International Journal of Engineering, Mathematical and Physical Sciences
ISSN: 2517-9934
Vol:4, No:5, 2010

Water Boundary Layer Flow Over Rotating Sphere with Mass Transfer

G. Revathi, P. Saikrishnan

Abstract—An analysis is performed to study the influence of non-uniform double slot suction on a steady laminar boundary layer flow over a rotating sphere when fluid properties such as viscosity and Prandtl number are inverse linear functions of temperature. Non-similar solutions have been obtained from the starting point of the streamwise co-ordinate to the exact point of separation. The difficulties arising at the starting point of the streamwise co-ordinate, at the edges of the slot and at the point of separation have been overcome by applying an implicit finite difference scheme in combination with the quasi-linearization technique and an appropriate selection of the finer step sizes along the stream-wise direction. The present investigation shows that the point of ordinary separation can be delayed by non-uniform double slot suction if the mass transfer rate is increased and also if the slots are positioned further downstream. In addition, the investigation reveals that double slot suction is found to be more effective compared to a single slot suction in delaying ordinary separation. As rotation parameter increase the point of separation moves upstream direction.

Keywords—boundary layer, suction, mass transfer, rotating sphere.

I. Introduction

A detailed analysis of boundary layer flow problems taking non-similarity into account has become significantly important in recent past. In an earlier study, a review on the non-similarity solution methods along with the relevant publications is given by Dewey and Gross [1]. Subsequently, many attempts have been made to provide non-similar solutions of boundary layer flow problems by finite difference method [2], [3] and an implicit finite difference method in combination with quasi-linearization technique [4], [5]. Fluid viscosity and thermal conductivity are the main governing fluid properties in the laminar water boundary layer forced flow and hence their variations can be expected to affect separation. Further, mass transfer through a slot strongly influences the development of a boundary layer along a surface and in particular can prevent or at least delay separation of the viscous region. Different studies [7], [6], [8], [9] show the effect of single slot suction (injection) into steady compressible and water boundary layer flows over two dimensional and axi-symmetric bodies. Moreover, Roy [10] and Subhashini et.al [11] gave investigated the influence of non-uniform multiple slot suction (injection) on compressible boundary layer flows over cylinder and yawed cylinder, respectively. Also, in more recent studies, Roy et.al. [12] and Roy and Saikrishnan [13] have reported the influence of non-uniform double slot suction (injection) on an incompressible boundary layer flow over a slender cylinder and sphere, respectively.

In the present investigation, the effect of non-uniform double slot suction on the steady laminar non-similar boundary layer flow over rotating sphere is considered. The non-similar solutions have been obtained starting from the origin of the stream-wise coordinate to the point of separation (zero skin friction in the stream-wise direction) using quasi-linearization technique with an implicit finite difference scheme. The present analysis may be useful in understanding many boundary layer flow problems of practical importance, for example, in suppressing recirculation bubbles and controlling transition and/or delaying the boundary layer separation over control surfaces.

II. Mathematical Formulation

Consider a steady laminar non-similar boundary layer forced convection flow (of water) with temperature-dependent viscosity and Prandtl number over a rotating sphere when the non-uniform mass transfer (suction in a slot) vary with the axial distance (x) along the surface. The sphere, rotating with the constant angular velocity Ω_o, is placed in a uniform stream with its axis of rotation parallel to the free stream velocity. An orthogonal curvilinear coordinate system (see Fig.1) has been chosen in which coordinate x measures the distance from the forward stagnation point along a meridian, y represents the distance in the direction of rotation and z is the distance normal to the body surface. The radius of a section normal to the axis of the sphere at a distance x along the meridian from the pole is $r(x)$ and it is assumed that $r(x)$ is large compared with the boundary layer thickness. The fluid is assumed to flow with moderate velocities, and the temperature difference between the wall and the free stream is small ($< 40^oC$). In the range of temperature considered (*i.e.*, $0^oC - 40^oC$), the variation of both density (ρ) and specific heat (C_p), of water, with temperature is less than 1% (see Table 1) and hence they are taken as constants. However, since the variations of viscosity (μ) and thermal conductivity (k) [and hence Prandtl number (Pr)] with temperature are quite significant, the viscosity and Prandtl number are assumed to vary as an inverse function of temperature (T) [5, 6]:

$$\mu = \frac{1}{b_1 + b_2 T} \qquad \text{and} \qquad Pr = \frac{1}{c_1 + c_2 T} \tag{1}$$

where

$$b_1 = 53.41, \quad b_2 = 2.43, \quad c_1 = 0.068 \quad \text{and} \quad c_2 = 0.004. \tag{2}$$

G. Revathi, Research student, Department of Mathematics, National Institute of Technology, Trichirappalli - 620 015.
P. Saikrishnan, Department of Mathematics, National Institute of Technology, Trichirappalli - 620 015.

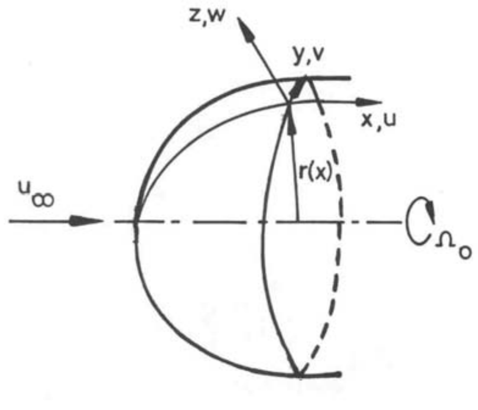

Fig. 1. Flow geometry

The numerical data, used for these correlations, are taken from [14]. The relations (1) - (2) are reasonably good approximations for liquids such as water, particularly for small temperature differences between the wall and ambient fluid. The fluid at the edge of the boundary layer is maintained at a constant temperature T_∞ and the body has a uniform temperature T_w ($T_w > T_\infty$). The blowing rate of the fluid is assumed to be small and it does not affect the inviscid flow at the edge of the boundary layer. Under the above mentioned assumptions, the boundary layer equations governing the flow can be written as (3) - (6):

$$(ru)_x + (rw)_z = 0, \tag{3}$$

$$uu_x + wu_z - r^{-1}v^2 r_x = u_e(u_e)_x + \rho^{-1}(\mu u_z)_z, \tag{4}$$

$$uv_x + wv_z + uvr^{-1}r_x = \rho^{-1}(\mu v_z)_z, \tag{5}$$

$$uT_x + wT_z = \rho^{-1}\left(\frac{\mu}{Pr}T_z\right)_z + \frac{\mu}{\rho C_p}(u_z^2 + v_z^2). \tag{6}$$

The boundary conditions are given by:

$$u(x,0) = 0, \qquad v(x,0) = \Omega_o r(x),$$
$$w(x,0) = w_w(x), \qquad T(x,0) = T_w = constant,$$
$$u(x,\infty) = u_e(x),\ v(x,\infty) = 0,\ T(x,\infty) = T_\infty = constant. \tag{7}$$

Applying the following transformations

$$\xi = \int_0^x \left(\frac{u_e}{u_\infty}\right)\left(\frac{r}{R}\right)^2 R^{-1}dx,$$
$$\eta = \left(\frac{Re}{2\xi}\right)^{1/2}\left(\frac{u_e}{u_\infty}\right)\left(\frac{r}{R}\right)\left(\frac{z}{R}\right),$$
$$\psi(x,z) = u_\infty R\left(\frac{2\xi}{Re}\right)^{1/2} f(\xi,\eta), \qquad ur = R\frac{\partial\psi}{\partial z},$$
$$wr = -R\frac{\partial\psi}{\partial x},$$
$$Re = \frac{u_\infty R\rho}{\mu_\infty}, \qquad G = \frac{T - T_w}{T_\infty - T_w},$$
$$v(x,z) = \Omega_o\ r(x)S(\xi,\eta) \tag{8}$$

to Eqs. (3) - (6), we find that Eq. (3) is identically satisfied and Eqs. (4) - (6) reduce to nondimensional form given by

$$(NF_\eta)_\eta + fF_\eta + \beta(\xi)(1-F^2) + \alpha(\xi)S^2 = 2\xi(FF_\xi - f_\xi F_\eta), \tag{9}$$

$$(NS_\eta)_\eta + fS_\eta - \alpha_1(\xi)FS = 2\xi(FS_\xi - S_\eta f_\xi), \tag{10}$$

$$(NPr^{-1}G_\eta)_\eta + fG_\eta + NEc\left(\frac{u_e}{u_\infty}\right)^2[F_\eta^2 + \lambda S_\eta^2] = 2\xi(FG_\xi - f_\xi G_\eta), \tag{11}$$

where

$$N = \frac{\mu}{\mu_\infty} = \frac{b_1 + b_2T_\infty}{b_1 + b_2T} = \frac{1}{a_1 + a_2G},$$
$$Pr = \frac{1}{c_1 + c_2T} = \frac{1}{a_3 + a_4G},$$
$$a_1 = \frac{b_1 + b_2T_w}{b_1 + b_2T_\infty}, \qquad a_2 = \frac{b_2(T_\infty - T_w)}{b_1 + b_2T_\infty},$$
$$a_3 = c_1 + c_2T_w, \qquad a_4 = c_2(T_\infty - T_w),$$
$$\beta(\xi) = \frac{2\xi}{u_e}\frac{du_e}{d\xi}, \quad \alpha_1(\xi) = \frac{4\xi}{r}\frac{dr}{d\xi}, \quad \lambda = \left(\frac{\Omega_o r}{u_e}\right)^2,$$
$$\alpha(\xi) = \frac{2\xi}{r}\frac{dr}{d\xi}\lambda, \quad Ec = \frac{u_\infty^2}{C_p(T_\infty - T_w)},$$
$$\triangle T_w = (T_w - T_\infty), \qquad u = u_e f_\eta = u_e F,$$
$$w = -\frac{ru_e}{R(2\xi R_e)^{1/2}}\{f + 2\xi f_\xi + (\beta(\xi) + \frac{\alpha_1(\xi)}{2} - 1)\eta F\}.$$

The transformed boundary conditions are

$$F(\xi,0) = 0, \qquad S(\xi,0) = 1, \qquad G(\xi,0) = 0,$$
$$F(\xi,\infty) = 1, \qquad S(\xi,\infty) = 0, \qquad G(\xi,\infty) = 1, \tag{12}$$

where $f = \int_0^\eta F d\eta + f_w$ and f_w is given by

$$f_w = -\xi^{-1/2}\left(\frac{Re}{2}\right)^{1/2}\int_0^{\bar{x}}\left(\frac{r}{R}\right)\frac{1}{u_\infty}w_w(\bar{x})d\bar{x} \tag{13}$$

The set of Eqs. (9) - (11) reduces to that of the classical nonsimilar flow over a stationary sphere for $\lambda = 0$. Hence Eq. (10) becomes redundant as the velocity component in the y- direction v=0 (*i.e.,* S=0) for $\lambda = 0$.

In the case of a sphere of radius R, the velocity at the edge of the boundary layer and non-uniform surface mass transfer being functions of $\bar{x}$, give rise to non-similarity. The velocity at the edge of the boundary layer and the radius of revolution $r(x)$ are given by [6]

$$\frac{u_e}{u_\infty} = \frac{3}{2}\sin\bar{x}, \qquad \frac{r}{R} = \sin\bar{x}, \qquad \bar{x} = \frac{x}{R}.$$

Consequently, the expressions for $\xi, \beta(\xi), \alpha(\xi), \alpha_1(\xi)$ and f_w can be expressed as

$$\xi = \frac{1}{2}P_1^2P_3, \qquad \beta = \frac{2}{3}P_3P_2^{-2}\cos\bar{x}, \qquad \alpha = \lambda\beta,$$
$$\alpha_1 = 2\beta, \qquad \lambda = \left(\frac{\Omega_o r}{u_e}\right)^2 = \frac{4}{9}\left(\frac{\Omega_o R}{u_\infty}\right)^2, \tag{14}$$

International Journal of Engineering, Mathematical and Physical Sciences
ISSN: 2517-9934
Vol:4, No:5, 2010

$$f_w = \begin{cases} 0 & , \quad \bar{x} \leq \bar{x}_o \\ AP_1^{-1}P_3^{-1/2}C(\bar{x},\bar{x}_o) & , \quad \bar{x}_o \leq \bar{x} \leq \bar{x}_o^* \\ AP_1^{-1}P_3^{-1/2}C(\bar{x}_o^*,\bar{x}_o) & , \quad \bar{x}_o^* \leq \bar{x} \leq \bar{x}_1 \\ AP_1^{-1}P_3^{-1/2}\{C(\bar{x}_o^*,\bar{x}_o)+C(\bar{x},\bar{x}_1)\} & , \quad \bar{x}_1 \leq \bar{x} \leq \bar{x}_1^* \\ AP_1^{-1}P_3^{-1/2}\{C(\bar{x}_o^*,\bar{x}_o)+C(\bar{x}_1^*,\bar{x}_1)\} & , \quad \bar{x} \geq \bar{x}_1^* \end{cases} \quad (1.$$

where $C(\bar{x},\bar{x}_o) = \frac{\sin\{(\omega^*-1)\bar{x}-\omega^*\bar{x}_o\}+\sin\bar{x}_o}{(\omega^*-1)} - \frac{\sin\{(\omega^*+1)\bar{x}-\omega^*\bar{x}_o\}-\sin\bar{x}_o}{(\omega^*+1)}$, $P_1 = 1-\cos\bar{x}, P_2 = 1+\cos\bar{x}$ and $P_3 = 2+\cos\bar{x}$.
Here, $w_w(\bar{x})$ (in (13))is taken as

$$w_w(\bar{x}) = \begin{cases} 0 & , \quad \bar{x} \leq \bar{x}_o \\ -u_\infty(\frac{Re}{2})^{-1/2}2^{1/2}A\sin\{\omega^*(\bar{x}-\bar{x}_o)\} & , \quad \bar{x}_o \leq \bar{x} \leq \bar{x}_o^* \\ 0 & , \quad \bar{x}_o^* \leq \bar{x} \leq \bar{x}_1 \\ -u_\infty(\frac{Re}{2})^{-1/2}2^{1/2}A\sin\{\omega^*(\bar{x}-\bar{x}_1)\} & , \quad \bar{x}_1 \leq \bar{x} \leq \bar{x}_1^* \\ 0 & , \quad \bar{x} \geq \bar{x}_1^* \end{cases}$$

where ω^* , $\bar{x}_o$ and $\bar{x}_1$ are the three free parameters which determine the slot length and slot locations, respectively. The function $w_w(\bar{x})$ is continuous for all values of $\bar{x}$ and it has a non-zero value only in the intervals $[\bar{x}_o,\bar{x}_o^*]$ and $[\bar{x}_1,\bar{x}_1^*]$. The reason for taking such a function is that it allows the mass transfer to change slowly in the neighbourhood of leading and trailing edges of the slot. The parameter $A > 0$ or $A < 0$ according to whether there is a suction or an injection. It is convenient to express Eqs. (9) - (11) in terms of $\bar{x}$ instead of ξ. Equation (14) gives the relation between ξ and $\bar{x}$ as

$$\xi\frac{\partial}{\partial\xi} = B(\bar{x})\frac{\partial}{\partial\bar{x}}, \qquad (16)$$

where $B(\bar{x}) = 3^{-1}\tan(\frac{\bar{x}}{2})P_3P_2^{-1}$.

Substituting Eq. (16) in Eqs. (9) - (11), we obtain

$$(NF_\eta)_\eta + fF_\eta + \beta(\bar{x})(1-F^2) + \alpha(\bar{x})S^2 = 2B(\bar{x})(FF_{\bar{x}} - f_{\bar{x}}F_\eta), \qquad (17)$$

$$(NS_\eta)_\eta + fS_\eta - \alpha_1(\bar{x})FS = 2B(\bar{x})(FS_{\bar{x}} - S_\eta f_{\bar{x}}), \qquad (18)$$

$$(NPr^{-1}G_\eta)_\eta + fG_\eta + NEc(\tfrac{u_e}{u_\infty})^2[F_\eta^2 + \lambda S_\eta^2] = 2B(\bar{x})(FG_{\bar{x}} - f_{\bar{x}}G_\eta), \qquad (19)$$

where $N = \frac{1}{a_1+a_2G}$, and $Pr = \frac{1}{a_3+a_4G}$.
The boundary conditions become

$$F(\bar{x},0)=0, \quad S(\bar{x},0)=1, \quad G(\bar{x},0)=0,$$
$$F(\bar{x},\infty)=1, \quad S(\bar{x},\infty)=0, \quad G(\bar{x},\infty)=1, \qquad (20)$$

where $f = \int_0^\eta F d\eta + f_w$.
The skin friction coefficients in x- and y- directions can be expressed in the form:

$$C_f(Re)^{1/2} = \frac{9}{2}\sin\bar{x}\ P_2P_3^{-1/2}N_w\ (F_\eta)_w, \qquad (21)$$

$$\bar{C_f}(Re)^{1/2} = \frac{9}{2}\lambda^{1/2}\sin\bar{x}\ P_2P_3^{-1/2}N_w\ (S_\eta)_w. \qquad (22)$$

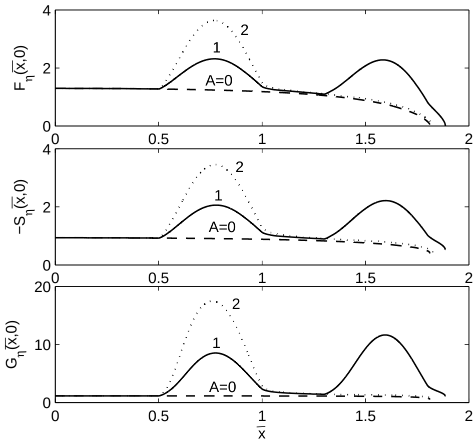

Fig. 2. Effect of suction ($A > 0$) on the velocity and temperature gradients for $T_\infty = 18.7^oC, \triangle T_w = 10^oC, w^* = 2\pi$ and $\lambda = 1$. $- - - - - - - -$ no slot. ...., $\bar{x}_o = 0.5$. ________, $\bar{x}_o = 0.5, \bar{x}_1 = 1.3$

Similarly, the heat transfer coefficient in terms of Nusselt number can be written as

$$Nu(Re)^{-1/2} = \frac{3}{2}P_2P_3^{-1/2}(G_\eta)_w, \qquad (23)$$

where $C_f = 2\frac{[\mu(\frac{\partial u}{\partial z})]_w}{\rho u_\infty^2}$, $\bar{C_f} = 2\frac{[\mu(\frac{\partial v}{\partial z})]_w}{\rho u_\infty^2}$, $Nu = \frac{R(\frac{\partial T}{\partial z})_w}{(T_\infty - T_w)}$, and $N_w = \frac{1}{a_1+a_2G_w} = constant$.

III. Results and Discussion

The set of equations (17) and (19) under the boundary conditions (20) have been solved numerically using an implicit finite difference scheme in combination with the quasilinearization method as discussed by Inoye and Tate [16]. Computations were carried out for various values of $A(-0.6 < A < 2.0)$ and $\lambda(0 \leq \lambda \leq 4)$. The effect of non-uniform suction of a single slot located at $\bar{x}_o = 0.5$ on the skin friction is compared with that over a non-uniform suction of double slot situated at $\bar{x}_o = 0.5$ and $\bar{x}_1 = 1.30$ in Fig.2. It is observed that the separation gets delayed and the point of separation moves further downstream due to the double slot suction than that of a single slot suction. Hence, double slot suction is more effective in delaying separation than the single slot suction when sphere is rotating with constant angular velocity Ω_o.

The effects of non-uniform double slot suction parameter ($A > 0$) on velocity gradients and temperature gradient $(F_\eta(\bar{x},0), -S_\eta(\bar{x},0), G_\eta(\bar{x},0))$ in the case of non-uniform double slot suction is presented in Figure 2. In both, double and single slot cases, the skin frictions gradually increase from the leading edges of the slots, attain a maximum and then start decreasing at the rear end the slots. Finally, the velocity and temperature gradients $(F_\eta(\bar{x},0), -S_\eta(\bar{x},0), G_\eta(\bar{x},0))$ decrease from their maximum values and reaches zero but

International Journal of Engineering, Mathematical and Physical Sciences
ISSN: 2517-9934
Vol:4, No:5, 2010

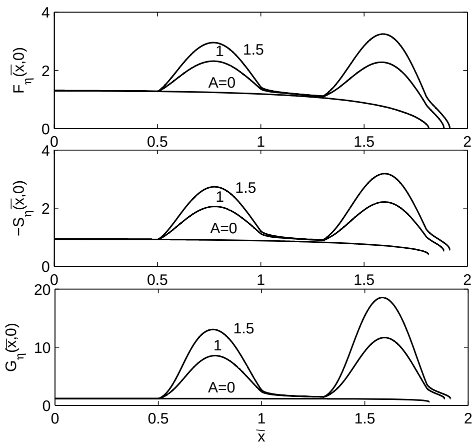

Fig. 3. Effect of suction ($A > 0$) on the velocity and temperature gradients for $T_\infty = 18.7^oC, \triangle T_w = 10^oC, \bar{x}_o = 0.5, \bar{x}_1 = 1.3, w^* = 2\pi$ and $\lambda = 1$.

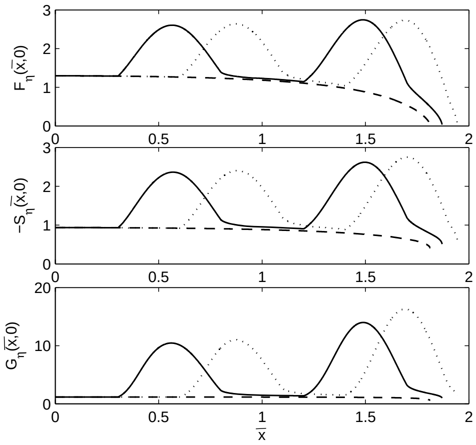

Fig. 4. Effect of slot movement on the velocity and temperature gradients for $A = 1.25, T_\infty = 18.7^oC, \triangle T_w = 10^oC, w^* = 2\pi$ and $\lambda = 3$. ...., $\bar{x}_o = 0.3, \bar{x}_1 = 1.2$. ________, $\bar{x}_o = 0.6, \bar{x}_1 = 1.4$. – – – – – –, no slot.

the velocity gradient in y direction $(S_\eta(\bar{x}, 0))$ remains finite. This implies that ordinary separation occurs at this point. For the problem under consideration, singular separation does not occur (i.e. for no value of $\bar{x}$, both $F_\eta(\bar{x}, 0), S_\eta(\bar{x}, 0)$ reach zero value simultaneously). Hence we have used the word separation to denote ordinary separation.

The effect of mass transfer parameter on the velocity gradients and temperature gradient $(F_\eta(\bar{x}, 0), -S_\eta(\bar{x}, 0), G_\eta(\bar{x}, 0))$ in the case of non-uniform double slot suction is presented in Fig. 3. The velocity and temperature gradients $(F_\eta(\bar{x}, 0), -S_\eta(\bar{x}, 0), G_\eta(\bar{x}, 0))$ increase with the increase in mass transfer rates. Further, the point of separation moves further downstream with the increase of A.

It is noticed from figure 4, that the point of separation moves downstream when positions of the slots are moved further downstream. Thus the point of separation can be delayed by non-uniform double slot suction ($A > 0$) and also by positioning the slots further downstream.

The effects of rotation parameter (λ) on velocity gradients and temperature gradient $(F_\eta(\bar{x}, 0), -S_\eta(\bar{x}, 0), G_\eta(\bar{x}, 0))$, with parameter $A = 0.5$ in the case of non uniform double slot suction is plotted in figure 5. Increasing value of rotation parameter results, the point of separation moves upstream direction.

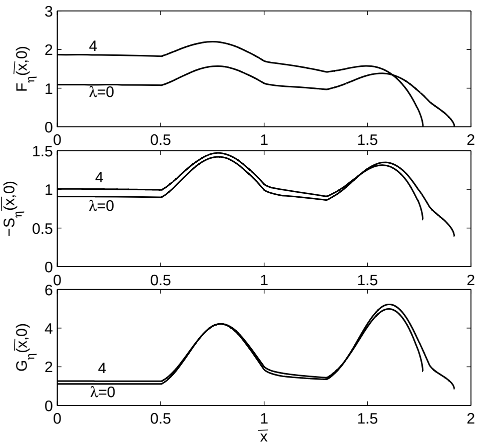

Fig. 5. Effect of rotation parameter of velocity and temperature gradients for $A = 2.0, T_\infty = 18.7^oC, \triangle T_w = 10^oC, \bar{x}_o = 0.5, \bar{x}_1 = 1.3, w^* = 2\pi$.

IV. Conclusions

Non-similar solution of a steady laminar incompressible (water) boundary layer flow over a rotating sphere with non-uniform double slot suction has been obtained starting from the origin of streamwise coordinate to the exact point of separation. The present study effectively compares the significance of non-uniform single and double slot suction of laminar water boundary layer flows over a rotating sphere. The numerical investigation shows that the point of separation can be delayed using non-uniform double slot suction and also by increasing the mass transfer rate in the slots. Further, it is noticed that by moving the double slot further downstream with ($A > 0$). The increasing value of rotation parameter results, the point of separation moves upstream direction.

ACKNOWLEDGMENT

G.Revathi is thankful to Jawaharlal Nehru Memorial Fund (Ref: SU/1/1047/2010-11/1090) for provinding scholarship to pursue Ph.D studies at NIT Trichy, India.

REFERENCES

[1] C. F. Dewey,J. F. Gross, Exact solutions of the Laminar Boundary Layer Equations, Advances in heat transfer, Vol. IV, Academic Press, New York, 1967, p.317.
[2] T. Davis, G.Walker, On solution of the compressible laminar boundary layer equations and their behaviour near separation, J. Fluid mech, 80, (1977) 279-292.
[3] B.J.Venkatachala, G. Nath, Non-similar laminar incompressible boundary layers with vectored mass transfer, Proc. Indian Acad. Sci. (Eng. Sci) 3 (1980) 129 - 142.
[4] A. T. Eswara and G. Nath, Unsteady two-dimensional and axi-symmetric water boundary layers with variable viscosity and Prandtl number, Int. J. Eng. Sci., 32 (1994) 267 - 279.
[5] S. Roy, Non-uniform mass transfer or wall enthalphy into a compressible flow over yawed cylinder, Int. J. Heat and Masss Transfer 44 (2001) 3017 - 3024.
[6] S. Roy and H.S. Takkar, Compressible boundary layer flow with non-uniform slot injection (or suction) over (i) a cylinder and (ii) a sphere, Heat Mass Transfer 39 (2003) 139 - 146.
[7] P. Saikrishnan, S. Roy., Non-uniform slot injection (suction) into water boundary layers over (i) a cylinder and (ii) a sphere, Int. J. Engg. Sci., 41 (2003)1351 - 1365.
[8] S. Roy, P. Saikrishnan, Non-uniform slot injection (suction) into steady laminar water boundary layer flow over a rotating sphere, Int. J. Heat Mass Transfer 46 (2003) 3389 - 3396.
[9] S.V. Subhashini, H.S. Takhar, G.Nath,Non-uniformmass transfer orwall enthalpy into a compressible flow over a rotating sphere, Heat Mass Transf. 43 (2007) 11331141.
[10] S. Roy, Non-uniform multiple slot injection (suction) or wall entthalphy into steady compressible laminar boundary layer, Acta Mech, 143 (2000) 113 - 128.
[11] S.V. Subhashini, H.S. Takhar, G. Nath, Non-uniform multiple slot injection (suction) or wall enthalphy into a compressible flow over a yawed circular cylinder, Int.J.Therm. Sci. 42 (2003) 749 - 757.
[12] S. Roy, Prabal Datta, R. Ravindran, E. Momoniat, Non-uniform double slot injection (suction) on a forced flow over a slender cylinder, Int. J. Heat Mass Transfer 50 (2007) 3190 - 3194.
[13] S. Roy, P. Saikrishnan, Bishun D. Pandey, Influence of double slot suction (injection) into water boundary layer flows over sphere, Int. Comm. in Heat and Mass Transfer 36 (2009) 646 - 650.
[14] D.R.Lide (Ed), CRC Handbook of Chemistry and Physics, 71st ed, CRC Press, Boca Ration, Florida, 1990.
[15] R. E. Bellman, R. E. Kalaba, Quasilinearization and non-linear boundary value problem, American Elsevier Publishing Co. Inc., New York (1965).
[16] K. Inouye, A. Tate, Finite difference version of quasilinearization applied to boundary layer equations, AIAA J. 12 (1974) 558 - 560.

G. Revathi doing Ph.D, in the department of mathematics, National Institute of Technology, Trichirappalli - 620015.

Dr. P. Saikrishnan working in the department of mathematics, National Institute of Technology, Trichirappalli - 620015. E-mail: psai@nitt.edu