International Journal of Business, Human and Social Sciences
ISSN: 2517-9411
Vol:8, No:3, 2014

Volatility Switching between Two Regimes

Josip Visković, Josip Arnerić, Ante Rozga

***Abstract*—**Based on the fact that volatility is time varying in high frequency data and that periods of high volatility tend to cluster, the most successful and popular models in modeling time varying volatility are GARCH type models. When financial returns exhibit sudden jumps that are due to structural breaks, standard GARCH models show high volatility persistence, i.e. integrated behavior of the conditional variance. In such situations models in which the parameters are allowed to change over time are more appropriate. This paper compares different GARCH models in terms of their ability to describe structural changes in returns caused by financial crisis at stock markets of six selected central and east European countries. The empirical analysis demonstrates that Markov regime switching GARCH model resolves the problem of excessive persistence and outperforms uni-regime GARCH models in forecasting volatility when sudden switching occurs in response to financial crisis.

***Keywords*—**Central and east European countries, financial crisis, Markov switching GARCH model, transition probabilities.

I. Introduction

In the last few decades there has been enormous interest in forecasting of returns fluctuations at the financial markets. The first autoregressive conditional heteroscedasticity model (ARCH) was proposed as in [1]. The ARCH model was extended by its generalized version (GARCH) as in [2]. However, GARCH(1,1) model usually indicate high persistence in the conditional variance, which may originate from structural changes in the variance process. Hence the estimates of a GARCH model suffer from a substantial upward bias in the persistence parameters. Therefore, models in which the parameters are allowed to change over time may be more appropriate for volatility modeling. The main feature of regime switching model is the possibility for some or all the parameters of the model to switch across different regimes according to a Markov process, which is governed by a state variable s_t. Markov regime switching GARCH models allow different speeds of mean reversion of innovation process on different levels of variance in different time periods. Hence, in this paper Markov regime switching GARCH model, i.e. MRS-GARCH(1,1) is analyzed to describe structural changes in returns of referent stock indices caused by financial crisis at six stock markets from six different central and east European countries: Zagreb Stock Exchange (CROBEX), Prague Stock Exchange (PX 50), Budapest Stock Exchange (BUX), Ljubljana Stock Exchange (SBI 20), Bucharest Stock Exchange (BETI) and Sofia Stock Exchange (SOFIX).

Josip Visković is with the University of Split, Faculty of Economics, Cvite Fiskovića 5, 21000 Split, Croatia (phone: 00 385 21 430 751; fax: 00 385 21 430 751; e-mail: jviskovi@efst.hr).

Josip Arnerić is with the University of Zagreb, Faculty of Economics, Trg J. F. Kennedya 6, 10000 Zagreb, Croatia (e-mail: jarneric@efzg.hr).

Ante Rozga is with the University of Split, Faculty of Economics, Cvite Fiskovića 5, 21000 Split, Croatia (e-mail: rozga@efst.hr).

II. Changes in Conditional Variance Process

The most widespread approach to volatility modelling consists of the GARCH model [2] and its numerous extensions that can account for the volatility clustering and excess kurtosis found in financial time series. The accumulated evidence from empirical research suggests that the volatility of financial markets can be appropriately captured by standard GARCH(1,1) model:

$$\begin{aligned} r_t &= \mu_t + \varepsilon_t \\ \varepsilon_t &= u_t \cdot \sqrt{\sigma_t^2} \\ u_t &\sim i.i.d.(0,1) \\ \sigma_t^2 &= \alpha_0 + \alpha_1 \cdot \varepsilon_{t-1}^2 + \beta_1 \cdot \sigma_{t-1}^2, \end{aligned} \tag{1}$$

where μ_t is the conditional mean of return process $\{r_t\}$, whereas $\{\varepsilon_t\}$ is the innovation process with its multiplicative structure of identically and independently distributed random variables u_t. The last equation in (1) is conditional variance equation with GARCH(1,1) specification which means that variance of returns is conditioned on the information set I_{t-1} consisting of all relevant previous information up to period $t-1$. According to ARMA(1,1) representation of GARCH(1,1) it follows that GARCH(1,1) model is covariance-stationary if and only if $\alpha_1 + \beta_1 < 1$. In particular, GARCH(1,1) model usually indicates high persistence in the conditional variance, i.e. integrated behavior of the conditional variance when $\alpha_1 + \beta_1 = 1$ (IGARCH) as in [3]. The reason for the excessive GARCH forecasts in volatile periods may be the well known high persistence of individual shocks in those forecasts. Lamouoreux and Lastrapes [4] show that this persistence may originate from structural changes in the variance process, i.e. shifts in the unconditional variance lead to biased estimates of the GARCH parameters suggesting high persistence. High volatility persistence means that a long time period is needed for shocks in volatility to die out (mean reversion period). Haas et al. [5] demonstrates that existence of shifts in the variance process over time can induce volatility persistence. A popular approach to endogenize changes in the data generating process is the Markov regime switching model. Hamilton [6] introduces this model to describe the U.S. business cycle. In the context of stock market volatility, a Markov process can be used to govern the switches between regimes with different variances. In practical applications, the structural changes are rarely observed in advance and state variable cannot be predetermined. In addition the excess kurtosis implies that conditional distribution has fatter tails than the normal distribution, which means that large observations occur much more often than one might expect for a normally distributed variable as presented in [7]. Therefore, Markov switching models can be used for modeling possible time varying kurtosis when degrees of freedom are state dependent variable assuming Student t-distribution.

III. Regime Switching Garch Model

First application of regime switching parameters by combining Markov switching model with ARCH specification was proposed in [8]. Markov switching ARCH model was designed to capture regime changes in volatility with unobservable state variable S_t following the first order Markov Chain with constant transition probabilities. Hamilton and Susmel [8] used an ARCH specification instead of a GARCH to avoid infinite path dependence problem. It should also be noted that most regime switching models appear certain difficulties in parameters estimation. Therefore, several models with certain modifications and restrictions are proposed. The most general form of MRS-GARCH(1,1) model can be written as:

$$\begin{aligned} r_t &= \mu_{S_t} + \varepsilon_t \\ \varepsilon_t &= u_t \cdot \sigma_{t,S_t} \\ u_t &\sim i.i.d.(0,1) \\ \sigma^2_{t,S_t} &= \alpha_{0,S_t} + \alpha_{1,S_t} \cdot \varepsilon^2_{t-1} + \beta_{1,S_t} \cdot \sigma^2_{t-1}, \end{aligned} \tag{2}$$

where the unobserved state variable S_t is assumed to evolve according to the first-order Markov chain.

Transition probability $\Pr(S_t | S_{t-1}, S_{t-2}, ..., S_1, I_{t-1}) = \Pr(S_t = j | S_{t-1} = i)$ indicates the probability of switching from state i at moment $t-1$ into state j at moment t. When only two regimes are considered $S_t = \{1, 2\}$, i.e. low and high volatility regimes, these probabilities can be grouped together into probability transition matrix:

$$P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} = \begin{bmatrix} p & (1-p) \\ (1-q) & q \end{bmatrix}. \tag{3}$$

Based on transition probabilities $\Pr(S_t = j | S_{t-1} = i)$ the conditional probabilities $\Pr(S_t = j | I_{t-1})$ can be generated to define the likelihood function. Hence, in order to maximize the likelihood function the stochastic behavior of discrete state variable must be assumed. Probability density function for each observation is presented as a weighted sum of the conditional distribution functions for both regimes. Associated weights $\Pr(S_t = j | I_{t-1})$ are interpreted as the probability that the process at the moment t is in the state j, conditioned by a set of past information up to the moment $t-1$ as in [9]. These probabilities are called ex ante probabilities. Since the procedure of maximizing the likelihood function for parameters estimation is iterative in each new iteration the conditional probabilities $\Pr(S_t = j | I_{t-1})$ can be updated using Kim's filter (smoothing algorithm) [10]. For initial probabilities Hamilton [6] proposed unconditional probabilities of each state, i.e. steady state probabilities:

$$\begin{aligned} \pi_1 &= \Pr(S_0 = 1 | I_0) = \frac{1-p}{2-p-q}, \\ \pi_2 &= \Pr(S_0 = 2 | I_0) = \frac{1-q}{2-p-q}. \end{aligned} \tag{4}$$

Based on the transition probabilities the expected duration of the j state regime can be calculated as:

$$d_1 = \frac{1}{1-p} \quad , \quad d_2 = \frac{1}{1-q}. \tag{5}$$

For the initial value of the variance σ_0^2 by recursive substitution in the conditional variance equation of (2) it can be obtained:

$$\sigma_t^2 = \sum_{i=0}^{t-1} \left[\alpha_{0,S_{t-i}} + \alpha_{1,S_{t-i}} \cdot \varepsilon^2_{t-1-i} \right] \prod_{j=0}^{i-1} \beta_{1,S_{t-j}} + \sigma_0^2 \prod_{i=0}^{t-1} \beta_{1,S_{t-i}} . \tag{6}$$

Equation (6) shows that σ_t^2 depends on the entire history of regimes up to moment t, i.e. there is an infinite path dependence problem, which means that conditional variance at moment t depends on the conditional variance at moment $t-1$ and state variable at moment t whereas the conditional variance at moment $t-1$ depends on the conditional variance at moment $t-2$ and state variable at moment $t-1$ and so on, which makes the estimation infeasible in practice. In order to solve the problem of path dependence, it is proposed that conditional variance σ^2_{t-1} is given by information at moment $t-2$, i.e to substitute σ^2_{t-1} with conditional expected value $E_{t-2}(\sigma^2_{t-1})$ as in [11]. Reference [12] introduces modification of Gray's MRS-GARCH(1,1) model [11] by using conditional variance of ε_{t-1} given the information $t-1$, i.e. he suggests to substitute σ^2_{t-1} with conditional expected value $E_{t-1}(\sigma^2_{t-1} | S_t)$. In [12], Klaassen's MRS-GARCH(1,1) model is compared to standard GARCH(1,1) model.

IV. Empirical Results

The data set analyzed in this paper is the stock market daily closing price indices from six central and east European countries. The sample period is from January 02, 2006 to

December 28, 2012 for a total of 1500 observations according to trading days. Both the conditional mean and the conditional variance equations are estimated jointly by maximizing the log-likelihood function which is computed as the natural logarithm of the product of the conditional densities of the innovations, which are assumed to follow Student-t distribution. When estimating MRS-GARCH(1,1) model each regime takes different degrees of freedom of a Student t-distribution. Parameters α_1 and β_1 are also allowed to switch between two regimes, i.e. low volatility regime when $S_t = 1$ and high volatility regime when $S_t = 2$ as in [13]. Maximum likelihood estimates are obtained with Broyden, Fletcher, Goldfarb and Shanno (BFGS) numerical quasi-Newton optimization algorithm in the Time Series Modelling package (TSM 4.29). Specifically, in the standard GARCH(1,1) model 4 parameters were estimated, whereas degrees of freedom are pre-estimated using the method of moments (related to the kurtosis). In the case of MRS-GARCH(1,1) model, 10 parameters are estimated whereas degrees of freedom were estimated jointly with other unknown parameters depending on the two regimes.

The estimation results show that high volatility persistence ($\alpha_1 + \beta_1$) is associated with the standard uni-regime GARCH(1,1) model, which suggests that a long time is needed for shocks in volatility to die out (approximately 74 days for CROBEX, or even never for PX 50, SBI 20 and SOFIX). Although the covariance-stationarity condition is not satisfied for three stock indices the finite unconditional variance is overestimated (3.22% for CROBEX, 3.59% for BUX and 3,10% for BETI), due to the bias in the parameters.

TABLE I
ESTIMATED PARAMETERS AND DIAGNOSTIC TESTS OF GARCH(1,1) AND MRS-GARCH (1,1) MODELS WITH T-DISTRIBUTION FOR THE CROBEX AND PX 50 RETURNS

Parameters	CROBEX		PX 50	
	GARCH (1,1)	MRS-GARCH (1,1)	GARCH (1,1)	MRS-GARCH (1,1)
ϕ_0	0.00079**	0.00076**	0.00129**	0.00124**
α_0	0.00003*	0.000025*	0.00005*	0.00003**
α_1	0.1438**	0.2176**	0.2018**	0.1764**
β_1	0.8469**	0.6298*	0.8181**	0.8235**
df	4.6	4.93**	4.4	6.55**
$\lim_{t\to\infty} \sigma_t$	3.22%	1.81%	∞	1.77%
p	-	0.98165**	-	0.98853**
q	-	0.95904**	-	0.97466**
$\alpha_{1,2}$	-	0.0258**	-	0.1491*
$\beta_{1,2}$	-	0.9553**	-	0.8263**
df_2	-	4.70**	-	6.10**
$\ln L^*$	3358.9	3361.3	3250.2	3248.0
AIC	-6709.8	-6702.6	-6492.5	-6476.0
$SBIC$	-6689.8	-6652.5	-6472.4	-6425.9
$Q(10)$	5.4478	7.7996	19.9039*	17.3096
$Q^2(10)$	5.9526	9.7704	8.1053	8.0618
$LM(10)$	5.8366	9.6348	7.7673	7.5236
π_1	-	0.3094	-	0.3116
π_2	-	0.6906	-	0.6884
d_1	-	54	-	87
d_2	-	24	-	39

* indicates significance at 5% level, ** indicates significance at 1% level
Source: Author's calculation

TABLE II
ESTIMATED PARAMETERS AND DIAGNOSTIC TESTS OF GARCH(1,1) AND MRS-GARCH (1,1) MODELS WITH T-DISTRIBUTION FOR THE BETI AND SOFIX RETURNS

Parameters	BETI		SOFIX	
	GARCH (1,1)	MRS-GARCH (1,1)	GARCH (1,1)	MRS-GARCH (1,1)
ϕ_0	0.000029	0.00031	0.00056*	0.00059*
α_0	0.000023*	0.00006**	0.000004*	0.000073*
α_1	0.3211**	0.2639**	0.3477**	0.4353**
β_1	0.6543**	0.7244**	0.6989**	0.5661**
df	6.5	6.60**	5.1	7.31**
$\lim_{t\to\infty}\sigma_t$	3.10%	1.95%	∞	1.64%
p	-	0.98334**	-	0.98091**
q	-	0.96503**	-	0.94550**
$\alpha_{1,2}$	-	0.0010*	-	0.0212*
$\beta_{1,2}$	-	0.9989**	-	0.9787**
df_2	-	6.50*	-	5.26**
$\ln L^*$	2975.7	2981.3	3496.8	3452.3
AIC	-5943.4	-5942.6	-6985.6	-6884.6
$SBIC$	-5923.3	-5892.4	-6965.6	-6834.4
$Q(10)$	24.8700	13.3286	19.1780	12.9041
$Q^2(10)$	8.3520	4.6406	7.1900	6.5117
$LM(10)$	8.4418	5.3218	7.6255	5.7754
π_1	-	0.3227	-	0.2594
π_2	-	0.6773	-	0.7406
d_1	-	60	-	52
d_2	-	29	-	18

* indicates significance at 5% level, ** indicates significance at 1% level
Source: Author's calculation

Obviously, persistence volatility is lower in the regimes of low volatility compared to the persistence in the regime of high volatility. For example, this means that in the regime of low volatility of CROBEX index the mean reversion period is 4 days, whereas in the high volatility regime 36 days are needed for shocks in volatility to die out. However, in the regime of low volatility of BUX index the mean reversion period is 2 days, whereas in the high volatility regime 38 days are needed for shocks in volatility to die out. Also, estimated transition probabilities indicate that the expected duration of the regime of low volatility for CROBEX of 54 days is more than the expected duration of the regime of high volatility (24 days).

TABLE III
ESTIMATED PARAMETERS AND DIAGNOSTIC TESTS OF GARCH(1,1) AND MRS-GARCH (1,1) MODELS WITH T-DISTRIBUTION FOR THE BUX AND SBI 20 RETURNS

Parameters	BUX		SBI 20	
	GARCH (1,1)	MRS-GARCH (1,1)	GARCH (1,1)	MRS-GARCH (1,1)
ϕ_0	0.000651	0.00037	0.000191	0.00029
α_0	0.000007*	0.00007*	0.000002*	0.000053*
α_1	0.1304**	0.1927**	0.2472**	0.6143**
β_1	0.8691**	0.4678*	0.7679**	0.3289**
df	4.8	6.56**	4.8	9.61**
$\lim_{t\to\infty}\sigma_t$	3.59%	2.3%	∞	1.15%
p	-	0.99108**	-	0.97490**
q	-	0.97580**	-	0.91463**
$\alpha_{1,2}$	-	0.0988*	-	0.0226*
$\beta_{1,2}$	-	0.8832**	-	0.9509**
df_2	-	5.46**	-	8.48**
$\ln L^*$	3025.3	3031.1	3811.8	3816.7
AIC	-6042.7	-6042.2	-7615.5	-7613.4
$SBIC$	-6022.7	-5992.1	-7595.4	-7563.2
$Q(10)$	22.437*	12.0680	10.2000	10.9301
$Q^2(10)$	10.4400	7.9551	6.1200	6.6165
$LM(10)$	8.6545	7.3852	6.1908	6.7862
π_1	-	0.2693	-	0.2272
π_2	-	0.7307	-	0.7728
d_1	-	112	-	40
d_2	-	41	-	12

* indicates significance at 5% level, ** indicates significance at 1% level
Source: Author's calculation

Ljung-Box statistics $Q(10)$, $Q^2(10)$ and Lagrange multiplier test $LM(10)$ are not statistically significant, suggesting that there is no ARCH effects or autocorrelation of standardized residuals (mean and variance equations are correctly specified). The value of AIC and SBIC information criteria is less in a standard GARCH(1,1) model compared with MRS-GARCH(1,1) model, because 10 parameters are estimated in regime switching model.

V. CONCLUSION

Due to the effects of structural changes in the return series, caused by the financial crisis, sum of the parameters $\alpha_1 + \beta_1$ in standard GARCH(1,1) model indicates high volatility persistence. In such situations models in which the parameters are allowed to change over time are more appropriate, i.e. Markov switching GARCH model. Regime switching

GARCH models allow different speeds of mean reversion of innovation process on different levels of variance in different time periods. Estimation results show that volatility persistence is lower in the regime of low volatility compared to the persistence in the regime of high volatility in selected central and east European countries. Estimated transition probabilities indicate that the expected duration of the regime of low volatility is more than the expected duration of the regime of high volatility. It is also important to point out that the variability of degrees of freedom of the Student t-distribution in MRS-GARCH(1,1) model suggests that degrees of freedom for a period of low volatility are greater in comparison to a period of high volatility. Specially, the estimated lower degrees of freedom in periods of increased volatility indicate that the occurrence of extremely low (negative) returns, as well as extremely high (positive) returns is more frequent. The empirical analysis demonstrates that the Markov regime switching GARCH model resolves the problem of excessive persistence and outperforms uni-regime GARCH models in forecasting volatility when sudden switching occurs in response to financial crisis.

References

[1] F. R. Engle, “Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of U.K. Inflation”, *Econometrica,* Vol. 50, pp. 987-1108., 1892.

[2] T. Bollerslev, “Generalized Autoregressive Conditional Heteroskedasticity”, *Journal of Econometrics*, Vol 31. pp. 307-327, 1986).

[3] F. R. Engle “The Use of ARCH/GARCH Models in Applied Econometrics”, *Journal of Economic Perspectives*, Vol. 15, No. 4, pp. 157-168., 2001.

[4] C. G. Lamouoreux and W. D. Lastrapes (1990) “Heteroskedasticity in Stock Return Data: Volume versus GARCH Effects”, *Journal of Finance*, Vol 45. No. 1., pp. 221-229, 1990

[5] M. Haas, S. Mittnik and S. M. Paolella (2004), “A New Approach to Markov-Switching GARCH Models”, *Journal of Financial Econometrics*, Vol. 2, No. 4, pp. 493-530.

[6] J. D. Hamilton, “A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle,” *Econometrica,* Vol 57, pp. 357-384, 1989.

[7] A. M. Carnero, “Persistence and Kurtosis in GARCH and Stohastic Volatility Models”, *Journal of Financial Econometrics*, Oxford University Press, Vol. 2, No. 2, pp. 319-342., 2004.

[8] J. D. Hamilton and R. Susmel “ARCH and Changes in Regime”, *Journal of Econometrics*, Vol. 64, pp. 307-333, 1994.

[9] J. Marcucci, “Forecasting Stock Market Volatility with Regime-Switching GARCH Models”, *Studies in Nonlinear Dynamics and Econometrics*, Vol 9, No. 4, pp. 1-42, 2005.

[10] C-J. Kim and C. R. Nelson, *State-Space Models with Regime Switching (Classical and Gibbs-Sampling Approaches with Applications)*, Massachusetts Institute of Technology Press, Cambridge, 1999.

[11] S. F. Gray, “Modeling the conditional distribution of interest rates as a regime-switching process, *Journal of Financial Economics*, Vol. 42, pp. 27-62, 1996.

[12] F. Klaassen, “Improving GARCH Volatility Forecasts with Regime-Switching GARCH”, *Empirical Economics*, Vol. 27, No. 2, pp. 363-394, 2002.

[13] L. Bauwnes Luc, A. Preminger and J. Romboust, “Regime Switching GARCH Models”, CORE Discussion Paper, No. 11, pp. 1-23. 2006.