Volterra Filtering Techniques for Removal of Gaussian and Mixed Gaussian-Impulse Noise

M. B. Meenavathi, and K. Rajesh

Abstract**—**In this paper, we propose a new class of Volterra series based filters for image enhancement and restoration. Generally the linear filters reduce the noise and cause blurring at the edges. Some nonlinear filters based on median operator or rank operator deal with only impulse noise and fail to cancel the most common Gaussian distributed noise. A class of second order Volterra filters is proposed to optimize the trade-off between noise removal and edge preservation. In this paper, we consider both the Gaussian and mixed Gaussian-impulse noise to test the robustness of the filter. Image enhancement and restoration results using the proposed Volterra filter are found to be superior to those obtained with standard linear and nonlinear filters.

Keywords**—**Gaussian noise, Image enhancement, Image restoration, Linear filters, Nonlinear filters, Volterra series.

I. Introduction

In the early development of image processing, linear filters were the primary tools for image enhancement and restoration. Their mathematical simplicity and the existence of some desirable properties made them easy to design and implement. Moreover, linear filters offered satisfactory performance in many applications. However, they have poor performance in the presence of non additive noise and in situations where system nonlinearities or Gaussian statistics are encountered [1]. In image processing applications, linear filters tend to blur the edges and do not remove Gaussian and mixed Gaussian impulse noise effectively. In order to generalize nonlinear system theory, the Volterra series have been recognized as a powerful mathematical tool leading to another class of nonlinear filters called polynomial filters [2]. Volterra series expansions represent an important model for the representation, analysis and synthesis of nonlinear systems. However, a significant problem with this approach to filter design is that the number of terms required to estimate grows exponentially with the order of expansion. In practice, therefore, we truncate the Volterra series to consist of, at most, second degree terms only. Quadratic filters are generalization of linear filters in which the filter output contains a linear combination of products of the input time series. Volterra filters are approximately equivalent to the product of a local mean estimator and a generalized high pass filter. In this paper, we designed a truncated polynomial filter based on the second order Volterra series.

The problem of eliminating noise from an image without causing degradation of image details has been considered frequently in recent literature. Volterra filters are proposed for edge preserving image restoration using block lexicographic matrices and symmetry conditions [3]. Ian J. Morrison and Peter J.W. Rayner [4] estimated the signals corrupted by additive non-Gaussian noise using nonlinear Wiener filters by extending the linear filter with nonlinear terms. The nonlinear terms are taken from the discrete Volterra series. This results in a nonlinear FIR filter which is linear in its coefficients. Inherently noise removal from image introduces blurring in many cases. An adaptive standard recursive low pass filter is designed by Klaus Rank and Rolf Unbehauen [5] considered the three local image features edge, spot and flats as adaptive regions with Gaussian noise. To reduce the number of parameters in Volterra series filter, different techniques are proposed using a tensor product basis approximation [6] and block lexicographic technique [3]. These techniques reduce the overall complexity of implementation and partial characterization of Volterra filter.

A class of nonlinear 2D filter is designed based on the truncated discrete Volterra series together with a matrix notation [11]. Exponential nonlinear Volterra filter is used for contrast sharpening in noisy images and the design of filter is based on the theory of Generalized Fock spaces (GF) of Volterra series [7]. Volterra filters are used for extraction and enhancement of edges in images also. Giovanni F. Ramponi [12] used nonlinear part of Volterra filters for edge extraction and is compared with the traditional Sobel edge operators. The isotropic property of nonlinear coefficients of Volterra filter is also used for edge extraction and enhancement and its performance is evaluated by comparing with Laplacian masks [8]. The linear filter corresponds to a first order Volterra kernel and quadratic filter corresponds to a second order Volterra kernel.

From the above discussions, it is evident that linear filters are most suitable for impulse noise cancellation only and they cause blur during filtering. Also, linear filters do not remove Gaussian and mixed Gaussian-impulse noise effectively. For this reason, nonlinear filters based on Volterra series are considered in our work. The proposed method deals with Gaussian and mixed Gaussian impulse noise. It is distinguished from the other methods by employing FIR widowing or constrained least mean square (LMS) algorithm [9], [10] to calculate filter coefficients. To reduce the Volterra

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parameters we used the block lexicographic matrix technique and symmetry conditions. The new filter works satisfactorily in removing Gaussian noise as well as mixed Gaussian-impulse noise effectively.

The paper is organized as follows. Section II describes the Volterra model which is the basic mathematical foundation for proposed work. Steps for the filter design and calculation of filter kernels are given in sections III and IV respectively. Experimental results and performance evaluation on Lena image are given in section V. Concluding remarks is presented in section VI.

II. Volterra Model

Volterra filters are discrete nonlinear time invariant systems with memory described by the discrete Volterra expansion as

$$y(n) = h_0 + \sum_{p=1}^{\infty} \hbar_p[x(n)] \tag{1}$$

where

$$\hbar_p[x(n)] = \sum_{m_1=0}^{N-1} \ldots\ldots \sum_{m_p=0}^{N-1} \hbar_p(m_1, m_2, \ldots\ldots m_p) \times$$

$$x(n-m_1)x(n-m_2)\ldots\ldots x(n-m_p)$$

Here h_0 is the offset term. $\hbar_p(m_1, m_2, \ldots. m_p)$ can be considered as a generalized p^{th} order impulse response characterizing the nonlinear behavior of the filter. This shows how the Volterra filter can viewed as a natural extension of the linear filter.

The truncated second order Volterra system has the form

$$y(n) = h_0 + \sum_{k1=0}^{\infty} h_1(k_1)u(n-k_1)$$

$$+ \sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty} h_2(k_2, k_1)u(n-k_1)u(n-k_2) \tag{2}$$

where

$\sum_{k1=0}^{\infty} h_1(k_1)u(n-k_1)$ and

$$\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty} h_2(k_2, k_1)u(n-k_1)u(n-k_2)$$

are called the linear and nonlinear terms respectively.

A Volterra series provides a description for dynamic systems in a similar way as the Taylor series does for static input/output relationships, i.e., the system output is split into linear, quadratic, cubic terms etc. The system model for the Volterra series is shown in Fig. 1.

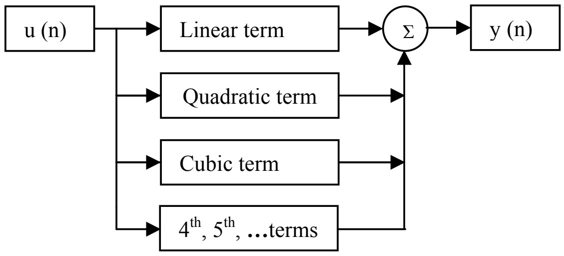

Fig. 1 System model

III. Filter Design

The second order truncated Volterra series with noise factor $\eta(n)$ can be written as

$$y(n) = h_0 + \sum_{k1=0}^{3} h_1(k_1)u(n-k_1)$$

$$+ \sum_{k_1=0}^{9}\sum_{k_2=0}^{9} h_2(k_2, k_1)u(n-k_1)u(n-k_2) + \eta(n) \tag{3}$$

By assuming the offset term $h_0 = 0$, the equation (3) can be written as

$$y(n) = y_1(n) + y_2(n) + \eta(n) \tag{4}$$

where

$$y_1(n) = \sum_{k_1=0}^{3} h_1(k_1)u(n-k_1)$$

$$y_2(n) = \sum_{k_1=0}^{9}\sum_{k_2=0}^{9} h_2(k_2, k_1)u(n-k_1)u(n-k_2)$$

and $h_1(k_1)$, $h_2(k_2, k_1)$ are filter coefficients which can be calculated using conventional FIR or LMS algorithms. In our research, we used FIR algorithms to calculate the filter coefficients. The input $u(n)$ and the noise $\eta(n)$ are zero mean independent stationary processes. The Volterra kernels $h_1(k_1)$ and $h_2(k_2,k_1)$ are causal and absolutely summable sequences. It is obvious that a low pass filter removes Gaussian noise well in an image, but it also degrades important features such as edges. The filter performance can be improved if the filter is made adaptive with respect to the local features [5]. This can be implemented by considering nonlinear terms of the Volterra series which acts as higher order high pass filter. The block diagram of basic second order Volterra series filter structure is shown in Fig. 2, which has a linear part acts as low pass filter and a nonlinear part acts as a high pass filter.

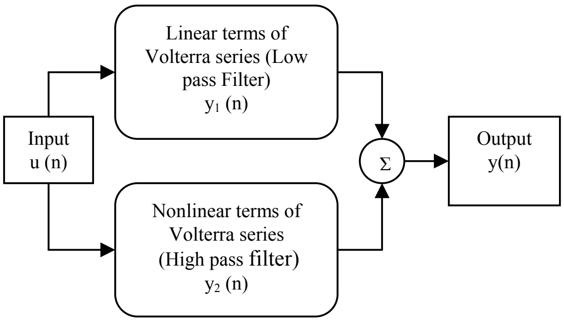

Fig. 2 Basic second order Volterra structure

The Proposed method has following steps.

Step-1: For the given image find the average intensity to select the appropriate cut-off frequency.

Step-2: Obtain the noisy image by including additive Gaussian and mixed Gaussian-impulse noise.

Step-3: Calculate the linear and nonlinear filter coefficients using FIR technique with suitable windowing method.

Step-4: Reduce the number of coefficients using Block Lexicographic matrix and symmetry conditions.

Step-5: Convolve the input noisy image with the filter coefficients to obtain the filtered image

Step-6: Evaluate the performance of the filter using different quantitative parameters.

IV. Design of Filter Kernels

The output sample of a 2-D second order Volterra filter for window $w = 3x3$ can be expressed as

$$y(i,j) = \sum_{i=1}^{9} h(i)u(i) + d\sum_{i=1}^{9}\sum_{j=1}^{9} \Psi(i,j)u(i)u(j) \qquad (5)$$

where d is a logic variable related to the output of the local decision algorithm. The value of d can be defined as

$$d = \begin{cases} 1 & if\ (\frac{1}{9}\sum_{i=1}^{9} u_i^2 - \bar{u}^2) \geq t \\ 0 & elsewhere \end{cases}$$

where t is the predefined threshold and it can be determined using the local variance estimator, and $u_i, \bar{u}$ are the input and mean value of the window w. The two conditions used to design the filter are

$$\sum_{i=1}^{9} h(i) = 1 \quad \text{and}$$

$$\sum_{i=1}^{9}\sum_{j=1}^{9} \Psi(i,j) = 0$$

The linear coefficients $h(i)$ of the proposed filter provide powerful noise cancellation in uniform gray zones. Since the sum of the linear coefficients of the filter is equal to one, it acts as a ideal low pass filter resulting in blur. The nonlinear coefficients $\Psi(i,j)$ compensate for the blurring due to the linear term and preserve the edges and high frequency components. The resulting image shows higher quality detail preserved image than that obtained by simple linear filtering.

In our work, we deal with $3x3$ complete window. The coefficients are formed using the FIR Hamming window technique. To reduce the number of coefficients of linear and nonlinear terms, we use the block lexicographic matrix technique [3]. In this technique Ψ is subdivided in to nine blocks of size $3x3$, with i fixed and j varying from 1 to 9.

To remove the redundancy in the coefficients the following symmetry conditions are imposed.

$$\Psi(i,j) = \Psi(j,i)$$

$$\Psi(|i|,|j|) = \Psi(i,j) \quad \text{and}$$

$$\Psi(|i|,|j|) = \Psi(|j|,|i|)$$

using the above symmetry conditions 9 linear and 81 nonlinear coefficients for the window $w = 3x3$ is reduced to 3 and 11 respectively. As an example, the reduced set of linear and nonlinear coefficients is determined for the cut-off frequency $w_c = 0.3$ are shown in Table I. The frequency response of the linear part, nonlinear part, ideal response and proposed Volterra filter for the calculated coefficients are shown in Fig. 3(a), 3(b), 3(c) and 3(d) respectively. The isotropy plots for the linear and nonlinear coefficients of the proposed Volterra filter shown in Fig. 3(a) and 3(b) are similar to the ideal response of low pass and high pass filters respectively. The response of these filters does not change for the reduced coefficients also. The overall behavior of the proposed Volterra filter shown in Fig. 3(d) does not deviate too much from the ideal response shown in Fig. 3(c). From the response of the proposed Volterra filter, we observe that it acts as low pass as well as high pass filter. The nonlinear response is controlled by the decision factor d and threshold t.

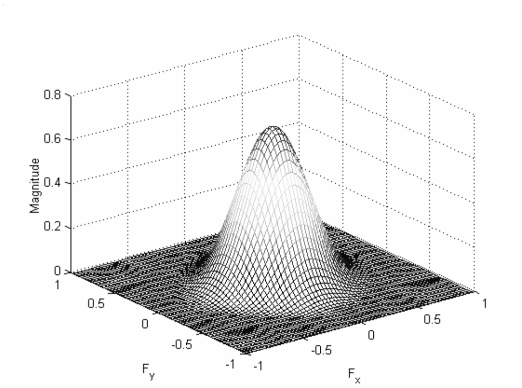

Fig. 3 (a) Linear part response of Volterra filter

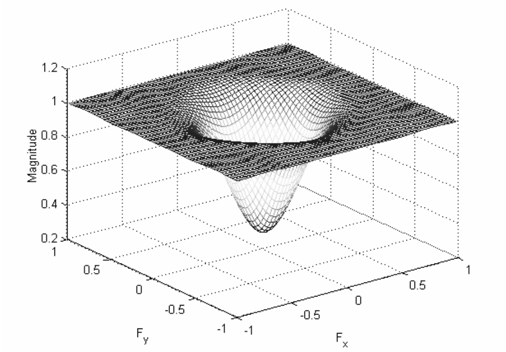

Fig. 3 (b) Nonlinear part response of Volterra filter

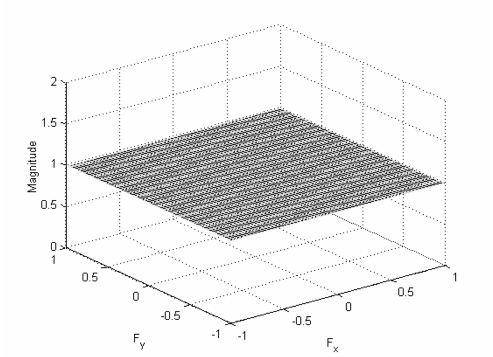

Fig. 3 (c) Ideal response of Volterra filter

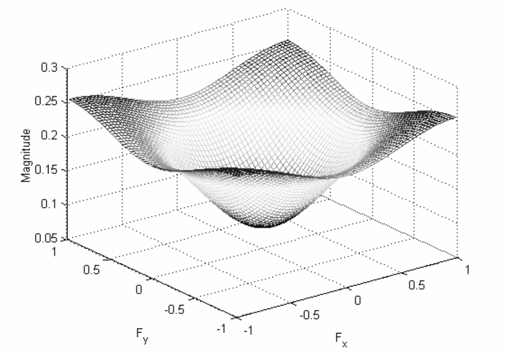

Fig. 3 (d) Response of proposed Volterra filter

TABLE I
FILTER COEFFICIENTS

(i, j)	Nonlinear coefficients $\Psi(i, j)$	(i)	Linear coefficients $h(i)$
(5,5)	0.7002	1(3,7,9)	0.0893
(2,8)	-0.2572	2(4,6,8)	0.2433
(2,5)	-0.1506	5	0.3276
(2,4)	-0.0324		
(2,2)	0.0457		
(1,9)	0.0255		
(1,6)	0.0103		
(1,5)	0.0078		
(1,3)	0.0029		
(1,2)	0.0007		
(1,1)	-0.0005		

Fig. 4 (a) Original image (b) Noisy image (σ = 0.02) (c) Low pass filter (d) High pass filter (e) Median filter (f) Volterra filter

V. RESULTS AND PERFORMANCE ANALYSIS

In this section we want to demonstrate the applications of Volterra filter to remove the Gaussian and mixed Gaussian impulse noise in real world images. The filter design in the previous sections was based on the idealized assumptions. To conduct the experiment we used the Lena image with 256x256 pixels and a dynamic range from 0 to 255, degrade by Gaussian and mixed Gaussian-impulse noise. Fig. 4 (a) shows the original image. The degraded image formed with Gaussian white noise and standard deviation $\sigma = 0.02$ is shown in Fig. 4 (b). The response of ideal Jong-sen low pass filter (JLPF), Jong-sen high pass filter (JHPF), median filter (MF) and Volterra filter (VF) are shown in Fig. 4 (c), 4 (d), 4 (e) and 4 (f) respectively.

In our work, we use the Jong-sen filter, described as

$$v(i, j) = m + k(u(i, j) - m) \tag{6}$$

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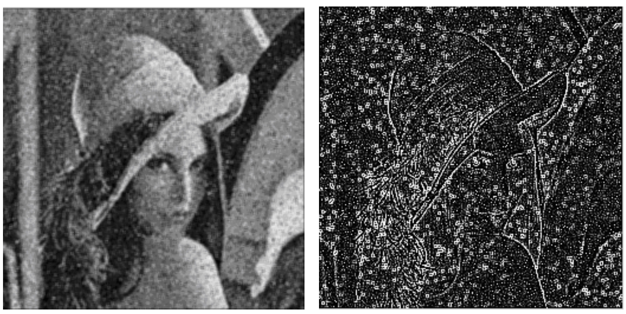
where $m = \frac{1}{N^2}\sum_{i=1}^{N}\sum_{j=1}^{N} u(i,j)$ is the mean and the value of k decides the nature of filtering operation. It acts as low pass filter for $k < 1$ and high pass filter for $k > 1$. From Fig. 4 (c) and 4 (d) we observe that the low pass filter tends to blur the spurious parts of the image and the high pass filter enhance the edges but suppress the prominent parts of the image. It is evident from Fig. 4 (e) that the median filter (MF) is not suitable for removing Gaussian noise. The output of proposed Volterra filter (VF) is shown in Fig. 4 (f) which removes the additive Gaussian noise effectively.

To test the robustness of the proposed Volterra filter, we consider the mixed Gaussian-Impulse noise with standard deviation $\sigma = 0.02$ and impulse noise density $\rho = 0.02$ shown in Fig. 5 (b). The output of low pass filters which smooths the mixed noise is shown in Fig. 5 (c). Since the high pass filter allows only the transition levels and attenuates uniform gray zones it is not suitable for impulse noise cancellation which is shown in Fig. 5 (d). The median filter is well suited for impulse noise but is not effective for Gaussian noise as observed in Fig. 5 (e). The output of the proposed Volterra filter is shown in Fig. 5 (f). From this we observe that the Volterra filter is most suitable for removing mixed noise also, because it exhibits the combined features of linear and nonlinear characteristics.

(a) (b)

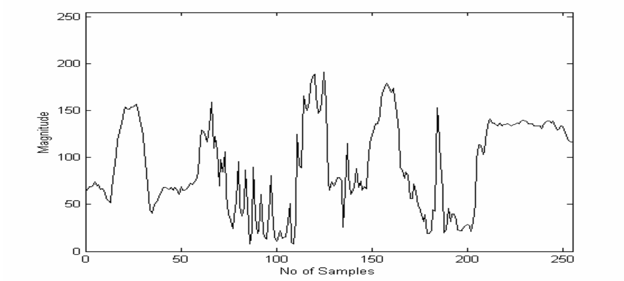


(c) (d)

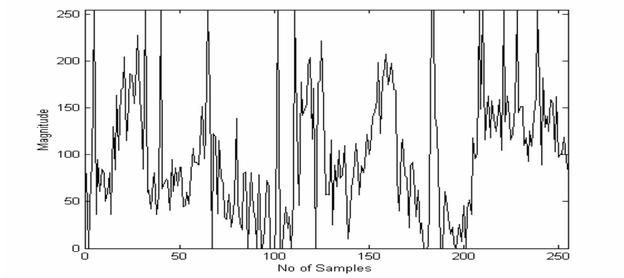
(e) (f)

Fig. 5 (a) Original image (b) Noisy image (σ = 0.02, ρ = 0.02) (c) Low pass filter (d) High pass filter (e) Median filter (f) Volterra filter

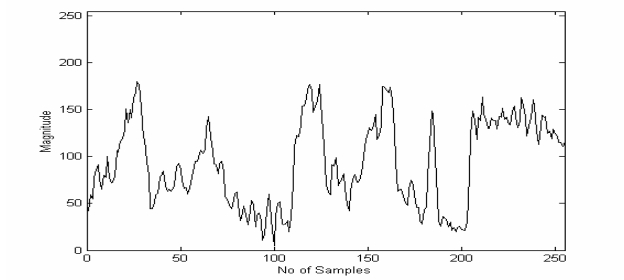
In order to appraise the noise cancellation behavior of the proposed filter, the luminance values of a row are graphically depicted in Fig. (6). The original noise free row number 128 is shown in Fig. 6 (a).The data affected by mixed noise are shown in Fig. 6 (b). The data processed by low pass filter, high pass filter and median filter are depicted in Fig. 6 (c), 6 (d) and 6 (e) respectively.



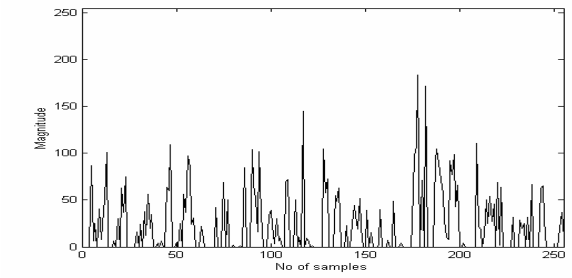
(a)



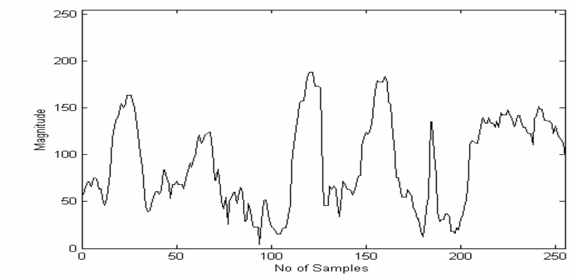
(b)



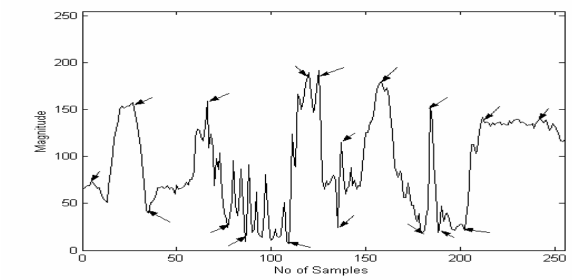
(c)



(d)



(e)



(f)

Fig. 6 Luminance values of the row 128: (a) Original image (b) Noisy image (σ = 0.02, ρ = 0.02) (c) Low pass filter (d) High pass filter (e) Median filter (f) Volterra filter

Finally, the result of proposed Volterra filter is shown in Fig. 6(f). Considering the data in Fig. 6(a) and 6(f), the good filtering behavior of the proposed filter is apparent. Mixed noise has been significantly reduced and image details have been satisfactorily preserved. The preserved and enhanced transitions are shown through arrows in Fig. 6(f).

The performance of the proposed Volterra filter is analyzed using the quantitative parameters such as signal to noise ratio (SNR) and mean square error (MSE). The SNR and MSE are computed as

$$SNR = 10 \log_{10} \left[\frac{\sum_{i=1}^{N} \sum_{j=1}^{N} v(i,j)^2}{\sum_{i=1}^{N} \sum_{j=1}^{N} (u(i,j) - v(i,j))^2} \right]$$

$$MSE = \frac{1}{NxN} \sum_{i=1}^{N} \sum_{j=1}^{N} (u(i,j) - v(i,j))^2$$

where NxN is the size of the image, $u(i,j)$ is the input image and $v(i,j)$ is the filtered image. Table II and Table III list these parameters for Gaussian and mixed Gaussian impulse noise. From these tables, we noticed that the proposed Volterra filter provide significant improvement in noise cancellation.

TABLE II
QUANTITATIVE ANALYSIS OF DIFFERENT FILTERS OUTPUT FOR GAUSSIAN NOISE

Parameters	Gaussian noise with (σ = 0.02)			
	JLPF	JHPF	MF	VF
MSE	0.4324	0.4793	0.3953	0.2168
SNR(dB)	26.2325	14.3018	26.3952	28.7987

TABLE III
QUANTITATIVE ANALYSIS OF DIFFERENT FILTERS OUTPUT FOR MIXED GAUSSIAN-IMPULSE NOISE

Parameters	Mixed Gaussian - Impulse noise with (σ = 0.02, ρ = 0.02)			
	JLPF	JHPF	MF	VF
MSE	0.4322	0.4885	0.4006	0.2675
SNR(dB)	24.2355	15.1567	24.7362	27.9756

VI. CONCLUDING REMARKS

In this paper, the proposed second order Volterra filter is designed by truncating the Volterra series. The coefficients are formed using FIR widowing technique. The coefficients are ordered using block lexicographic matrix technique. The redundant coefficients are removed using symmetry conditions. From the experimental results shown in Table II and Table III, the linear filters are not effective for Gaussian and mixed Gaussian impulse noise. The effect of median based filter is restricted for impulse noise only. The proposed $9x9$ mask for nonlinear part of Volterra filter is well suited for edge preserving and Gaussian noise cancellation. The $3x3$ linear mask smoothes the impulse noise effectively. The proposed filter has the linear and nonlinear behavior. We conclude that it is well suited for improving the image accuracy when the images are corrupted by impulse and Gaussian noises.

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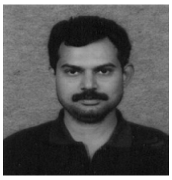

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