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Weakly generalized closed map

R. Parimelazhagan †, N. Nagaveni‡

Abstract: In this paper we introduce a new class of mg-continuous mapping and studied some of its basic properties.We obtain some characterizations of such functions. Moreover we define sub minimal structure and further study certain properties of mg-closed sets.

Keywords: m-structure, mg-continuous mapping, minimal structure, mg $T_2 space$,
sub minimal structure, $T_{\frac{1}{2}}$ space, mg-compact set

1. Introduction

Levine [9] introduced the concept of g-closed sets and studied their properties. A subset A of a space X is g-closed if and only if $cl(A) \subset O$ whenever $A \subset O$ and O is open. Hence every closed set is a g-closed set. The union and intersection of two g-closed set is g-closed set. Regular open sets and stronger regular open sets have been introduced and investigated by Stone[19] and Tang[21] respectively. Complements of regular open sets and strong regular open sets are called regular closed sets and strong regular closed sets. Andrijecvic [1], Arya and Nour[2], Bhattacharya and Lahiri[5], Levine[9],[10],Mashour et al[13] and Njastad[17] introduced and investigated semi-preopen sets, generalized semi open sets, semi generalized open sets, generalized open sets, semi-open sets, pre-open sets, generalized open set, semi-open sets pre-open sets and α-open sets which are some of the weak forms of open sets and the complements of theses sets are called the same types of closed sets respectively. Ganster and Reilly [8] have introduced locally closed sets which are weaker than both open and closed sets. Cameron[6] has introduced regular semi-open sets which are weaker than regular open sets.

*Received: 16 sept ember 2008
†Department of Science and Humanities, Karpagam college of Engineering, coimbatore -32. Tamil Nadu India Fax: 04222619046 e-mail:pari_tce@yahoo.com
‡Department of Mathematics, Coimbatore Institute of Technology, coimbatore

2. Preliminaries

In this section we begin by recalling some definitions and properties.

Let (X, τ) be a topological spaces and A be a subset. The closure of A and interior of A are denoted by $cl(A)$ and $int(A)$ respectively. We recall some generalized open sets.

Definition [9] 2.1: A subset A of a space X is g-closed if and only if $cl(A) \subset G$ whenever $A \subset G$ and G is open.

Definition [20]2.2: A map $f : X \to Y$ is called g-closed if each closed set F of $X, f(F)$ is g-closed in Y.

Definition[18]2.3: A map$f : X \to Y$ is called semi-closed if each closed set F of $X, f(F)$ is semiclosed in Y.

Definition [15] 2.4 : A map $f : X \to Y$ is called α-open if each open set F of X, f(F) is α-set in Y.

Definition [7]2.5 : A map $f : X \to Y$ is called pre-closed if for each closed map F of X, f(F) is pre-closed in Y.

Definition [12]2.6: A map $f : X \to Y$ is called regular-closed if for each set F of $X, f(F)$ is regular closed in Y.

Definition (11)2.7: A map $f : X \to Y$ is said to be strongly continuous if $f^{-1}(V)$ is both open and closed in X for each subset V of Y.

Definition [4] 2.8:A map $f : X \to Y$ is said to be generalized continuous if $f^{-1}(V)$ is g-open in X for each set V of Y

Definition [15] 2.9 A subset $A$of a topological space X is said to be weakly generalized closed (wg-closed) set in X if G contains $cl(int(A))$ whenever G

contains A and G is open in X.

Definition[9] 2.10A topological space X is said to be *T1/2*-space if every g-closed set is closed.

Remark:2.11: The following diagram are well known.

$$closed \Rightarrow g - closed \quad w - closed$$

$$\swarrow$$

$$regularclosed \Rightarrow wg - closed \quad \Leftarrow \alpha - closedset$$

$$\swarrow \nwarrow$$

$$gsp - closedset \qquad Pre - closedset$$

3.Properties of Weakly generalized closed

In this section we studied some of wg-closed sets properties.

Definition 3.1: A map $f : X \to Y$ is called wg-closed map if for each closed set F of X, f(F) is wg-closed set.

Remark 3.2: Every g-closed map is a wg-closed map and the converse is need not be true from the following example.

Example3.3: Let $X = \{a,b,c\}$ and $\tau_1 = \{\phi, x, \{a\}, \{b\}, \{a,b\}\}$,$\tau_2 = \{\phi, X, \{a\}, \{a,b\}\}$ be topologies on X. Let $\{a,c\}$ is T_1-closed but not T_2-closed.

Theorem 3.4: A map $f : X \to Y$ is wg-closed if and only if for each subset S of Y and for each open set U containing $f^{-1}(S)$ there is a wg-open set V of Y such that $S \subset V$ and $f^{-1}(V) \subset U$

Proof: Suppose f is wg-closed. Let S be a subset of Y and U is an open set of X such that $f^{-1}(S) \subset U$. Then $V = Y - f^{-1}(X-U)$is a wg-open set containing S such that $f^{-1}(V) \subset U$.

For the converse suppose that F is a closed set of X. Then $f^{-1}(Y - f(F)) \subset X - F$ and $X - F$ is open. By hypothesis there is wg-open set V of Y such that $Y - f(F) \subset V$ and $f^{-1}(V) \subset X - F$. Therefore $F \subset X - f^{-1}(V)$. Hence $Y - V \subset f(F) \subset f(X - f^{-1}(V)) \subset Y - V$ which implies $f(F) = Y - V$. Since $Y - V$ is wg-closed if $f(F)$ is wg-closed and thus f is a wg-closed map.

Theorem 3.5: If $f : X \to Y$ is continuous and wg-closed and A is a wg-closed set of X then $f(A)$ is wg-closed.

proof: Let $f(A) \subset O$ where O is an open set of Y. Since f is g-continuous, $f^{-1}(O)$ is an open set containg A. Hence $cl(A) \subset f^{-1}(O)$ is A is wg-closed set. since f is wg-closed, $f(cl(A))$ is a wg-closed set contained in the open set O which implies than $cl(f(Cl(A)) \subset O$ and hence $clf(cl(A)) \subset O$ and hence $cl(f(A)) \subset O$ so f is a wg-closed set.

corollary 3.6: If $f : X \to Y$is g-continuous and closed and A is g-closed set of X the $f(A)$is wg-closed.

Corollary 3.7: If $f : X \to Y$ is wg-closed and continuous and A is wg-closed set of X then $f_A : A \to Y$ is continuous and wg-closed set.

Proof Let F be a closed set of A then F is wg-closed set of X. From above theorem 3.5 follows that $f_A(F) = f(F)$ is wg-closed set of Y. Here f_A is wg-closed and continuous.

Theorem 3.8 If a map $f : X \to Y$ is closed and a map $g : Y \to Z$ is wg-closed then $f : X \to Z$is wg-closed.

Proof Let H be a closed set in X. Then $f(H)$ is closed and $(g \circ F)(H) = g(f(H))$ is wg-closed as g is wg-closed. Thus $g \circ f$ is wg-closed.

Theorem 3.9: If $f : X \to Y$ is continuous and wg-closed and A is a wg-closed set of X then $f_A : A \to Y$is continuous and wg-closed.

Proof: If F is a closed set of A then F is a wg-closed set of X. From Theorem 3.4, It follows that $f_A(F) = f(F)$ is a wg-closed set of Y. Hence f_A is wg-closed. Also f_A is continuous.

Theorem 3.10: If $f : X \to Y$ is wg-closed and $A = f^{-1}(B)$ for some closed set B of Y then $f_A : A \to Y$ is wg-closed .

Proof: Let F be a closed set in A. Then there is a closed set H in X such that $F = A \cap H$. Then $f_A(F) = f(A \cap H) = f(H) \cap f(B)$. Since f is wg-closed $f(H)$ is wg-closed in Y. so $f(H) \cap B$ is wg-closed in Y. Since the intersection of a wg-closed and a closed set is a wg-closed set. Hence f_A is wg-closed.

Remark 3.11: If B is not closed in Y then the above theorem is not hold from the following example.

Example 3.12: Take $B = \{a, b\}$. Then $A = f^{-1}(B) = \{a, b\}$ and $\{a\}$ is closed in A but$f_A(\{a\}) = \{a\}$ is not wg-closed in $Y.\{a\}$ is also not wg-closed in B.

4. Normal and Regularity

In this section we introduce the new class of wg-regular and studied some of its properties.

Theorem 4.1: If $f : X \to Y$is continuous , wg-closed map from a normal space X onto a space Y then Y is normal.

Proof: Let A, B be disjoint closed sets in Y. Then $f^{-1}(A), f^{-1}(B)$ are disjoint closed sets of X. Since X is normal there are disjoint open sets U, Vin X such that $f^{-1}(A) \subset U$ and $f^{-1}(B) \subset V$. Since f is wg-closed by theorem 3.4, there are wg-open sets G, H in Y such that $A \subset G, B \subset H$ and $f^{-1}(G) \subset U$and $f^{-1}(H) \subset V$. Since U, V are disjoint $intG, intH$ are disjoint open sets. Since G is wg-open, A is closed and $A \subset G, A \subset intG$. similarly $B \subset intH$. Hence Y is normal.

Theorem 4.2: If $f : X \to Y$ is an open continuous wg-closed surjection, where X is regular then$Y$ is regular.

Proof: Let U be an open set containing a point P in Y. Let X be a point of X such that$f(X) = P$. Since X is regular and f is continuous there is an open set U such that $x \in V \subset cl(V) \subset f^{-1}(V)$. Hence $P \in f(V) \subset f(Cl(V)) \subset U$. Since f is wg-closed $f(Cl(V))$ is wg-closed set contained in the open set U. It follows that$cl(f(Cl(V)) \subset U$ and hence$p \in f(V() \subset cl(f(V)) \subset U$and $f(V)$ is open. Since f is open. Hence Y is regular.

Remark 4.3: The normality is preserved under regular closed, continuous and surjective.

Example 4.4: In the example 3.12. It is shown that f is wg-closed $\{a, b\}$ is a regular closed set in $(X, \tau_1)$and it is not closed in (X, τ_2). Hence f is not regular closed.

Example 4.5 Let T_1 be the countable complement topology on the real line R and T_2 be the usual topology on R and $f : (R, T_1) \to (R, T_2)$ be the identity map. Then f is regular closed by the remark immediately after the above example. But f is not wg-closed. For if$A = \{1/n, n \in N\}$ then A is closed in (R, T_1) and $f(A) = A$ is not wg-closed as $f(A) \subset (0, 2)$ and $(0, 2)$ is open in (R, T_2). But $clf(A) \subset (0, 2)$.

Theorem 4.6: If A is wg-closed set of a space X then $IndA \leq IndX$

Proof: It suffices to show that if $IndX \leq n$ and A is wg-closed set of X then $IndA \leq n$. We prove this theorem by induction. The result holds trivially for n=1. Assume that for every wg-closed set A of X ind $X \leq n-1 \Rightarrow Ind \leq n-1$.

Let X be space with $Ind \leq n$. Let A be a wg-closed set of X. Let E be a closed set of A and G be an open set of A such that $E \subset G$. Then there exist a closed set F of X and an open set H of X such that $E = A \cap F$ and $G = A \cap H$. Since E is closed in A and A is wg-closed. Since $IndX \leq n$, there is an open set V of X such that $clE \subset V \subset H$and $Indbd(V) \leq n-1$. Then $V \cap A$ is an open set of A such that $E \subset V \cap A \subset G$ and $bd_A(V \cap A) \subset bd(V)$. Now $bd_A(V \cap A)$ is a wg-closed set of $bd(V)$. By induction hypothesis and $Indbd_A(V \cap A) \leq n-1$. Hence $IndA \leq n$.

Theorem 4.7: If A is a wg-closed set of a space X then dime $A \leq dimX$.

Proof If $dimX = 0$ then $dimA \leq 0 = dimX$. Hence $dimA \leq dimX$.
If $dimX \leq 0$ then $dimX = n$, where n is an integer greater than or equal to -1. If $n = -1 dimX = -1$ which implies that$X = \phi$ and hence $A = \phi$ and $dimA = -1 = dimX$ and thus $dimA \leq dimX$.
Next suppose $dimX = n$ where $n \geq -1$ and let A be a wg-closed set of X. Let $\{u_1, u_2, u_3, ...u_k\}$ be a finite open cover of A. Then for$i = 1, 2, 3, ...K$there exist open sets.V_1 of X such that $u_1 = A \cap V_1$. Since A is wg-closed and $\bigcup_k^{i=1} v_i$ is an open set containing $A, clA \subset \bigcup_{i=1}^{K} pv_i$Since $cl(A)$ is a closed set,$dimcl(A) \leq n$ so the finite open cover $\{clA \cap v_i, i = 1, 2, 3, ..k\}cl(A)$ has a refinement $cl(A) \cap w_i, i = 1, 2, 3, ..k$ or order at most $n+1$, where each w_1 is open in X and $clAw_1 \subset clA \cap V_1$

for each i. Then $\{A \cap w_i) : i = 1, 2, ...\}$ is an open cover of A refining $\{u_i, i = 1, 2, 3, ...k\}$ and of order not exceeding $n + 1$. Hence $dimA \leq n$ which implies that $dimA \dim X$.

Theorem 4.8: If A is a wg-closed set of a space X then $DindA \leq DindX$.

Proof Let X be a space such that $DindX = n$ and A be a wg-closed set of X. By using the notations of the above thoerem, $clA \subset \bigcup V_i$. Since clA is a closed set,$DindA \leq n$. Hence for every open cover $V_i \cap clA, i = 1, 2, 3...k$ there is a disjoint family $W_i, J = 1, 2, 3, ...k$ of open sets clA refining $V_i \cap clA, i = 1, 2, 3, ...k$ and such that $Dind(clA - \bigcup_{j=1}^{k} W_j) \leq n - 1$. But $A - \bigcup_{j=1}^{k} W_j \subset clA - \bigcup_{j=1}^{k} W_j$and $A - \bigcup_{j=1}^{k} W_j = A \cap (clA - \bigcup_{j=1}^{k} w_j)$is a wg-closed set of clA as the intersection of wg-closed set and closed set is a wg-closed set. By induction hypothesis $Dind(A - \bigcup_{j=1}^{k} W_j) \leq n - 1$. Also $W_j \cap A, j = 1, 2, 3...k$is a disjoint family of open sets of A refining $u_1, U_2, ...U_k$. Thus $DindA \leq n$and the theorem is proved.

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- R.Parimelazhagan completed his under graduate and postgraduate study at st.Joseph college Trichy. Later he acquired M.Phil degree from Bharathiar University, Coimbatore and qualified himself in B.Ed from Annamalai University, Chidambaram India. His major field of studies Mathematics throughout

higher education. He has part in 13 years of teaching experience as a Lecturer, Senior Lecturer, Assistant Professor and Head of the department (Science and Humanities/Applied Sciences). Currently he is working at Karpagam college of Engineering, Coimbatore, Tamil Nadu, India. He has published 17 books in Engineering Mathematics and presented research papers in national and international conferences.He is member of MISTE, ISCA and CSI.
E.Mail: pari_tce@yahoo.com

- Nagaveni. N received the B.Sc., M.Sc and B.Ed degrees in Mathematics from Bharathiar University, India in 1985,1987 and 1988 respectively. M.Phil degree from Avinashilingam University, India in 1989 and M.Ed degree from Annamalai University, India in 1992. And Ph.D degree in the area of Topology in Mathematics from Bharathiar University, India in 2000. Since 1992, She has been with the department of Mathematics coimbatore Institute of Technology, Coimbatore Tamil Nadu, India where she is currently as Assistant Professor. She is engaged as a research supervisor and her research interests includes Topology, Fuzzy sets and continuous function, data mining, distributed computing, Web mining and privacy preservalign in data mining. She is the member in Indian Science congress Association (ICSA). she has been presented many research papers in the annual conference of ICSA. She has been published many papers in the international and national Journals.