})^2 \right]^{1/2}, j = 1, 2, \dots, n$$

5. Calculation of project (j) relative closeness to the ideal solution of  $L_{\alpha_0}$  and  $R_{\alpha_0}$  by using the following equations:

$$C^* L_j = \frac{dL_j^-}{dL_j^+ + dL_j^-} \quad j = 1, 2, \dots, n$$

$$C^* R_j = \frac{dR_j^-}{dR_j^+ + dR_j^-} \quad j = 1, 2, \dots, n$$

6. Fuzzy desirability  $U_j$  in  $\alpha_0$  profile is defined as follows:

$$U_j = \{(C^* L_j, \alpha_0), (C^* R_j, \alpha_0)\}, \text{ if } C^* L_j < C^* R_j$$

$$U_j = \{(C^* R_j, \alpha_0), (C^* L_j, \alpha_0)\}, \text{ if } C^* L_j > C^* R_j$$

In other words, the right and left side value of fuzzy desirability  $U_j$  is obtained using  $C^* L_j$  and  $C^* R_j$ .

In the TOPSIS model, the priority depends on relative closeness of each choice to ideal solution. That is why the sixth step equation is used.

Since in our proposed method, the left and right distances of any fuzzy number from ideal and non-ideal solutions are used to measure desirability projects, and then the above equations are used as left and right sides of the ideal fuzzy solution.

By creating various profiles and repeating Steps (2) to (6), the desirability fuzzy solution for all projects is produced.

V. CONCLUSION

The evaluation and selection of industrial projects before investment decision is customarily done using, technical and financial information. In this paper, we proposed a new methodology to provide a simple approach to assess alternative projects and help the decision maker to select the best one. By using four common methods of comparing investment alternatives as criteria in a TOPSIS technique, we support project selection decisions. Also this paper uses the improved TOPSIS to make comparison more intuitionitic and reduce or eliminate assessment.

REFERENCES

[1] J. Meredith and S. Mantle, "Project Management: A Managerial Approach," fourth ed. Wiley, New York, 2000.  
[2] N. Danila, "Strategic evaluation and solution of R&D projects," R&D Management 19, 47-62, 1999.

- [3] A. Shpak and D. Zaporozhan, "Working out R&D program via multicriteria analysis," *Computer Science of Moldova* 4 (2 (11)), pp. 239–259, 1996.
- [4] A. Mehrez and Z. Sinuany-Stern, "An interactive approach to project selection," *Journal of Operational Research Society* 34, pp. 621–626, 1983.
- [5] R. Khorramshahgole and H. M. Steiner, "Resource analysis in project evaluation," *A multicriteria approach. Journal of Operational Research Society* 95 (2), pp. 795–803, 1998.
- [6] P. Chu, Y. Hsu and M. Fehling, "A decision support system for project portfolio selection," *Computer in Industry* 32 (2), pp. 141–149, 1996.
- [7] N.R. Baker and J. Freeland "Recent Advances in R&D Benefit Measurement and Project Selection Methods," *Management Science*, 21, pp.1164-1175, 1975.
- [8] G. Lockett and M. Straford "Ranking of Research Projects, Experiments with Two Methods," *Omega* 15, pp. 395-400, (1987).
- [9] A. Lewandowski and M. Grauer, "The Reference Point Optimisation Approach, Multiobjective and Stochastic Optimisation," *European Journal of Operational Research*, 84 (1), pp. 353-376, 1995.
- [10] A.P. Wierzbicki, *The Use of Reference Objectives in Multiobjective Optimization. In Multiple Criteria Decision Making Theory and Application*. pp. 468-486, Springer, Berlin. 1980.
- [11] F. Ghasemzadeh, N. P. Archer and P. Iyogun, "A zero-one model for project portfolio selection and scheduling," *Journal of Operational Research Society* 50 (7), p.p 745–755, 1999.
- [12] S. Kumar, J. Ordóñez and A. Nasseriana, " Integrated project evaluation and selection using multiple-attribute decision-making technique," *International Journal of Production Economics*, 103 (1), pp. 90-103, 2006.
- [13] P. K. Dey and S. S. Gupta "Feasibility analysis of cross country pipeline projects: A quantitative approach", *Project Management Journal* 32 (4), pp. 50–58, 2001.
- [14] S. A. Mian, and N.D. Christine, "Decision-making over the project life cycle: An analytical hierarchy approach," *Project Management Journal* 30 (1), 40–52. 1999.
- [15] W. L. Eddie and Li. Heng, "Analytic Network Process Applied to Project Selection," *Journal of Construction Engineering and Management*, Vol. 131, No. 4, pp. 459-466, April 2005.
- [16] R. P. Mohanty, R. Agarwal, A. K. Choudhury and M. K. Tiwari, "International Journal of Production Research," *International Journal of Production Research*, Vol. 43, No. 24, pp. 5199-5216(18), December 2005.
- [17] D. Y. Chang,d, "Extent analysis and synthetic decision," *Optimization Techniques and Application*, Vol (1), p. 352, 1992.