# Versatile Dual-Mode Class-AB Four-Quadrant Analog Multiplier 

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#### Abstract

Versatile dual-mode class-AB CMOS four-quadrant analog multiplier circuit is presented. The dual translinear loops and current mirrors are the basic building blocks in realization scheme. This technique provides; wide dynamic range, wide-bandwidth response and low power consumption. The major advantages of this approach are; its has single ended inputs; since its input is dual translinear loop operate in class-AB mode which make this multiplier configuration interesting for low-power applications; current multiplying, voltage multiplying, or current and voltage multiplying can be obtainable with balanced input. The simulation results of versatile analog multiplier demonstrate a linearity error of $1.2 \%$, a -3 dB bandwidth of about 19 MHz , a maximum power consumption of 0.46 mW , and temperature compensated. Operation of versatile analog multiplier was also confirmed through an experiment using CMOS transistor array.


Keywords-Class-AB, dual-mode CMOS analog multiplier, CMOS analog integrated circuit, CMOS translinear integrated circuit.

## I. Introduction

ANALOG multiplier are important nonlinear analog signal processing function finding application of a wide variety in adaptive filtering, modulation, frequency translation, automatic gain controlling, neural network, etc. At present, the power consumption is a key parameter in the designing of high performance mixed-signal integrated circuit. As CMOS technology is widely recognized as the most desirable technology for integrated circuits implementation [1]. Therefore, some of CMOS multipliers [2]-[6] are not very optimal lowvoltage low-power applications. Several techniques of reducing power consumption, CMOS analog multiplier circuit, have recently been described. They are the usable floatinggate MOS [7]-[9], bulk driven MOS [10], [11], subthreshold mode [11]-[13], or class-AB mode [14], [15]. However, these analog multiplier circuits have been proposed with multiplying either in voltage or current form.

In this paper, we present a versatile dual-mode analog multiplier circuit using dual translinear loops. The resulting input signal of the analog multiplier circuit can be voltage and/or current. Namely, it properly combines a voltage multi-

[^0]plier, a current multiplier, a current and a voltage multiplier into a circuit (we called the versatile analog multiplier circuit [16], [17]). The class-AB technique will be chosen to reduce the power consumption circuit in a standard CMOS technology. The advantages of this technique are;

1) special processing and calibration steps associated with floating gate devices are avoided;
2) the input transistor pair gain reduction and input impedance reduction associated with a bulk driven avoided;
3) high frequency response reduction associated with device operate in subthreshold mode are avoided;
4) standard CMOS technology can be used.

In addition, based on the dual translinear loop allows the design of analog multiplier circuit that exhibit wide bandwidth, high dynamic range and high speed [18], [19]. The proposed analog multiplier has single ended input, which no requires additional hardware and consequently more silicon area. Thus, it can be more easily in a large system. PSPICE simulation and experimental results are confirmed to characterize the performance of the proposed circuit.

## II. Circuit Descriptions

## A. Class-AB CMOS Squaring Circuit

An MOS version of the dual translinear loop for realizing the proposed versatile dual-mode analog multiplier circuit is shown in Fig. 1. The circuit consists of a dual translinear loop (M1-M4) and a current mirror (M5-M6). Generally the drain current $I_{D}$ of an MOS transistor operated in saturation by neglecting the second order effect such as mobility reduction and channel-length modulation can be expressed as
or

$$
\begin{align*}
& I_{D}=K\left(V_{G S}-V_{T}\right)^{2}  \tag{1.1}\\
& V_{G S}=V_{T}+\sqrt{\frac{I_{D}}{K}} \tag{1.2}
\end{align*}
$$

where $\mathrm{K}=0.5 \mu_{\mathrm{o}} \mathrm{C}_{\mathrm{OX}}(\mathrm{W} / \mathrm{L})$ is transconductance parameter of transistor, $\mu_{\mathrm{O}}$ is the electron mobility, $\mathrm{C}_{\mathrm{OX}}$ is the gate oxide capacitance per unit area, W/L is the transistor aspect ratio, $\mathrm{V}_{\mathrm{GS}}$ is the gate-to-source voltages and $\mathrm{V}_{\mathrm{T}}$ is threshold voltage of the MOS transistor, respectively.


Fig. 1 Basic building block


Fig. 2 Squaring and copy current into a single circuit
Consider a loop of MOS transistor M1 to M4, summing the gate-source voltages around the loop gives [20]:

$$
\begin{equation*}
V_{G S 1}+V_{G S 3}=V_{G S 2}+V_{G S 4} \tag{2}
\end{equation*}
$$

Assuming that MOS transistors M1 to M4 from a dual translinear loop in Fig. 1 are biased in the saturation region and all transistors are well matched and having the same value of the transconductance parameter, that is $\mathrm{K}_{\mathrm{N}}=\mathrm{K}_{\mathrm{P}}$, then using (1)-(2), the relationship of the current $\mathrm{I}_{\mathrm{O}}, \mathrm{I}_{\mathrm{D} 2}$ and $\mathrm{I}_{\mathrm{D} 4}$ in the loop can be described as

$$
\begin{equation*}
2 \sqrt{I_{O}}=\sqrt{I_{D 2}}+\sqrt{I_{D 4}} \tag{3}
\end{equation*}
$$

The drain current $\mathrm{I}_{\mathrm{D} 1}$ and $\mathrm{I}_{\mathrm{D} 3}$ are equal constant current source $\mathrm{I}_{\mathrm{O}}$, thus from the class-AB principle [14] and [15], it can be shown that the current $\mathrm{I}_{\mathrm{D} 2}$ and $\mathrm{I}_{\mathrm{D} 4}$ are equal;

$$
\begin{equation*}
I_{D 2}=I_{O}-\frac{I_{i n}}{2}+\frac{I_{i n}^{2}}{16 I_{O}} \tag{4}
\end{equation*}
$$

and

$$
\begin{equation*}
I_{D 4}=I_{O}+\frac{I_{i n}}{2}+\frac{I_{i n}^{2}}{16 I_{O}} \tag{5}
\end{equation*}
$$

The $V_{\text {in }}$ and $\mathrm{I}_{\text {in }}$ are the input voltage at port Y and the input current at port X , respectively, $\mathrm{R}_{\mathrm{X}}$ is conversions resistance. Assume the current mirror (M5, M6) has a perfect unity current gain. The current $\mathrm{I}_{\mathrm{D} 2}$ adds to the current $\mathrm{I}_{\mathrm{D} 2}$, the output current $\mathrm{I}_{\text {out }}$ of the circuit in Fig. 1 can be written as

$$
\begin{equation*}
I_{\text {out }}=2 I_{O}+\frac{I_{\text {in }}^{2}}{8 I_{O}} \tag{6}
\end{equation*}
$$

It can see from (6) this results is the current squarer of input signal, a basic building block of the class-AB CMOS analog multiplier. It should be noted that, in a voltage input mode, the input voltage will be applied at the port Y. Since the dual translinear loop is also working as a voltage-to-current converter where $\mathrm{V}_{\text {in }}=\mathrm{V}_{\mathrm{X}}=\mathrm{V}_{\mathrm{Y}}$ [20] and $\mathrm{R}_{\mathrm{X}}$ is the conversion resistance, the output current $I_{\text {out }}$ in this case can be given by

$$
\begin{equation*}
I_{\text {out }}=2 I_{O}+\frac{V_{\text {in }}^{2}}{8 R_{\text {in }}^{2} I_{O}} \tag{7}
\end{equation*}
$$

From (6) and (7), we see that the input signal can be voltage or current. The circuit is shown in Fig. 1, it is modified to combine a squarer and a copier into a single circuit while maintaining the class- AB operates, which can be shown in Fig. 2. Assume it is well matched current mirror (M8, M10), therefore it can be given as

$$
\begin{equation*}
I_{S Q}=2 I_{O}+\frac{I_{i n}^{2}}{8 I_{O}} \tag{8}
\end{equation*}
$$

By setting the aspect ratio (W/L) of the M5 twice of M7, therefore, the current $\mathrm{I}_{\mathrm{Z}}$ is rewritten as

$$
\begin{equation*}
I_{Z}=I_{i n} \tag{9}
\end{equation*}
$$

The circuit as shown in Fig. 2 is a circuit having both a squarer and a copier into a single circuit that is used for key building block of the proposed class-AB CMOS analog multiplier.

## B. The Proposed Analog Multiplier Circuit

The basic principle of operation of the proposed multiplier is based on the square-difference identity: $(\mathrm{X}+\mathrm{Y})^{2}-\mathrm{X}^{2}-$ $\mathrm{Y}^{2}=2 \mathrm{XY}$. The proposed analog multiplier circuit is shown in Fig. 3. It is based on the class-AB squaring circuit of the Fig. 2 and three dual translinear loops was used. First loop (M1 to M4) provides a X input function ( $\mathrm{I}_{\mathrm{X}}$ or $\mathrm{V}_{\mathrm{X}}$ ) to squarer function $\mathrm{X}^{2}$, second loop (M4 to M6) provides a Y input function ( $\mathrm{I}_{\mathrm{Y}}$ or $\mathrm{V}_{\mathrm{Y}}$ ) to be squarer function $\mathrm{Y}^{2}$ and providing a squarer function $(\mathrm{X}+\mathrm{Y})^{2}$ for the third loop.


Fig. 3 Proposed versatile dual-mode class-AB analog CMOS multiplier


Fig. 4 Linear MOS transistor
Assuming current mirrors, M21-M25 and M19-M20 has a perfect unity current gain, thus
and

$$
\begin{gather*}
I_{D 20}=2 I_{O}+\frac{\left(I_{X}+I_{Y}\right)^{2}}{8 I_{O}}  \tag{10}\\
I_{D 25}=4 I_{O}+\frac{I_{X}^{2}+I_{Y}^{2}}{8 I_{O}} \tag{11}
\end{gather*}
$$

From Kirchhoff's current law, it is written that

$$
\begin{equation*}
I_{\text {out }}=I_{D 20}-I_{D 25}+I_{\text {off }} \tag{12}
\end{equation*}
$$

Substitute (10), (11) into (12), results the output current of the analog multiplier can be defined and expressed as

$$
\begin{equation*}
I_{\text {out }}=\frac{I_{X} I_{Y}}{4 I_{O}}-2 I_{O}+I_{\text {off }} \tag{13}
\end{equation*}
$$

To compensate the bias current $2 \mathrm{I}_{\mathrm{O}}$ with constant current $\mathrm{I}_{\text {off }}$ ( $\mathrm{I}_{\text {off }}=2 \mathrm{I}_{\mathrm{O}}$ ), the current $\mathrm{I}_{\text {out }}$ can be rewritten as

$$
\begin{equation*}
I_{o u t}=\frac{I_{X} I_{Y}}{4 I_{O}} \tag{14}
\end{equation*}
$$

It is clearly seen that the current $\mathrm{I}_{\text {out }}$ is in the form of a multiplier value as required. Consider (14), the multiplier has function as the current multiplier when $\mathrm{V}_{\mathrm{X}}=\mathrm{V}_{\mathrm{Y}}=0$ or connecting voltage input to ground.

By using (7) and setting $R_{X}=R_{Y}=R_{L}$, give $I_{X}$ and $I_{Y}$ is connected to grounded resistor $\mathrm{R}_{\mathrm{X}}$ and $\mathrm{R}_{\mathrm{Y}}$, respectively, the multiplier has function as the voltage multiplier.

$$
\begin{equation*}
V_{\text {out }}=\frac{V_{X} V_{Y}}{4 I_{O}} \tag{15}
\end{equation*}
$$

If $V_{X}$ is attached to ground, $I_{Y}$ is connected to grounded resistor or $\mathrm{V}_{\mathrm{X}}$ is attached to ground, $\mathrm{I}_{\mathrm{X}}$ is connected to grounded resistor, the multiplier has function as the current and voltage multiplier.
and

$$
\begin{equation*}
V_{\text {out }}=\frac{I_{X} V_{Y}}{4 I_{O}} \tag{16}
\end{equation*}
$$

From the multi-function of the multiplier, the proposed multiplier called the versatile analog multiplier as same in [16], [17]. It should be noted from the equations (14)-(17) that, since no terms of thansconductance parameters and threshold voltages of MOS transistors that cause of temperature effect are included in the output current $I_{\text {out }}$ function and output voltage $V_{\text {out }}$ function. It means that the output current and voltage are less sensitive than temperature.

For the input case is voltage form, an analog multiplier is re-quired to grounded resistors for V-I conversion. The MOS implementation of resistors which is used instead of their resistors ( $\mathrm{R}_{\mathrm{X}}, \mathrm{R}_{\mathrm{Y}}$ ), can be obtained using only two MOS transistor, is shown in Fig. 4 [21]. The resistance values can be given as:

$$
\begin{equation*}
R_{e q}=\frac{1}{4 K\left(V_{D D}-V_{T N}\right)} \tag{18}
\end{equation*}
$$

TABLE I
Functions for Versatile Analog Multiplier

| Function | Condition |
| :---: | :---: |
| $I_{\text {out }}=\frac{I_{X} I_{Y}}{2 I_{O}}$ | $V_{X}=V_{Y}=G N D$ |
| $V_{\text {out }}=\frac{V_{X} V_{Y}}{2 I_{O}}$ | $I_{X}=I_{Y}=R$ |
| $V_{\text {out }}=\frac{I_{X} V_{Y}}{2 I_{O}}$ | $V_{X}=G N D, I_{Y}=R$ |
| $V_{\text {out }}=\frac{V_{X} I_{Y}}{2 I_{O}}$ | $I_{X}=R, V_{Y}=G N D$ |

when assume that MOS transistor MR1 and MR2 are matched, operated in saturation region, and have the same characteristic; $\mathrm{K}=\mu_{\mathrm{O}} \mathrm{C}_{\mathrm{OX}} \mathrm{W} / L, \mathrm{~V}_{\mathrm{DD}}=-\mathrm{V}_{\mathrm{SS}}$ and $\mathrm{V}_{\mathrm{TN}}$.

It can be seen from equations (14) and (17) that for an analog multiplier circuit the voltage or current can be applied. Addition, the output signal can be product of any of current and voltage form which shown in (16) and (17). All functions are listed in Table I. Finally, the circuit is suitable for IC implementation when resistors can be used instead of MOS resistors that shown in Fig. 4.

## III. Performance Analysis

In this section, the characteristics of analog multiplier will be analyzed in Fig. 3. Input range, input and output impedance, mismatched and second-order effects, and frequency responses are discussed.

## A. Mismatch and Second-Order Effects

In this section, the contribution consideration of the component mismatches on the distortion of the analog multiplier circuit. The transconductance and threshold voltage mismatch and second-order effect will be discussed.

The mismatching between the transconductance parameters, $K_{N}$ and $K_{P}$, of the $n$-channel and p-channel MOS transistors of the class -AB configuration of Fig. 1 is, therefore, it is considered as the major contributor to the errors from the ideal per-formance [12]. Under this assumption, (3) becomes;

$$
\begin{equation*}
\sqrt{\frac{I_{O}}{K}}+\sqrt{\frac{I_{O}}{K+\Delta K}}=\sqrt{\frac{I_{D 2}}{K}}+\sqrt{\frac{I_{D 4}}{K+\Delta K}} \tag{19}
\end{equation*}
$$

with $\mathrm{K}_{\mathrm{N}}=\mathrm{K}, \mathrm{K}_{\mathrm{P}}=\mathrm{K}+\Delta \mathrm{K}$, using the approximation $\left(1+\mathrm{I}_{\mathrm{in}}\right)^{-1 / 2} \approx 1-$ $(1 / 2) \mathrm{I}_{\mathrm{in}}$ if $\mathrm{I}_{\mathrm{in}}<1$, ignoring terms containing $\Delta^{2}$. Assume that $\left.4(1-\Delta) \mathrm{I}_{\mathrm{in}}\right\rangle(\Delta / 2) \mathrm{I}_{\mathrm{in}}$, it can be rewritten the (4) and (5) as follows;
and

$$
\begin{align*}
I_{D 2} & \approx I_{O}-\left(\frac{1}{2}-\frac{1}{\Delta}\right) \frac{I_{i n}}{2}+\left(1-\frac{4}{\Delta}\right) \frac{I_{i n}^{2}}{16 I_{O}}  \tag{20}\\
I_{D 4} & \approx I_{O}+\left(\frac{1}{2}-\frac{1}{\Delta}\right) \frac{I_{i n}}{2}+\left(1-\frac{4}{\Delta}\right) \frac{I_{i n}^{2}}{16 I_{O}} \tag{21}
\end{align*}
$$

Therefore, the output current $\mathrm{I}_{\text {out }}$ of the circuit in Fig. 1 can be rewritten as;

$$
\begin{equation*}
I_{o u t} \approx 2 I_{O}+\frac{I_{i n}^{2}}{8 I_{O}}+\frac{I_{i n}}{\Delta}-\frac{I_{i n}^{2}}{4 \Delta I_{O}} \tag{22}
\end{equation*}
$$

From (24) implies to the first order, transconductance mismatch will affect to slope and the offset in DC squaring characteristic.
For the proposed analog multiplier that shown in Fig. 3, using (20) and (21), (10) and (11) can be rewritten as follows;

$$
\begin{equation*}
I_{D 20} \approx 2 I_{O}+\frac{\left(I_{X}+I_{Y}\right)^{2}}{8 I_{O}}+\frac{I_{X}+I_{Y}}{\Delta}-\frac{\left(I_{X}+I_{Y}\right)^{2}}{\Delta 4 I_{O}} \tag{23}
\end{equation*}
$$

and $\quad I_{D 25} \approx 4 I_{O}+\frac{I_{X}^{2}+I_{Y}^{2}}{8 I_{O}}+\frac{I_{X}+I_{Y}}{\Delta}-\frac{I_{X}^{2}+I_{Y}^{2}}{\Delta 4 I_{O}}$

Therefore, the output current $\mathrm{I}_{\text {out }}$ of the analog multiplier circuit can be rewritten as follows:
and

$$
\begin{align*}
& I_{o u t} \approx \frac{I_{X} I_{Y}}{4 I_{O}}-\frac{I_{X} I_{Y}}{\Delta 4 I_{O}}  \tag{25}\\
& V_{\text {out }} \approx \frac{V_{X} V_{Y}}{4 I_{O}}-\frac{V_{X} V_{Y}}{\Delta 4 I_{O}} \tag{26}
\end{align*}
$$

From (27) and (28) imply that the transconductance mismatch will dominate in first order harmonic distortion.

The second order effects such as body effect, mobility reduction, and channel-length modulation are considered next. The second source of nonlinear is mobility degradation. The mobility variation is reduced by maximizing the gate-source voltage of the class-AB MOS transistors. Channel length modulation is another source of nonlinear. To reduce this effect, it is necessary to stabilize the drain-source voltage of the class-AB transistors by some effect, such as using a sufficiently large channel length throughout the circuit or cascading this also improves the current mirroring accuracy. The body effect can be reduced by connecting source and bulk of MOS transistor.

## B. Input Range and Input/Output Resistance

The voltage input range of the multiplier is restricted by dual translinear loop, M1-M4 and M5-M8, and MOS transistors M13, M16, M21, operate in the saturation region. If we assume that the $\mathrm{V}_{\mathrm{DS}}$ of each MOS transistors are high enough to satisfy the saturation condition. From the class-AB principle, the MOS transistor M2 and M4 alternating operate (M1off, M3-on or M2-on, M4-off). Therefore, the voltage input range of input $\mathrm{V}_{\mathrm{X}}$ can be written as
$V_{S S}+2 \sqrt{\frac{I_{O}}{K_{13}}}-\sqrt{\frac{I_{O}}{K_{3}}}+V_{T 13} \leq V_{X} \leq V_{D D}+2 \sqrt{\frac{I_{O}}{K_{21}}}-\sqrt{\frac{I_{O}}{K_{1}}}+V_{T 21}$

$$
\begin{equation*}
\frac{I_{\text {out }}}{V_{X}}=\frac{\left(\frac{g_{m 2}\left(R_{X}\left(C_{g s 1}+C_{g s 2}\right) s+1\right)}{1+R_{X} g_{m 2}}\right)}{\left(\frac{\left(C_{g s 1}+C_{g s 2}\right) s}{g_{m 2}}-1\right)\left(\frac{\left(C_{g s 19}+C_{g s 20}\right) s}{g_{m 20}}-1\right)\left(\frac{\left(C_{g 521}+C_{g s 22}\right) s}{g_{m 22}}-1\right)\left(\frac{R_{X} C_{g 52} s}{1+R_{X} g_{m 2}}+1\right)} \tag{34}
\end{equation*}
$$

whereas, the input $\mathrm{V}_{\mathrm{Y}}$ restricts the MOS transistor M5 and M6 is alternating operates (M6-off, M8-on or M5-on, M7-off). Thus, input voltage range of input $\mathrm{V}_{\mathrm{Y}}$ can be written as

$$
\begin{equation*}
V_{S S}+2 \sqrt{\frac{I_{O}}{K_{16}}}-\sqrt{\frac{I_{O}}{K_{7}}}+V_{T 16} \leq V_{Y} \leq V_{D D}+2 \sqrt{\frac{I_{O}}{K_{21}}}-\sqrt{\frac{I_{O}}{K_{5}}}+V_{T 21}(2 \tag{28}
\end{equation*}
$$

Again, the value of input currents, $\mathrm{I}_{\mathrm{X}}$ and $\mathrm{I}_{\mathrm{Y}}$, is restricted by the saturation condition of an MOS transistor of the dual trans-linear loop (M1 toM8). Therefore the input current range of proposed multiplier should be given by

$$
\begin{equation*}
\left|I_{X}\right|=\left|I_{Y}\right| \leq 2 I_{O} \tag{29}
\end{equation*}
$$

The analog multiplier in Fig. 3 can fine the input and output impedance by take it into the small-small circuit. Assume that; $\mathrm{r}_{\mathrm{Io}}<1 / \mathrm{g}_{\mathrm{ml}}, 1 / \mathrm{g}_{\mathrm{m} 5}<1 / \mathrm{gm}^{2}$ and $1 / \mathrm{g}_{\mathrm{m} 7}$, where $\mathrm{r}_{\mathrm{Io}}$ is inner resistance of current source $\mathrm{I}_{\mathrm{O}}, \mathrm{r}_{\mathrm{dj}}$ and $\mathrm{g}_{\mathrm{mj}}$ are drain-source resistances and transconductances, respectively, of MOS transistor Mj , Therefore, the input impedances at port $\mathrm{V}_{\mathrm{X}}$ and $\mathrm{V}_{\mathrm{Y}}$ can be given as;

$$
\begin{equation*}
r_{V_{X}}=r_{V_{Y}} \cong \frac{r_{I_{O}}}{2} \tag{30}
\end{equation*}
$$

The input impedance at port $\mathrm{I}_{\mathrm{X}}$ and $\mathrm{I}_{\mathrm{Y}}$ can be derived as;
and

$$
\begin{align*}
& r_{I_{X}}=\frac{1}{\left(g_{m 2}+g_{m 4}\right)+\left(\frac{g_{m 4} g_{m 14} g_{d 2}}{g_{m 21} g_{d 13}}\right)} \\
& r_{I_{Y}}=\frac{1}{\left(g_{m 6}+g_{m 8}\right)+\left(\frac{g_{m 8} g_{m 15} g_{d 6}}{g_{m 21} g_{d 16}}\right)} \tag{32}
\end{align*}
$$

The output impedance $\mathrm{r}_{\text {out }}$ can be calculated from M20, M25 as follows;

$$
\begin{equation*}
r_{\text {out }}=r_{d 20} / / r_{d 25} \tag{33}
\end{equation*}
$$

From (33), $\mathrm{r}_{\mathrm{V}_{\mathrm{X}}}, \mathrm{r}_{\mathrm{V}_{\mathrm{Y}}}$ depend on inner resistance of current source $I_{0}$. Typically, the inner resistance of the current source is very high. Thus, as indicated by (33), the input resistances $\mathrm{V}_{\mathrm{X}}, \mathrm{V}_{\mathrm{Y}}$ of proposed circuit is very high.

For example, the CMOS with $\mathrm{V}_{\mathrm{TN}}=0.62 \mathrm{~V}, \mathrm{~V}_{\mathrm{TP}}=-0.58 \mathrm{~V}$, $\mu_{\mathrm{O}} \mathrm{C}_{\mathrm{OX}}=53 \times 10^{-7} \mathrm{AV}^{-2}$. Let $\mathrm{V}_{\mathrm{DD}}=-\mathrm{V}_{\mathrm{SS}}=1.5 \mathrm{~V}$ and $\mathrm{I}_{\mathrm{O}}=10 \mu \mathrm{~A}$, then the input range $\mathrm{V}_{\mathrm{X}}, \mathrm{V}_{\mathrm{Y}}$ and $\mathrm{I}_{\mathrm{X}}, \mathrm{I}_{\mathrm{Y}}$ of multiplier is approximately equal to $|20 \mu \mathrm{~A}|$ and $|400 \mathrm{mV}|$, respectively.

## C. Frequency Response

From the analog multiplier in Fig. 3, the input voltage has considered under condition operation in class-AB. Therefore, MOS transistor M1, M2, M17, M18, M23, and M24 is the pass of signal that chosen for calculation. For high-frequency, the major high frequency limitation of the proposed circuit results for the bandwidth of three current mirrors M1-M2, M21-M22 and M19-M20, the transfer function of the multiplier circuit can be expressed as equation (32), where $g_{m i} g_{d i}$ denote the transconductances and output conductance and $\mathrm{C}_{\mathrm{gsi}}$ denotes the gate-to-source capacitances, respectively, of the MOS transistor Mi. Based on (34), assume $\mathrm{g}_{\mathrm{m} 1} \approx \mathrm{~g}_{\mathrm{m} 5}$ and $\mathrm{g}_{\mathrm{m} 2} \approx \mathrm{~g}_{\mathrm{m} 6}$, the transfer function of $\mathrm{I}_{\text {out }} / \mathrm{V}_{\mathrm{Y}}$ can be given as $\mathrm{I}_{\text {out }} / \mathrm{V}_{\mathrm{Y}} \approx \mathrm{I}_{\text {out }} / \mathrm{V}_{\mathrm{X}}$. From equation (34), the cutoff frequency can be approximately given by $1 /\left(2 \pi \mathrm{R}_{\mathrm{x}}\left(\mathrm{C}_{\mathrm{gs} 1}+\mathrm{C}_{\mathrm{gs} 2}\right)\right)$.
The input current is considered next. The major high frequency limitation results from the bandwidth of negative current mirror M21-M22 and positive current mirror M19M20. It means the input current of circuit can be operated wider bandwidth than input voltage. The dominant poles of the two current mirrors is approximately given by $1 /\left(1+\mathrm{s}\left(\mathrm{C}_{\mathrm{g} 521} / \mathrm{g}_{\mathrm{m} 21}\right)\right), 1 /\left(1+\mathrm{s}\left(\mathrm{C}_{\mathrm{gs} 1} / \mathrm{g}_{\mathrm{m} 19}\right)\right)$.

## IV. Simulation and Experimental Results

## A. Simulation Results

The performance of the analog multiplier circuit as shown in Fig. 3 are simulated using PSPICE with level 3 model of $0.5 \mu \mathrm{~m}$ CMOS parameter of MIETEC as shown in Table II. The transistors aspect ratios as given in Table III and the supply voltage are $\mathrm{V}_{\mathrm{DD}}=-\mathrm{V}_{\mathrm{SS}}=1.5 \mathrm{~V}, \mathrm{I}_{\mathrm{O}}$ is set to $10 \mu \mathrm{~A}$ and $\mathrm{R}_{\mathrm{L}}=$ $\mathrm{R}_{\mathrm{X}}=\mathrm{R}_{\mathrm{Y}}=10 \mathrm{k} \Omega$.

Fig. 5 shows the DC transfer characteristics of the proposed analog multiplier when input are currents. The output current swings between $-20 \mu \mathrm{~A}$ to $+20 \mu \mathrm{~A}$ for the input current range of $\pm 20 \mu \mathrm{~A}$, while the $\mathrm{V}_{\mathrm{X}}$ and $\mathrm{V}_{\mathrm{Y}}$ are connected to ground.
Fig. 6 shows the DC transfer characteristic of multiplier under condition by varying $\mathrm{V}_{\mathrm{X}}$ from -300 mV to 300 mV against varying $\mathrm{V}_{\mathrm{Y}}$ from -300 mV to 300 mV at 100 mV per step, while the input $\mathrm{I}_{\mathrm{X}}$ and $\mathrm{I}_{\mathrm{Y}}$ are connected to $\mathrm{R}_{\mathrm{X}}=\mathrm{R}_{\mathrm{Y}}=10 \mathrm{k} \Omega$. The input and output swing can be increased by using large input and output resistors.


Fig. 5 Simulated DC transfer characteristics for inputs are currents


Fig. 6 Simulated DC transfer characteristics for inputs are voltages


Fig. 7 Simulated DC transfer characteristics for inputs are voltage and current


Fig. 8 Total harmonic distortion versus input current


Fig. 9 Total harmonic distortion versus input voltage


Fig. 10 Simulated DC transfer characteristic for $I_{Y}=10 \mu \mathrm{~A}$ and temperatures $=50^{\circ} \mathrm{C}, 75^{\circ} \mathrm{C}$ and $100^{\circ} \mathrm{C}$

TABLE II
MOS Model used in the Simulation
.MODEL NM NMOS LEVEL=3 UO=460.5 TOX=1.0E-8 TPG=1 VTO=0.62 JS=1.08E-6 XJ=0.15U RSH=2.73 LD=0.04U VMAX=130E3 NSUB=1.71E17 $\mathrm{PB}=0.761$ ETA $=00$ THETA $=0.129$ PHI $=0.905$ GAMMA $=0.69$ KAPPA $=0.10$ $\mathrm{CJ}=76.4 \mathrm{E}-5 \quad \mathrm{MJ}=0.357 \quad \mathrm{CJSW}=5.68 \mathrm{E}-10 \quad$ MJSW $=0.302 \quad$ CGSO $=1.38 \mathrm{E}-10$ CGDO $=1.38 \mathrm{E}-10 \quad \mathrm{CGBO}=3.45 \mathrm{E}-10 \quad \mathrm{KF}=3.07 \mathrm{E}-28 \quad \mathrm{AF}=1 \quad \mathrm{WD}=+0.11 \mathrm{U}$ DELTA $=+0.42$ NFS $=1.2 \mathrm{E} 11$ DELL=0U LIS= 2 ISTMP $=10$ TT=0.1E- 9
.MODEL PM PMOS LEVEL=3 UO $=100$ TOX $=1.0 \mathrm{E}-8 \quad \mathrm{TPG}=1 \quad \mathrm{VTO}=-0.58$ JS=0.38E-6 XJ=0.10U RSH=1.81 LD=0.03U VMAX=113E3 NSUB=2.08E17 $\mathrm{PB}=0.911 \quad \mathrm{ETA}=00 \quad$ THETA $=0.120 \quad \mathrm{PHI}=0.905 \quad \mathrm{GAMMA}=0.76 \quad \mathrm{KAPPA}=2$ $\mathrm{CJ}=85 \mathrm{E}-5 \quad \mathrm{MJ}=0.429 \quad \mathrm{CJSW}=4.67 \mathrm{E}-10 \quad \mathrm{MJSW}=0.631 \quad \mathrm{CGSO}=1.38 \mathrm{E}-10$ $\mathrm{CGDO}=1.38 \mathrm{E}-10 \mathrm{CGBO}=3.45 \mathrm{E}-10 \mathrm{KF}=1.08 \mathrm{E}-29 \mathrm{AF}=1 \mathrm{WD}=0.14 \mathrm{U}$ DELTA $=0.81$ $\mathrm{NFS}=0.52 \mathrm{E} 11$ DELL=$=0 \mathrm{U}$ LIS $=2$ ISTMP $=10 \mathrm{TT}=0.1 \mathrm{E}-9$

TABLE III
MOS TRANSISTOR SIZES

| Transistor |  |
| :--- | :---: |
| W/L $(\mu \mathrm{m} / \mu \mathrm{m})$ |  |
| M1, M2, M5, M6, M9, M10, M13, M14, M15, <br> M16, M19, M20 | $20 / 4$ |
| M3, M4, M7, M8, M11, M12, M21, M22, M23, <br> M24, M25 | $93 / 4$ |
| M17, M18 | $40 / 4$ |

From PSPICE simulations, the linearity error was $1.2 \%$. The total harmonic distortion (THD) versus input current signal at 100 kHz and 1 MHz is shown in Fig. 8, and versus input voltage is shown in Fig 9. The dependence of the output current on temperature was studied and shown in Fig. 10. The simulation results for the variation of the output current, $\mathrm{I}_{\text {out }}$, due to the change of temperature are plotted for the operating tempera- tures $=50^{\circ} \mathrm{C}, 75^{\circ} \mathrm{C}$ and $100^{\circ} \mathrm{C}$. Let the input signal current $\mathrm{I}_{\mathrm{Y}}=10 \mu \mathrm{~A}$ and $\mathrm{I}_{\mathrm{X}}$ varied from $-20 \mu \mathrm{~A}$ to $20 \mu \mathrm{~A}$. This simulation results demonstrated the realization, the temperature depen- dence of the transconductances and the threshold voltages of MOS transistors can be compensated.


Fig. 11 Measured frequency doubler with a 10 kHz sine wave inputs. Lower track: $400 \mathrm{mV}_{\text {P-P }}$ sinusoidal signal of $20 \mathrm{kHz}(200 \mathrm{mV} / \mathrm{div})$, Upper track: output waveform ( $200 \mathrm{mV} / \mathrm{div}$ ), horizontal scale is $25 \mu \mathrm{~s} / \mathrm{div}$


Fig. 12 Measured output waveform of a 50 kHz sinusoid and a 3 kHz triangular wave. Lower track: $400 \mathrm{mV}_{\text {P-P }}$ sine signal of 3 kHz $(200 \mathrm{mV} / \mathrm{div})$, Upper track: output waveform $(200 \mathrm{mV} / \mathrm{div})$, Horizontal scale is $100 \mu \mathrm{~s} / \mathrm{div}$


Fig. 13 Measured output spectrum of multiplier, for the case in which a $400 \mathrm{mV} V_{\text {P-P }}$ sinusoidal signal of 50 kHz and a 400 mV DC voltage are applied to $\mathrm{V}_{\mathrm{X}}$ and $\mathrm{V}_{\mathrm{Y}}$, respectively [horizontal scale is $100 \mu \mathrm{~s} / \mathrm{div}$; vertical scale is 10 dBm ]

## B. Experimental Results

The circuit of Fig. 3 is prototyped the form of complementtary MOS transistor pair (MC14007). In this experiment, $V_{D D}=-V_{S S}=3 V$ and $R_{L}=1 \mathrm{k} \Omega$ where they are used for the multiplier. The input $I_{X}$ and $I_{Y}$ are connected with $R_{X}=R_{Y}=10 \mathrm{k} \Omega$ and the input $\mathrm{V}_{\mathrm{X}}$ and $\mathrm{V}_{\mathrm{Y}}$ are used. Fig. 11 shows the performance of the multiplier as a frequency doubler, the input signals are two in-phase 10 kHz sine wave $400 \mathrm{mV}_{\text {P-P }}$ (Lower trace $200 \mathrm{mV} / \mathrm{div}$ ) that applied to $\mathrm{V}_{\mathrm{X}}$ and $\mathrm{V}_{\mathrm{Y}}$, respectively. The output is a sine wave of twice the input signal frequency and has a magnitude of $320 \mathrm{mV}_{\text {P-P }}$ (Upper trace $200 \mathrm{mV} / \mathrm{div}$ ). Fig. 12 shows the multiplier being used for amplitude modulation, when inputs, a 3 kHz sinusoidal signal (Lower trace, $200 \mathrm{mV} / \mathrm{div}$ ) and a $50 \mathrm{kHz} 400 \mathrm{mV}_{\text {P-p }}$ sinusoidal signal are applied to $\mathrm{V}_{\mathrm{X}}$ and $\mathrm{V}_{\mathrm{Y}}$, respectively. Fig. 13 shows the
multiplier output on spectrum analyzer when a $400 \mathrm{mV}_{\text {P-P }}$ sinusoidal signal of 50 kHz and a 0.4 V DC voltage are applied to $V_{X}$ and $V_{Y}$, respectively. In Fig. 13 the measured THD is 59 dB with the second and third harmonics as main contribution to distortion.

## V. CONCLUSION

The class-AB CMOS four-quadrant multiplier was presented. The proposed circuit has distinguished in used class $A B$ mode, and its combine a voltage multiplier, a current multiplier, a current and a voltage multiplier into single ended circuit. The advantages of the proposed analog multiplier circuit over previous circuit are; (i) versatile analog multiplier since the input can be voltage, current or both voltage and current, (ii) high dynamic range and high speed, since input operated in class-AB, (iii) high bandwidth and high temperature stability, since the circuit realizing based on dual translinear loop [18], [19]. The multiplier can be used in analog VLSI cell for low power high dynamic range and high speed application, such as IF variable gain amplifier and adaptive filters. The experimental results have been shown to confirm the operation. The PSPICE simulation results of this analog multiplier are summarized in Table IV.

TABLE IV
Simulated Performance of Proposed Multiplier

| Parameter | Values |
| :--- | :---: |
| Technology | $0.5 \mu \mathrm{~m} \mathrm{CMOS}$ |
| Supply voltage | $\pm 1.5 \mathrm{~V}$ |
| Bias current | $10 \mu \mathrm{~A}$ |
| Input voltage range $@ \mathrm{R}_{\mathrm{X}}=\mathrm{R}_{\mathrm{Y}}=10 \mathrm{k} \Omega$ | $\pm 300 \mathrm{mV}$ |
| Input current range | $\pm 20 \mu \mathrm{~A}$ |
| Bandwidth $(-3 \mathrm{~dB})$ | 19 MHz |
| Nonlinear error for $\mathrm{I}_{\mathrm{X}} @ \mathrm{I}_{\mathrm{Y}}=20 \mu \mathrm{~A}$ | $1.2 \%$ |
| Nonlinear error for $\mathrm{V}_{\mathrm{X}} @ \mathrm{~V}_{\mathrm{Y}}=300 \mathrm{mV}$ | $1.2 \%$ |
| THD for @ $\mathrm{I}_{\mathrm{X}}=20 \mu \mathrm{~A}_{\mathrm{P}-\mathrm{P}}, 100 \mathrm{kHz} ; \mathrm{I}_{\mathrm{Y}}=20 \mu \mathrm{~A}_{\mathrm{DC}}$ | $2 \%$ |
| THD for $@ \mathrm{~V}_{\mathrm{X}}=300 \mathrm{mV} \mathrm{V}_{\mathrm{P} . \mathrm{P}}, 100 \mathrm{kHz} ; \mathrm{V}_{\mathrm{Y}}=300 \mathrm{mV}_{\mathrm{DC}}$ | $1.9 \%$ |
| Power consumption $($ Max. $)$ | 0.46 mW |

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