

Using the Simple Fixed Rate Approach to Solve Economic Lot Scheduling Problem under the Basic Period Approach

Yu-Jen Chang, Yun Chen, Hei-Lam Wong

Abstract—The Economic Lot Scheduling Problem (ELSP) is a valuable mathematical model that can support decision-makers to make scheduling decisions. The basic period approach is effective for solving the ELSP. The assumption for applying the basic period approach is that a product must use its maximum production rate to be produced. However, a product can lower its production rate to reduce the average total cost when a facility has extra idle time. The past researches discussed how a product adjusts its production rate under the common cycle approach. To the best of our knowledge, no studies have addressed how a product lowers its production rate under the basic period approach. This research is the first paper to discuss this topic. The research develops a simple fixed rate approach that adjusts the production rate of a product under the basic period approach to solve the ELSP. Our numerical example shows our approach can find a better solution than the traditional basic period approach. Our mathematical model that applies the fixed rate approach under the basic period approach can serve as a reference for other related researches.

Keywords—Economic Lot, Basic Period, Genetic Algorithm, Fixed Rate.

I. INTRODUCTION

THE Economic Lot Scheduling Problem (ELSP) is a valuable mathematical model that can support decision makers to make scheduling decisions. The ELSP has been applied for production planning and inventory control in industries such as plastics extrusion, metal stamping, textile manufacturing, bottling, printing and packing [12].

The ELSP is concerned with the scheduling of cyclical production of $n \geq 2$ products on a single facility in equal lots over an infinite planning horizon, assuming stationary and known demands for each product. The objective of the ELSP is to determine the lot size and the schedule of production of each product so as to minimize the total cost incurred per unit time. The costs considered include the setup cost and inventory holding cost. For solving the ELSP, an optimal solution must minimize the average total cost and also generate a feasible production schedule. The ELSP has been shown as a NP-hard problem [10].

Elmaghraby [6] suggested that the solution methodologies for the ELSP may be divided into two major categories, namely, analytical approaches and heuristics. The analytical approaches include the Independent Solution (IS) approach, the

Common Cycle (CC) approach, the Basic Period (BP) approach and the Extended Basic Period (EBP) approach. One may easily obtain the solution of the IS approach by summing the objective function values from the optimal solutions of the Economic Production Quantity (EPQ) model of n products (which can be viewed as a lower bound of the cost for the ELSP). But, the IS approach is not able to ensure the feasibility of the production schedule on a facility. The CC approach is a simple method that can guarantee the feasibility of its solution. It assumes that all the products must be produced using the same cycle time. It means that all products must be produced once during the length of the time T_{cc} . Therefore, a common cycle time T_{cc} must be large enough to accommodate the production lots of all products. The solution of the CC approach can be considered as the upper bound of the cost for the ELSP.

A popular category of research for the ELSP is the so-called “basic period-based cyclic schedule” which uses a basic period B as the base for production planning and scheduling. It includes the BP approach and the EBP approach. A basic period B is an interval of time devoted to the setup and production of a subset of (or all) the products. A solution of the ELSP under the basic period-based approaches is usually given in the form of $(B, k_1, k_2, \dots, k_n)$ which implies that the replenishment cycle of product i (denoted by T_i) is equal to $k_i B$, i.e., $T_i = k_i B$.

The BP approach restricts all products must be produced at the first period, but each of all products may or may not be produced at other periods. Also, for any product i , its cycle time is an integer multiplier k_i of a basic period B . If the cyclic multipliers $\{k_i\}$ of all products are equal to 1 under the BP approach, this solution can be viewed as a solution under the CC approach. So, the solution of the CC approach can viewed as a special solution of the BP approach. In general, the BP approach can find better solutions than the CC approach. Its disadvantage is that B must be large enough in order to product all products at the first period. So there may exist much idle time at other periods. When applying the BP approach, researchers [1] used dynamic programming to solve the ELSP. Using the BP approach to solve the ELSP, [7] provided an algorithm that finds an optimal solution.

The EBP approach is the most complex solving approach for the ELSP. The EBP approach is similar to the BP approach. But, the former allows the flexibility of scheduling the production of a product not being started at the first basic period. If the cyclic multiplier k_i of product i is equal to 2, product i can be produced at period 1, 3, 5,..... or period 2, 4,

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6..... Because the EBP approach does not constrain which period is the first starting production period for product i , the BP approach may be considered as a special case of the EBP approach. Also, the EBP approach always obtains better solutions than the BP approach. Table I lists feasible production schedules of two products under different solving approaches for solving the ELSP. However, Table I shows that a solution $\{k_i\}$ can generate a number of possible schedules under the EBP approach. So the EBP approach must spend much time judging whether a schedule is feasible or not. If the sum of the production time and setup time of a subset of (or all) the products that are produced within a period on a facility is less than or equal to the length B of a basic period, the cyclic schedule is referred as feasible when using the EBP approach.

TABLE I
THE PRODUCTION SCHEDULES OF DIFFERENT SOLVING APPROACHES

period		1	2	3	4	5	6
the CC approach	product 1	$k_i=1$	V	V	V	V	V
	product 2	$k_i=1$	V	V	V	V	V
the BP approach	product 1	$k_i=2$	V		V		V
	product 2	$k_i=3$	V			V	
the EBP approach	product 1	$k_i=2$		V		V	V
	product 2	$k_i=3$			V		V
.....	product 1	$k_i=2$	V		V		V
	product 2	$k_i=3$		V			V

V: product i must be produced at period j

When a schedule generated by a solution $\{k_i\}$ is not feasible, product i must change its first starting production period. For example, the cyclic multiplier k_i of product i is equal to 3, the first starting production period of product i can be adjusted from period 1 to period 2 or 3 in order to obtain a feasible schedule. Then product i is produced for every 3 periods. Because the EBP approach must spend much time judging the feasibility of a solution and adjust the first starting production period for each product. So the EBP is time-consuming solving approach in order to obtain better solutions.

These analytic approaches, however, take long run time to solve relatively 'small' (say, 10-product) problems. The solution of large-scale ELSP problems seems to be out of reach for these analytical approaches.

A few researches [12] discussed the advantages of the Time Varying Lot Sizes (TVLS) approach for solving the ELSP. The TVLS approach does not constrain that every period must have the same cyclic length and product i must produce the same lot size every time. If the TVLS approach can satisfies a special constraint, it can obtain easily a feasible and better solution than the CC approach. However, the TVLS approach is an unstable solving approach. The setup setting of all products often affects significantly the quality of its solutions [4].

A number of studies extend the ELSP to discuss the related interesting topics. Soman et al. [15] studied the ELSP with considering self-life under the basic period approach. They used a like-brand-bound solving approach to search a set of cyclic multipliers $\{k_i\}$ that can generate a near-optimized solution. Tang and Tenter [17] studied some demands can be satisfied by repairing or remanufacturing the defective items.

These items are returned from outside a factory. This ELSP is called as the ELSP with returns. Chang and Yao [5] discussed the ELSP with reworks. They discussed some defective items are generate during some processes inside a factory. These items must be repaired or remanufactured on the same facility. Chang and Yao developed a genetic algorithm approach to find a near-optimal production schedule for solving the ELSP with reworks. Yao et al. [20] studied the economic lot inspection and scheduling problem. It must generate a feasible inspection and production schedule concurrently. A facility is responsible for inspecting and another facility is responsible for repairing. Yao et al. [20] developed a junction point solving approach to search for cyclic multiplier $\{k_i\}$ that can generate a near-optimized inspection and production schedule.

In general, most researches that studied the ELSP often made an important assumption. This assumption supposes that a product is produced by using its maximum production rate. But [3] indicated if a facility has extra idle times, a product can lower its production rate to reduce its inventory holding cost. A product can change its production rate once during a cycle time T . Then this product uses a new production rate to be produced. This approach is called as the fixed rate approach.

Fig. 1 shows the concept of the fixed rate approach. Product i can utilize the idle time of a facility to lower its production rate in order to reduce the average total cost. Fig. 1 represents the gray triangle area is the inventory holding cost savings for product i . Silver [14] pointed that only one product with the largest $d_i h_i$ value is necessary to be adjusted its production rate under the CC approach. d_i and h_i is the demand rate and inventory holding cost of product i respectively. The other products still use the maximum production rate to be produced. Moon and Christy [11] pointed that the lower bound of the production rate of product i is equal to its demand rate, the original production rate can be viewed as the upper bound of the production rate of product i . In addition, the production rates of one or more products increases, the average total cost will increase. Khouja [10] discussed the fixed rate approach for solving an imperfect production system. The result showed that the deterioration of the quality level will increase the lot sizes and production rates of all (or some) products. Yang et al. [18] discussed the scheduling problem of two products on a facility. They adjusted the production rates of some products to look for a near-optimal production and setup schedule. They used dynamic programming and the Hamilton–Jacobi–Bellman equation to ensure the optimality of a solution.

The ELSP researches that applied the fixed rate approach usually solved the ELSP under the CC approach. However, the BP approach can get better solutions than the CC approach. To the best knowledge of the authors, this research is the first paper to study how to apply the fixed rate approach to solve the ELSP under the BP approach. Because there are different product mixes at each period, it would have three difficult problems must be solved for this study. The first problem is how to compute the size of the idle time in each period. The second problem is how to select which product to lower its production rate at a period. Remember, a product may or may not be produced at a period. The third problem is how to compute the

average total cost when a product has different production rates in different periods. The third problem is the most difficult one. For solving the ELSP, the average total cost per unit time can be used to determine the quality of a solution. However, when a product has different production rates in different periods, how to compute the average total cost per unit time of product i can be more complex.

This research provides a mathematical model of the ELSP under the BP approach. This model develops a method to compute the average total cost per unit time when a product has different production rates. This research also develops a genetic algorithm approach that applies the fixed rate approach in order to search the optimal $\{k_i\}$. When the genetic algorithm find out a candidate solution $\{k_i\}$, this approach can apply the fixed rate approach to adjust the production rate of one product in order to compute corresponding average total cost for this solution $\{k_i\}$. Using the solving approach proposed by this research, it can find better solutions than the traditional BP approach.

II. THE MATHEMATICAL MODEL

We first introduce the assumptions and notations in our mathematical model of the ELSP as follows.

Assumptions:

1. A facility can produce only one product at any time point.
2. A facility has enough capacity to produce the demand of the produced items during a production cycle.
3. The setup costs and setup times of the products are independent of their production sequence on a facility.
4. No shortage is allowed.
5. The parameters for each product are known and fixed at any time point.
6. There is only one lot for product i during a production cycle $k_i * B$.

Notation:

- a_i : The setup cost of product i .
- h_i : The holding cost of product i .
- p_i : The production rate of product i .
- $p_{i,z}$: The production rate of product i at period z .
- s_i : The setup time of product i .
- d_i : The demand rate of product i .
- n : The number of the products.
- I : The idle time of each period that can be utilized to adjust the production rate of a product.
- $TC_{i,z}$: The total cost per unit time of product i at period z .
- X_z : Product that is selected to lower its production rate at period z .

Decision Variables:

- k_i : The cyclic multiplier of product i .
 - B : The time length of a basic period.
- Applying the fixed rate approach to solve the ELSP under the basic period approach is a complex problem. Because there are different product mixes at different periods, it results in two problems as follows.
1. The length of the idle time for each period may be different.
 2. The product with the largest $d_i h_i$ value that adjusts its

production rate may be different.

The basic period approach requires all products must be produced at the first period. So the time length of the idle time for each period is larger than one of the first period. So this research supposes the time length of the idle time of the first period is used as one of other periods. Based on this assumption, this research develops a simple solving approach of the ELSP that applies the fixed rate approach to adjust the production rate of one product. It means a product is selected to lower its production rate at different periods; this product has the same new production rate at different periods because of the same length of the idle time at each period.

Then, this research uses a simple example to explain how to compute the average total cost per unit time. It is the most difficult problem when applying the fixed rate approach to solve the ELSP under the BP approach. A product may have different production rates at different periods after applying the fixed rate approach. Table II shows an example of two products. This research supposes that product 2 has higher $d_i h_i$ value than product 1. A product with the highest $d_i h_i$ value should has higher priority to be adjusted its production rate to obtain more cost improvement. So, product 2 should be selected to lower its production rate at period 1 and 3. And product 1 is selected to lower its production rate at period 2 and 4.

TABLE II
THE PRODUCTION SCHEDULE OF TWO PRODUCTS UNDER THE BASIC PERIOD APPROACH

Product	Period			
	1	2	3	4
1($k_i=1$)	V	V	V	V
2($k_i=2$)	V		V	
which product is selected to lower its production rate	2	1	2	1

V means that product i should be produced at this period

$p_{i,z}$ is the production rate of product i at period z and $TC_{i,z}(p_{i,z})$ is the average total cost per unit time of product i at period z . $TC_{i,z}(p_{i,z})$ can be expressed as (1). Table III lists the production cost of two products at each period. Because product 1 can be selected to lower its production rate at period 2 and 4, it is obvious that $TC_{1,1}(p_{1,1}) = TC_{1,3}(p_{1,3})$ and $TC_{1,2}(p_{1,2}) = TC_{1,4}(p_{1,4})$. The least common multiplier of the cyclic multiplier $\{k_i\}$ of two products is equal to 2. It means that product 1 and 2 will cyclically be produced for every 2 periods. Therefore, for product 1, there are different average total costs at period 1 and 2. However, for product 1, there is the same sum of the average total cost for every 2 periods. For example, $TC_{1,1}(p_{1,1}) + TC_{1,2}(p_{1,2})$ is equal to $TC_{1,3}(p_{1,3}) + TC_{1,4}(p_{1,4})$.

TABLE III
THE PRODUCTION COST OF PRODUCT I AT EACH PERIOD

Period	Product	1	2	3	4
1		$TC_{1,1}(p_{1,1})$	$TC_{1,2}(p_{1,2})$	$TC_{1,3}(p_{1,3})$	$TC_{1,4}(p_{1,4})$
2		$TC_{2,1}(p_{2,1})$	$TC_{2,2}(p_{2,2})$	$TC_{2,3}(p_{2,3})$	$TC_{2,4}(p_{2,4})$

So, for product 1 (or 2), we can average the production cost at period 1 and 2 as the average total cost per unit time. Therefore, the average total cost per unit time of product i can be expressed as (2). K is the least common multiplier of the cyclic multiplier $\{k_i\}$ for all products.

$$TC_i = \frac{\sum_{z=1}^K TC_{i,z}(p_{i,z})}{K} \tag{2}$$

Based on the above assumptions, the mathematical model that applies the fixed rate approach to solve the ELSP under the BP approach can be shown as (3)-(9) in Table IV. Equation (3) is the objective function of this research that is used to compute the average total cost for the ELSP. A product has different production rates at different periods, so it is possible that there are different production costs at each period for product i . Therefore it cannot use the tradition basic period approach to compute the average total cost. Because the basic assumption of traditional basic period approach is that product i has the same production cost at every period.

Equation (4) is used to compute the average total cost per unit time $TC_{i,z}$ of product i at period z . If product i is not be produced at period z , $TC_{i,z} = 0$. Equation (5) is used to judge whether the time length B of a period satisfies the requirement of production and setup for all products. The BP approach requires that all products are produced at the first period. So, for a solution, if the time length B of a period satisfies the requirement of capacity at the first period, this solution is feasible.

However, a product is cyclic produced for every K periods. It means that the sum of the cost of every K periods is the same. K is the least common multiple of the cycle multiplier of all products. Therefore, for all products, the average total cost can be expressed as (3) that is the objective function of this research. Equation (3) sums the average production cost TC_i of all products. For product i , TC_i is the average production cost per unit time of K periods that can be expressed as (2).

Equation (6) is used to compute the new production rate of product i at period z . There are three situations that can express the new production rate of product i .

$$TC_{i,z}(p_{i,z}) = \frac{a}{k_i B} + \frac{k_i B d_i h_i}{2} \left(1 - \frac{d_i}{p_{i,z}} \right) \tag{1}$$

TABLE IV
THE ELSP MATHEMATIC MODEL UNDER THE BASIC PERIOD APPROACH

$$\min TC = \sum_i^n TC_i \tag{3}$$

$$TC_{i,z} = \begin{cases} 0 & \text{if product } i \text{ is not produced at period } z \\ \frac{a}{k_i B} + \frac{k_i B d_i h_i}{2} \left(1 - \frac{d_i}{p_{i,z}} \right) & \text{if product } i \text{ is produced at period } z \end{cases} \tag{4}$$

$$\sum_i \left(s_i + \frac{u_i v_i}{p_i} B \right) \leq B$$

$$(5) \quad p_{i,z} = \begin{cases} 0 & \text{if product } i \text{ is not produced at period } z \\ p_i & \text{if product } i \text{ is produced at period } z, \text{ but product } i \text{ does not} \\ & \text{change its production rate} \\ p_{i,z} = \frac{d_i k_i B}{\frac{d_i k_i B}{p_i} + I} & \text{if product } i \text{ is produced at period } z, \text{ but product } i \text{ change its} \\ & \text{production rate} \end{cases} \tag{6}$$

$$I = \min_z I_z = I = B - \sum_i \left(s_i + \frac{d_i I}{I} \right) \tag{7}$$

$$k_i = 2^{v_i} \quad v_i \text{ is a non-negative integer} \tag{8}$$

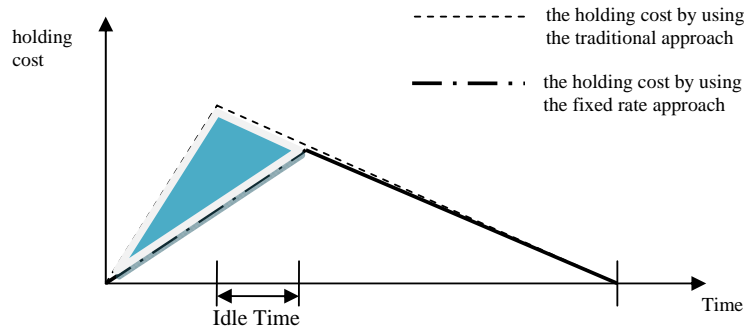


Fig. 1 The Holding Cost Function under Different Approaches

1. Product i is not produced at period z .
2. Product i is produced at period z and does not adjust its production rate.
3. Product i is produced at period z and is selected to lower its production rate.

I_z is the idle time at period z . Because the first period must produce all products, the first period should have the smallest idle time among all periods. It means that the idle time of the first period should not larger than one of other periods. Equation (7) is used to compute the idle time I_1 of the first period. Because this research is the first paper to discuss how to change the production rate of product i under the basic period approach, this research supposes that I_1 can be viewed as the idle time of other periods. When the idle time of a period is known, this research uses (9) to compute the new production rate of the selected product at period z .

$$\frac{d_i k_i B}{P_i} + I_1 = \frac{d_i k_i B}{P_{i,z}} \quad (9)$$

Product i can choose the general integer policy or power of two (PoT) policies to present its cyclic multiplier under the basic period approach. The general integer policy requires that k_i is a positive value greater than 0. The power of two policies requires that k_i is the power of 2. The power of two policies can reduce the search space and does not loss the quality of solutions. So many researchers suggest that using the power of two policies to determine k_i for solving the ELSP under the basic period approach. This research also uses power of two policies to search for $\{k_i\}$. Equation (8) expresses the definition of k_i .

III. THE GA APPROACH FOR SOLVING THE ELSP

Recall that the ELSP is NP-hard [8], and it is very difficult to employ commercial software to solve optimal solutions even for small-size problems. When applying the fixed rate approach to solve the ELSP under the BP approach, the key problem is how to find the optimal $\{k_i\}$ of all products. This research proposes a GA solving approach to find a near-optimized solution $\{k_i\}$. This research presents the major components of our GA approach that can be used to search for $\{k_i\}$ in section 3.1. In section 3.2, this research presents how to apply the fixed

rate approach to compute the average total cost for a given candidate solution $\{k_i\}$ obtained from the GA solving approach. **Lemma 1.** Let the cyclic multipliers of all products be equal to 1. Then $B_{CC}(\{k_i\})$ is the time length of a cycle time under the CC approach. $TC_{CC}(B_{CC}(\{k_i\}))$ is the average total cost under the CC approach. $TC_{CC}(B_{CC}(\{k_i\}))$ also can be viewed as the upper bound of the cost for solving the ELSP.

Lemma 1 indicates that if you cannot find a feasible solution except for the solution $\{k_i\} = 1$, you can use the solution of the CC approach as the optimal solution of the BP approach.

Recall that a solution for the ELSP is in the format of (B, k_1, \dots, k_n) under the BP approach. Our GA proposed in this section searches in the solution space of (k_1, \dots, k_n) , and obtain the time length B of a basic period to not only minimize the objective function value but also generate a feasible production schedule for a candidate solution $\{k_i\}$ obtained from the GA approach. Therefore, this research shall represent each multiplier k_i as a particular part of a chromosome. For instance, the first u_1 bits are used to encode the value of k_1 and the particular piece of chromosome from the $(u_1 + 1)^{\text{th}}$ bit to the $(u_1 + u_2)^{\text{th}}$ bit represents the value of k_2 , and so on.

Under *PoT* policy, each k_i is a power-of-two integer, i.e., $k_i = 2^{v_i}$, for some nonnegative integer v_i . In our GA under *PoT* policy, we represent k_i by its (integer) value of power v_i for encoding in the chromosome. For example, if we use $v_i = 3$ to represent all the possible values of k_i , then there exist $2^{v_i} = 2^3 = 8$ possible values of v_i , namely, $\{0, 1, 2, \dots, 7\}$ (in which they correspond to $(0,0,0)$, $(0,0,1)$, ..., $(1,1,1)$, respectively, in binary-coding). In such a case, we may use the binary strings $(0,1,0)$ and $(1,0,1)$ to represent $k_i = 2^2 = 4$ and $k_i = 2^5 = 32$, respectively.

In order to represent all the possible values of k_i for each product i , we need an upper bound on the value of k_i (consequently, on the value of integer-power, v_i) to comprise the chromosome representation in the GA. Lemma 2 propose a procedure to obtain the upper bound k_i^{\max} of the cyclic multiplier k_i for product i . However, [19] and [15] also developed different procedure to compute the upper bound of the cyclic multiplier k_i for product i .

Lemma 2. k_i^{\max} is the upper bound of the cyclic multiplier k_i for product i . k_i^{\max} can be obtained by using (10) and (11). The research use the capacity constraint to obtain k_i^{\max} . So, for product i , k_i is not larger than k_i^{\max} .

$$s_i + \frac{d_i k_i^{\max} B_{UB}}{p_i} \leq B_{UB} \quad (10)$$

$$k_i^{\max} = 2^{v_i} \leq \frac{B_{UB} - s_i}{d_i B_{UB} p_i} \quad (11)$$

On the other hand, by the definition of *PoT* policy, the lower bound on the value of k_i is 1, obviously. Therefore, the lower bound on each v_i is $v_i^{LB} = 0$.

Since we encode the value of k_i by binary strings of integer powers, we have to establish a mapping between each binary string and an integer. In fact, we need a binary string of u_i bits for encoding the mapping where u_i is the smallest integer such that $2^{u_i} \geq v_i^{UB}(B_{lb})$. Then, we may use (12) to express the mapping between the binary string and the value of k_i as:

$$\langle b_{u_i}, b_{u_i-1}, \dots, b_1 \rangle = \left(\sum_{x=1}^{u_i} b_x 2^{x-1} \right) \quad (12)$$

where $b_x \in \{0,1\}, \forall x$.

In case the value of the mapped integer in (12) is larger than $v_i^{UB}(B_{lb})$, we flip all bits in the binary string (equivalently, using compliment computation) to assure that it is no larger than $v_i^{UB}(B_{lb})$. Therefore, the total length of a chromosome in our GA is $\sum_{i=1}^n u_i$ bits.

Note that, our GA presented in this section first ignores the capacity constraints, searches in the solution space of (k_1, \dots, k_n) and tries to minimize the objective function value. For a given set of multipliers $\{k_i\}$, we may use (13) to obtain an optimal value of B .

$$B(\{k_i\}) = \sqrt{\frac{2 \sum_{i=1}^n (a_i / k_i)}{\sum_{i=1}^n (d_i h_i k_i (1 - d_i / p_i))}} \quad (13)$$

During the evolutionary process in GA, we collect chromosomes whose fitness value falls between a lower bound (from the IS approach) and an upper bound (from the CC approach). Then, we employ (5) to test the feasibility of (B, k_1, \dots, k_n) . If there exists a feasible production schedule for (B, k_1, \dots, k_n) , we record it as a candidate of the optimal solution. If no feasible production schedule exists for (B, k_1, \dots, k_n) , Chang and Yao [5] suggested that a *binary-search heuristic* can be used to locate a particular value of basic period $B(\{k_i\})$ that

enables $(B(\{k_i\}), \{k_i\})$ to secure a feasible production schedule with the minimal average total cost. Next, we shall use a procedure in Section III B to compute the objective function values of the candidate solutions by using (3) to conduct the selection mechanism in our GA. The value of the objective function by using (3) that can viewed as the fitness value of the corresponsive chromosome for a given multipliers $\{k_i\}$.

Since there may exist problems associated with fitness values when solving minimization problems, we propose to perform *fitness normalization* in our GA. Fitness normalization is a process of converting row fitness values to ones that behave better [9] and give high probability for selecting good solutions in new generations, while maintaining some chance of survival to poor solutions [2]. Fitness normalization can be carried out in three forms: (i) inversion normalization, (ii) linear ranking normalization, and (iii) nonlinear normalization. In linear ranking and nonlinear normalization, the term *selection pressure* (SP) represents the ratio of the probability of selecting the best individual to the average probability of selecting all chromosomes [13]. Ranking normalization was found to outperform inversion normalization (proportional assignment) with respect to scaling problems that arise when premature chromosomes appear within a generation and cause narrowing of the search domain. Also, [2] commented that the difference in fitness values between two chromosomes, either small (0.01) or big (1.0×10^7), has no impact on the normalized fitness values. Therefore, we have decided to use *linear ranking normalization* in our GA.

After normalizing fitness, the selection mechanism can be performed in two forms: (i) roulette wheel and (ii) tournament selection. In our GA, we used a *roulette wheel mechanism* for selecting individuals for reproduction. The reproduction probability P_{chrom} of each chromosome is proportional to its normalized fitness $eval_{chrom}$ (relative to the sum of the normalized fitness value of all the individuals) as expressed in (14) as:

$$P_{chrom} = \frac{eval_{chrom}}{\sum_{chrom=1}^{PS} eval_{chrom}} \quad (14)$$

PS is the number of the chromosomes at our GA. As one may observe, the larger the probability (corresponding to better fitness) for a chromosome, the higher the chance it will be reproduced in the next generation.

Those individuals, that survive the selection step, undergo the alternation by two genetic operators, namely, crossover and mutation, to generate the chromosomes in the next generation.

In the literature, single-point crossover, multi-point crossover and uniform crossover are three most-used operators for crossover operations. Single-point and multi-point crossovers define cross point(s) as places between loci where a chromosome can be split. Uniform crossover generalizes this scheme to make every locus a potential crossover point [16]. In this study, we test both two-point crossover and uniform crossover in our GA for its crossover operations since uniform crossover, like multi-point crossover, has been claimed to reduce the bias associated with the length of the binary

representation used and the particular coding for a given parameter set [13].

In a uniform crossover, we first create a crossover mask, which is the same length as the chromosome structure, at random. The parity of the bits in the mask indicates which parent will supply the offspring with which bits. Fig. 2 represents an example of an uniform crossover in which we consider two chromosomes with 17 binary bits. For each bit the parent who contributes to the offspring is chosen randomly with equal probability. Here, the offspring 1 is produced by taking the bit from parent 1 if the corresponding mask bit is 1 or the bit from parent 2 if the corresponding mask bit is 0. And, offspring 2 is created using the inverse of the mask.

chromosome 1	0 1 1 1 0 0 1 1 0 1 0 0 1 1 1 0 0
chromosome 2	1 0 1 0 1 1 0 0 1 0 1 1 0 1 0 1 1
mask 1	0 1 1 0 0 0 1 1 0 1 0 0 1 1 0 0 0
mask 1'	1 0 0 1 1 1 0 0 1 0 1 1 0 0 1 1 1
off spring 1	1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 0 1 1
off spring 2	0 0 1 1 0 0 0 0 0 0 0 0 0 0 1 1 0 0

Fig. 2 An Example of an Uniform Crossover

Next, we apply the mutation operator to the population that just experienced the crossover operator. The mutation operator randomly chooses ones among the genes of all chromosomes in the population with a fixed mutation rate. Then, the mutation operator flips the chosen genes.

Based on our numerical experiments, this research suggests that the population of the GA is $10 \cdot n$. This research tests two settings of the crossover rate (denoted by CR) and the mutation rate (denoted by MR) for the GA. For the first setting, both the crossover rate and the mutation rate are set to a fixed value as $CR = 0.6$ and $MR = 1/n$, respectively. (Note: the designated CR and MR are the best combinations which have resulted from our experiments.) In the second setting, the crossover rate and mutation rate vary linearly during the evolutionary process. In the beginning of the evolution, we set the crossover rate at a higher level ($CR = 0.9$) while the mutation rate is lower ($MR = 0.05$), so that our GA can take advantages of the chromosome characteristics. During the evolutionary process, the crossover rate decreases by 0.001 for each generation and the mutation rate increases by 0.01 after 100 generations. The crossover rate and the mutation rate stop their variation as they reach a specified level, i.e. $CR = 0.2$ and $MR = 0.2$, respectively. We hope that the chromosomes become similar to one another as the crossover rate decreases while the mutation rate increases, so that our GA could still explore new regions in the search space and raise the diversity of the population at the end of the evolution in such a varying parameter setting.

We may have a total of 4 combinations on selecting the crossover operators (namely, taking either the two-point crossover or uniform crossover operators) and the parameter settings of CR and MR (namely, using either fixed rates or varying rates). Our analysis from the design of experiments (DOE) suggests using the uniform crossover operator and fixed rates of CR and MR.

The GA ends the evolutionary process when the best-on-hand solution shows no improvement during the last 50 generations or the number of generations reaches 1000. It shows no significant difference between these two termination conditions when performing another DOE analysis. We decide to terminate Our GA after 1000 generations.

For a given solution (or chromosome) $\{k_i\}$, this research can use the following procedure to apply the fixed rate approach in order to solve the ELSP under the BP approach and then compute the corresponsive average total cost.

1. Use (13) to compute the time length B of the basic period for a given solution $\{k_i\}$.
2. Use (7) to obtain the idle time at the first period that can be viewed as the idle time at other periods.
3. Let z is equal to 1.
4. We can choose product i_z^* with the highest d_i/h_i value from products that be produced at period z . Use (9) to adjust product i_z^* 's new production rate.
5. If z is equal to K , go to Step 6; otherwise, $z = z + 1$, go to Step 4.
6. Use Step 3, 4 and 5 to obtain/adjust each product's production rate at every period. Then, we can use (2) to compute the average total cost of product i for every K periods.
7. Finally, we can use (3) to compute the corresponsive average total cost that can be viewed as the fitness value for a given solution $\{k_i\}$.

IV. A NUMERICAL EXAMPLE

This research proposes a genetic algorithm solving approach that applies the fixed rate approach to solve the ELSP under the basic period approach. A simple 5-product example is used to show the performance of our approach. This research uses a computer with Celeron 3.0G CPU and 4GB RAM to run the solving procedure of this example. The propose approach is written and executed in Java programming language.

Table V shows the parameters of all products in our example. The utilization rate of this example is 0.59 that can be obtained by using (15). The best cyclic multipliers $\{k_i\}$ obtained by using the GA approach is $\{2, 2, 2, 1, 2\}$ under the traditional basic period approach. The time length of basic period B is 1.684. Table VI list the production loads and production cost per unit time for each period under the traditional basic period approach. The cyclic multiplier of product 1 is 2. So the production load of product 1 must satisfy the demand at period 1 and 2. The production load of product 1 at period 1 can be obtained by using (16). The average production cost per unit time of product 1 is 8.432. In this example, the least common multiplier K of the cyclic multiplier $\{k_i\}$ for all products is 2. It means that all products must be produced once cyclically for two periods. This research sums the average total cost for all products, the average cost per unit time of this example is 71.268 under the traditional basic period approach.

TABLE V
THE PARAMETERS OF ALL PRODUCTS

Product No.	d_i	a_i	s_i	p_i	h_i
1	9	20	0.05	150	0.175
2	15	20	0.05	150	0.263
3	9	20	0.05	150	0.350
4	50	20	0.05	150	0.638
5	5	20	0.05	150	0.525

Applying the fixed rate approach adjusts the production rate of one product in order to get a better solution. The cyclic multipliers of the optimal solution by using our approach still are {2, 2, 2, 1, 2}. The optimal time length of a basic period is 1.684. The idle times of all basic periods is set equal to the idle time of the first basic period. So the length of the idle time at the first period is 0.446. After using the GA approach to search for optimal solution $\{k_i\}$ and applying the fixed rate approach to generate a feasible production schedule, Table VII lists the production load and schedule of all periods in our example. Theoretically, different product should be chosen to adjust its production rate at different basic periods because there are different products are produced at every period. However, Table VII shows that if the product with the highest value is produced at every period, the idle time at each period can be utilized to lower the average total cost. The average total cost per unit time of this example obtained by our approach is 64.154. Our fixed rate approach can obtain 9.98% cost improvement than the traditional approach. The computation time of our approach is less than 5 seconds.

$$UF = \sum_i (s_i + \frac{d_i k_i B}{p_i}) \tag{15}$$

$$s_i + \frac{d_i B k_i}{p_i} \tag{16}$$

V. CONCLUSION

In past 40 years, hundreds researches that studied the traditional ELSP were published. A few researchers also published the studies that apply the fixed rate approach to solve the ELSP. However, these researchers studied this topic only under the CC approach.

The solving approaches for the ELSP include the CC approach and the BP-based approaches, e.g. the BP and EBP approaches. The latter can get better solutions than the CC approach. But The EBP approach must judge whether the schedule generated by a solution is feasible or not. So the EBP approach is a difficult and time-consuming solving approach for the ELSP.

Applying the fixed rate approach can lower the average total cost of a production schedule to get better solutions. However, no researchers studied how to adjust the production rate of a product to solve the ELSP under the BP approach. Because a product has different production rates at different periods, the traditional mathematical model of the ELSP cannot be applied to compute the average total cost per unit time of the ELSP that applies the fixed rate approach under the BP approach. It may be a possible reason that no studies discussed how to apply the fixed rate approach to solve the ELSP under the BP approach.

TABLE VI
THE AVERAGE TOTAL COST AND PRODUCTION LOADS OF OUR EXAMPLE UNDER THE TRADITIONAL BASIC PERIOD APPROACH

	Product No.	k_i	Period 1	Period 2	Period 3	Period 4
the average total cost of each product at period z	1	2	8.432		8.432	
	2	2	11.917		11.917	
	3	2	10.925		10.925	
	4	1	29.782	29.782	29.782	29.782
	5	2	10.212		10.212	
the production load of each product at period z	1	2	0.252	0.000	0.252	0.000
	2	2	0.387	0.000	0.387	0.000
	3	2	0.252	0.000	0.252	0.000
	4	1	0.611	0.611	0.611	0.611
	5	2	0.162	0.000	0.162	0.000
total production load at period z			1.665	0.611	1.665	0.611

TABLE VII
THE AVERAGE TOTAL COST AND PRODUCTION LOADS OF OUR EXAMPLE THAT APPLIES THE FIXED RATE APPROACH UNDER THE BASIC PERIOD APPROACH

	Product No.	k_i	Period 1	Period 2	Period 3	Period 4
the average total cost of each product at period z	1	2	8.432		8.432	
	2	2	11.917		11.917	
	3	2	10.925		10.925	
	4	1	22.669	22.669	22.669	22.669
	5	2	10.212		10.212	
the production load of each product at period z	1	2	0.252	0.000	0.252	0.000
	2	2	0.387	0.000	0.387	0.000
	3	2	0.252	0.000	0.252	0.000
	4	1	1.064	0.611	0.611	0.611
	5	2	0.162	0.000	0.162	0.000
total production load at period z			1.665	0.611	1.665	0.611

As the best knowledge of the authors, this research should be the first study to discuss how to apply the fixed rate approach to solve the ELSP under the BP approach. For simplifying the complexity of this research, we use the length the idle time of each period must be equal to the one of the first period. This research provides a mathematical model that can compute average total cost if a product can adjust its production rate under the basic period approach. Our example shows the following findings. The first one is that product i^* with the highest d_i/h_i value among all products should be produced at each period in order to full utilize idle times. The second one is that product i^* should has the same lower production rate at different periods because of the first finding and the assumption about the idle time I_i at each period.

The numeric example in this research shows our approach can obtain nearly 9% of the improvements of the cost than the traditional ELSP solving approach under the BP approach. This research should be the first study to discuss how to apply the fixed rate approach to solve the ELSP under the BP approach. For developing a solving approach, this research suggests a more restrict constraint that there is the same length of the idle time at each period. How to release these constraints to obtain better cost improvement is the future research direction for this study.

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