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Use of Linear Programming for Optimal Production in a Production Line in Saudi Food Co.

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Abstract**—**Few Saudi Arabia production companies face financial profit issues until this moment. This work presents a linear integer programming model that solves a production problem of a Saudi Food Company in Saudi Arabia. An optimal solution to the above-mentioned problem is a Linear Programming solution. In this regard, the main purpose of this project is to maximize profit. Linear Programming Technique has been used to derive the maximum profit from production of natural juice at Saudi Food Co. The operations of production of the company were formulated and optimal results are found out by using Lindo Software that employed Sensitivity Analysis and Parametric linear programming in order develop Linear Programming. In addition, the parameter values are increased, then the values of the objective function will be increased.

Keywords**—**Parameter linear programming, objective function, sensitivity analysis, optimize profit.

I. Introduction

LINEAR programming is a technique used to find the best value by taking many linear inequalities relating to some situation. In real life salutation, linear programming (LP) is very important part in different areas of industry. A large number of companies are using LP to solve several kinds of practical problems. Organization supervisors frequently look with choices identifying with the utilization of restricted resource. These resources may incorporate people, materials and money. Applications include transportation problems, production planning, investment decision problems, blending problems, location and allocation problems, among many others. Sensitivity analysis is one of the most interesting areas in optimization. It is a technique used to determine how different values of an independent variable impact a dependent variable under a given set of assumptions. Sensitivity analysis (or post-optimality analysis) is used to determine how the optimal solution is affected by changes, within specified ranges, in the objective function coefficients or the right-hand side (RHS) values. The strategy is utilized inside particular limits that rely upon at least one information factors, for example, the impact that adjustments in loan fees have on security costs. Sensitivity analysis is used to quantify the effect of changes in the initial data of linear programs on the optimal value.

The plant selected in this project is Saudi Food Company. Saudi Food Company established in 1979 in Riyadh city, Saudi Arabia. It became one of the leading companies in the industry of milk, juices, ice cream, chocolate, coffee and tea baradians in the Kingdom of Saudi Arabia, which is engaged in the production and marketing of juices and dried fruits, milk and dairy products [1].

In a few years the company has grown significantly and has been able to obtain a significant increase in its market share and rapidly gained widespread fame for its products. The company's annual production capacity is about 250 million liters of juices, beverages and milk. The company markets its products in Saudi Arabia, Cooperation Council Through more than 32,000 outlets in and outside the Kingdom [1]. The company uses the latest technology to produce high-quality products. Saudi owns a state-of-the-art factory, one of the largest factories in the Middle East. It operates the latest high technology in the world with the most modern production lines that are fully compatible with the plans to meet the needs and requirements of the market. The company also has large warehouses and sales offices throughout the Kingdom.

II. Literature Review

There are many researches done in food industry. It is necessary to review some of these works. These days, the most often used decision making tools in industrial is LP which help mangers and administration for various features to deal with production and other industry operations [2]. Balogun et al. worked on to maximize profit with the LP model, they focus on the application of LP Problem to drive the maximum profit from production of soft drink for Nigeria Bottling Company Nigeria, Ilorin plant 2012 [3]. The result they found out that Niger Mills Company Calabar should produce 576 (50 kg) bags of flour per day, and stop presently to produce semolina and wheat offal, because of their non-optimality, to enable the company maximize profit. Saurav and Ashish worked to use sensitivity analysis in order to complete the study effects of parametric variations on the production volume [4].

III. Problem Statement

The Company has produced some kind of natural juice with different flavors, such as pineapple, orange, fruit cocktail, mango, apple, grape, pomegranate, strawberries, guava, and banana as shown in Table I. In addition, the average cost and sealing by dollar for all products are shown in Table II. The available quantity of raw materials in inventory that company used to produce is shown in Table III. There are some insufficient resources available, so the issue depends on the most proficient method to settle on which resources would be assigned to get the best outcome. This outcome may identify

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with profit, cost or both. This company also produces four different brands of mix juice which are watermelon and rose, berry mix, lemon and mint, and lime and kiwi. The data will be formulated into interval objective LP problem. Parametric linear programming LP method is used to solve the problem. The transformation of interval objective LP to parametric includes the parametric LP problems [5]. Parametric LP has many applications in production schedules [6].

TABLE I
THE AMOUNT OF RAW MATERIALS FOR ALL PRODUCTS

Flavours	Sodium (kg)	Carbohydrate (Kg)	Dietary Fibers (Kg)	Sugar (Kg)	Water (Litters)
Pineapple	0.01	0.012	0.005	0.012	6.822
Orange	0.01	0.008	0.005	0.008	7.552
Fruit Cocktail	0.01	0.015	0.01	0.015	4.824
Mango	0.005	0.015	0.005	0.015	7.621
Apple	0.005	0.012	0.01	0.012	6.539
Grape	0.01	0.014	0.005	0.014	7.602
Pomegranate	0.005	0.010	0.01	0.013	6.12
Strawberries	0.01	0.008	0.005	0.010	7.055
Guava	0.005	0.012	0.01	0.012	7.508
Banana	0.01	0.012	0.005	0.008	7.051

TABLE II
AVERAGE COST AND SELLING OF A CRATE OF EACH PRODUCT

Products	Average Cost (SR)	Average Selling (SR)	Profit (SR)
Pineapple	1	2.19	1.19
Orange	1	2.51	1.51
Fruit Cocktail	1	2.51	1.51
Mango	1	2.19	1.19
Apple	1	2.51	1.51
Grape	1	2.41	1.41
Pomegranate	1	2.10	1.10
Strawberries	1	2.19	1.19
Guava	1	2.15	1.15
Banana	1	2.08	1.08

TABLE III
QUANTITY OF RAW MATERIALS AVAILABLE IN STOCK

Raw materials	Quantity available
Sodium (kg)	4,037
Carbohydrate (kg)	23,651
Dietary Fibres (kg)	4,651
Sugar (kg)	46,712
Water (Liters)	16,376,630

IV. RESEARCH OBJECTIVES

LP Technique is considered as one of methods that can derive the maximum profit from production of natural juice for the company, deciding how limited resources, raw materials of company, and shows which items should be produced. One of the company goals is to have efficient and organized production schedules by using a new algorithm which is parametric LP sensitivity analysis.

V. DATA COLLECTION

The data were collected from the company website and management, including the amount of raw materials needed to produce a bottle of each products. Also, the available quantity of raw materials holds in stock, and the average cost and sealing in Saudi riyal (SR) for all products. Some assumptions have been made for unavailable data.

VI. FORMULATION OF PROBLEM

A. Define the Decision Variables

X_1 = Pineapple
X_2 = Orange
X_3 = Fruit Cocktail
X_4 = Mango
X_5 = Apple
X_6 = Grape
X_7 = Pomegranate
X_8 = Strawberry
X_9 = Guava
X_{10} = Banana

The definition of the decision variable implies:

B. Formulate Objective Function (OF)

The basic formula for profit is revenue minus cost. Thus, Profit = Revenue – Cost, and Profit = $Z_{max.}$ Therefore, both revenue and cost must be defined in relation to this problem. After defining revenue and cost, profit (Z_{max}) can be determined using the formula above. To maximize Z:

$$Z = 1.19\ X_1 + 1.51\ X_2 + 1.51\ X_3 + 1.19\ X_4 + 1.51\ X_5 + 1.41\ X_6 + 1.10\ X_7 + 1.19\ X_8 + 1.15\ X_9 + 1.08\ X_{10}$$

C. Formulate Constraints

Constraint 1: The amount of sodium used to produce products in Kg. The amount of the raw material should be less than the 4,037 Kg that is available for production.

$$0.01\ X_1 + 0.01\ X_2 + 0.01\ X_3 + 0.005\ X_4 + 0.005\ X_5 + 0.01\ X_6 + 0.005\ X_7 + 0.01\ X_8 + 0.005\ X_9 + 0.01\ X_{10} \leq 4037$$

Constraint 2: The amount of carbohydrate used to produce products in Kg. The amount of the raw material should be less than the 23,651 Kg that is available for production.

$$0.012\ X_1 + 0.008\ X_2 + 0.015\ X_3 + 0.015\ X_4 + 0.012\ X_5 + 0.014\ X_6 + 0.010\ X_7 + 0.08\ X_8 + 0.012\ X_9 + 0.012\ X_{10} \leq 23651$$

Constraint 3: The amount of Diet Fibers used to produce products in Kg. The amount of the raw material the raw material should be less than the 4,651 Kg that is available for production.

$$0.005\ X_1 + 0.005\ X_2 + 0.01X_3 + 0.005\ X_4 + 0.01X3 + 0.005\ X4 + 0.01X5 + 0.005X6 + 0.01X7\ 0.005\ X8 + 0.01X9 + 0.005\ X_{10} \leq 4651$$

Constraint 4: The amount of sugar used to produce products is in units of Kg. The amount of sugar should be less than the 46,012 Kg that is available for production.

$$0.012\ X_1 + 0.008\ X_2 + 0.015\ X_3 + 0.015\ X_4 + 0.012\ X_5 + 0.014\ X_6 + 0.013\ X_7 + 0.010\ X_8 + 0.012\ X_9 + 0.008\ X_{10} \leq 46012$$

Constraint 5: The amount of water used to produce from products is in units of Liters. The amount of water should be less than the 16,376,630 Liters that is available for production.

$$6.822\ X_1 + 7.552\ X_2 + 4.824\ X_3 + 7.671\ X_4 + 6.539\ X_5 + 7.602\ X_6 + 6.12\ X_7 + 7.055\ X_8 + 7.508\ X_9 + 7.051\ X_{10} \leq 16376630$$

D. Solution Using LONDO, Results Analysis and Discussion

LINDO software was used to solve the problems.

max 1.19 X1 + 1.51 X2 + 1.51 X3 + 1.19 X4 + 1.51 X5 + 1.41 X6 + 1.10 X7 + 1.19 X8 + 1.51 X9 + 1.08 X10
S.T.

0.01 X1 + 0.01 X2 + 0.01 X3 + 0.005 X4 + 0.005 X5 + 0.01 X6 + 0.005 X7 + 0.01 X8 + 0.005 X9 + 0.01 X10 <= 4037

0.012 X1 +0.008 X2 + 0.015 X3 + 0.015 X4 + 0.012 X5 + 0.014 X6 + 0.010 X7 + 0.08 X8 + 0.012 X9 + 0.012 X10 <= 23651

0.005 X1 + 0.005 X2 + 0.01X3 + 0.005 X4 + 0.01X5 + 0.005X6+ 0.01X7 0.005 X8 + 0.01X9 + 0.005 X10 <= 4651

0.012 X1 + 0.008 X2 + 0.015 X3 + 0.015 X4 + 0.012 X5 + 0.014 X6 + 0.013 X7 + 0.010 X8 + 0.012 X9 + 0.008 X10 <= 46012

6.822 X1 + 7.552 X2 + 4.824 X3 + 7.671 X4 + 6.539 X5 + 7.602 X6 + 6.12 X7 + 7.055 X8 + 7.508 X9 + 7.051 X10 <= 16376630

```
LP OPTIMUM FOUND AT STEP      3

       OBJECTIVE FUNCTION VALUE

       1)      1000102.

 VARIABLE        VALUE          REDUCED COST
       X1         0.000000          2.048100
       X2         0.000000          0.550000
       X3         0.000000          0.870000
       X4    684600.000000          0.000000
       X5    122800.000000          0.000000
       X6         0.000000          0.650000
       X7         0.000000          0.410000
       X8         0.000000          0.870000
       X9         0.000000          0.000000
      X10         0.000000          0.980000

      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.000000        174.000000
       3)     11908.400391          0.000000
       4)         0.000000         64.000000
       5)     34269.398438          0.000000
       6)  10322074.000000          0.000000

NO. ITERATIONS=       3
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Fig. 1 LINDO output for objective function and decision variable values

```
RANGES IN WHICH THE BASIS IS UNCHANGED:

                           OBJ COEFFICIENT RANGES
VARIABLE        CURRENT        ALLOWABLE        ALLOWABLE
                  COEF          INCREASE         DECREASE
      X1        0.011900        2.048100         INFINITY
      X2        1.510000        0.550000         INFINITY
      X3        1.510000        0.870000         INFINITY
      X4        1.190000        0.320000         0.183333
      X5        1.510000        0.550000         0.000000
      X6        1.410000        0.650000         INFINITY
      X7        1.100000        0.410000         INFINITY
      X8        1.190000        0.870000         INFINITY
      X9        1.510000        0.000000         INFINITY
     X10        1.080000        0.980000         INFINITY

                           RIGHTHAND SIDE RANGES
     ROW        CURRENT        ALLOWABLE        ALLOWABLE
                  RHS           INCREASE         DECREASE
       2     4037.000000      614.000000      1711.500000
       3    23651.000000        INFINITY     11908.400391
       4     4651.000000     3423.000000       614.000000
       5    46012.000000        INFINITY     34269.398438
       6 16376630.000000        INFINITY  10322074.000000
```

Fig. 2 LINDO output for objective function ranges

After using the parametric LP method, the problem was formulated as shown in Figs. 1 and 2. These figures show that two products that should be produced are mango and apple juice. The company should produce 684,600 crates of mango juice and 122,800 crates of apple juice to produce a maximum profit of SR 1,000,102.

VII. Sensitivity Analysis

Fig. 2 shows the output of sensitivity analysis. It provides the following information:

- Changing the objective function coefficient
- Determining which variables are basic and which ones are non-basic
- Changing the RHS of the constraints [7], [8].

The objective function range portion of the LINDO output gives the range of values for an objective function coefficient for which the current basis remains optimal. Currently, X_4= 684,600, X_9 = 122,800, and $X_1 = X_2 = X_4 = X_5 = X_6 = X_7 = X_8$ = 0. The current solution value for X_5 is 807,400 and the current objective function coefficient for X_4 is 1.19. The allowable increase that is shown in Fig. 3 illustrates that the coefficient of X_4 in the objective function lies between 1.006667 and 1.51. The values of the decision variables remain unchanged. Producing 1,000,102 crates of mango juice remains optimal even if the profit on mango juice is not actually 1.19 but lies between 1.006667 and 1.51.

The same considerations can be drawn for X_2, X_3… X_{10}. The variables that are currently zero must be improved before they become non-zero. For example, the non-basic variable X_2 has a reduced cost of SR 0.55. This implies that if the company increases X_2's objective function coefficient by exactly SR 0.55, then there will be alternative optimal solutions, at least one of which will have X_2 as a basic variable. If the company increases X_2's objective function coefficient by more than SR 0.55, then any optimal solution to the LP will have X_2 as a basic variable. The company must keep a close watch on X_2's objective function coefficient, because a slight increase will change the LP's optimal solution. The same considerations can be drawn for X_1… and X_{10}. This company also produces four different brands of mix juice, which are watermelon and rose, berry mix, lemon and mint, and lime and kiwi.

The data will be formulated into interval objective LP problem. The transformation of interval objective LP to parametric includes the parametric LP problems. Parametric LP has many applications in production schedules. This company has some constraints to produce this kind of mixed juice and wants to have efficient and organized production schedules. The method that this company should employ is the parametric LP algorithm for the customer demands [9]. Al Rabie Food Co. requires the production of four bottles of watermelon and rose and one bottle of berry mix will be at most ten bottles. Also, the production of one bottle of watermelon and rose and five bottles of berry mix will result in at most nine bottles of the natural juice. The production of five bottles of watermelon and rose, four bottles of berry mix,

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and three bottles of lemon and mint will result in at least twelve bottles of the natural juice. Also, the production of one bottle of lemon and mint minus five bottles of lime and kiwi will result in at most four bottles in total. Finally, the production of one bottle of lemon and mint and one bottle of lime and kiwi will result in at most eight bottles of the natural juice.

The estimated profit per bottle of watermelon and rose is between SR 0.1 and SR 0.13, berry mix is between SR 0.16 and SR 0.19, lemon and mint is between SR 0.14 and SR 0.16, and lime and kiwi is between SR 0.25 and SR 0.27. Determining the production schedule for Al Rabie Food Co. is shown in Table IV. Let X_1 be the number of bottles of watermelon and rose produced, X_2 be the number of bottles of berry mix produced, X_3 be the number bottles of lemon and mint produced, X_4 be the number of bottles of lime and kiwi produced.

A. Mixed Juice

$$\text{Max } Z = (0.1+0.03p)X_1 + (0.16+0.03p)X_2 + (0.14+0.02p)X_3 + (0.25+0.02p)X_4$$

Subject to;

$$4X_1 + X_2 \leq 10,$$
$$X_1 + 5X_2 \leq 9$$
$$5X_1 + 4X_2 + 3X_3 \geq 12$$
$$X_3 - 5X_4 \leq 4$$
$$X_3 + X_4 \leq 8$$
$$X_i \geq 0$$

LINDO software was used to solve problems when interval parametric value (p) was 0.

```
LP OPTIMUM FOUND AT STEP      0

      OBJECTIVE FUNCTION VALUE

      1)      2.434737

 VARIABLE        VALUE          REDUCED COST
       X1         2.157895          0.000000
       X2         1.368421          0.000000
       X3         0.000000          0.110000
       X4         8.000000          0.000000

      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.000000          0.017895
       3)         0.000000          0.028421
       4)         4.263158          0.000000
       5)        44.000000          0.000000
       6)         0.000000          0.250000

NO. ITERATIONS=       0

RANGES IN WHICH THE BASIS IS UNCHANGED:

                           OBJ COEFFICIENT RANGES
VARIABLE        CURRENT        ALLOWABLE        ALLOWABLE
                  COEF          INCREASE         DECREASE
      X1        0.100000        0.540000         0.068000
      X2        0.160000        0.340000         0.135000
      X3        0.140000        0.110000         INFINITY
      X4        0.250000        INFINITY         0.110000

                           RIGHTHAND SIDE RANGES
     ROW        CURRENT        ALLOWABLE        ALLOWABLE
                  RHS           INCREASE         DECREASE
       2       10.000000       26.000000         3.857143
       3        9.000000       41.000000         6.500000
       4       12.000000        4.263158         INFINITY
       5        4.000000        INFINITY        44.000000
       6        8.000000        INFINITY         8.000000
```

Fig. 3 Output of the company when interval is 0

Fig. 3 shows the output of the company when interval is 0, the company should produce three bottles of watermelon and rose, two bottles of berry mix, and eight bottles of lime and kiwi to get an estimated profit of SR 2.434737 and should stop producing lemon and mint. It is also presented that the coefficient of X_1 in the objective function lies between 0.032 and 0.64. The values of the decision variables remain unchanged. Producing of one bottle of watermelon and rose remains optimal even if the profit on watermelon and rose is not actually 0.1, but lies between 0.032 and 0.64. The same considerations can be drawn for X_2, X_3, and X_4.

The variables which are currently zero must be improved before they become non-zero. For example, the non-basic variable X_3 has a reduced cost of SR 0.11. This implies that if company increases X_3's objective function coefficient by exactly SR 0.11, then there will be alternative optimal solutions, at least one of which will have X_3 as a basic variable. If company increases X_3's objective function coefficient by more than SR 0.11, then any optimal solution to the LP will have X_3 as a basic variable. Company must keep a close watch on X_3's objective function coefficient, because a slight increase will change the LP's optimal solution as shown in Table IV.

LINDO software has been used to solve the parametric LP problem from 0.1 to 1.0 and got the results are shown in Table IV.

TABLE IV
THE PARAMETRIC LP PROBLEM FROM 0.1 TO 1

Parameter (p)	X_1	X_2	X_3	X_4	Max Z SR
0.1	3	2	0	8	2.287
0.2	3	2	0	8	2.311
0.3	3	2	0	8	2.335
0.4	3	2	0	8	2.359
0.5	3	2	0	8	2.379
0.6	3	2	1	8	2.407
0.7	3	2	1	8	2.411
0.8	3	2	1	8	2.432
0.9	3	2	1	8	2.453
1.0	3	2	1	8	2.475

VIII. SUMMARY

It is clear that when the parameter values are increased, the values of the objective function are increased. For example, the company should produce three bottles of watermelon and rose, two bottles of berry mix, and eight bottles of lime and kiwi to get the estimated profit. When the parameter is increased to 1, the company should produce two bottles of watermelon and rose, two bottles of berry mix, one bottle of lemon and mint, and eight bottles of lime and kiwi. In addition, the estimated profit will be increased to SR 2.475. The result shown in Table IV presents the value of objective function (Z) for each interval. It is shown that if the parameter values are increased, then the values of the objective function will be increased. When this problem was solved, it was noticed that this method of LP which is parametric LP would be a good option for any company that wants to have the perfect production schedule.

IX. Conclusion

Based on the analysis the data collected from Saudi Food Co., the company should produce more mango juice and apple juice to achieve customer satisfaction and meet customer requirements. In addition, producing more mango juice and apple juice will maximize the company's profit. Moreover, any factory that would like to maintain an organized production schedule should apply the parametric LP algorithm that was used in this paper. The problem was solved using LINDO software for the parametric LP method; however, another developed computer program should be used to identify the production schedule of a company faster and more accurately**.**

References

[1] Al Rabie Food Co, http://www.alrabie.com/english/products.php?id=28-local

[2] Fagoyinbo S., Akinbo Y., Ajibode A., and Olaniran A. (2013). Maximization of profit in manufacturing industry using linear programming techniques: Nigeria limited. Proceedings of the 1st International Technology, Education and Environment Conference.

[3] Divya N., (2016). Some Applications of Simplex Method. International Journal of Engineering Research and Reviews ISSN 2348-697X (Online) Vol. 4, Issue 1, pp: (60-63), Month: January - March 2016, Available at: www.researchpublish.com.

[4] Saurav D. and Ashish S, (2016). Sensitivity analysis and optimal production scheduling as a dual phase simplex model. Indian journal of science and technology, vol 9(39), doi: 10.17485/ijst/2016/v9i39/100788, October 2016.

[5] Huang, Rujun, Lou, Xinyuan (2012), A Simplex Based Parametric Programming Method for the Large Linear Programming Problem. Proceeding of the International Multiconference of Engineers and computer scientists 2012 Vol II, IMECS 2012, March 14-16,2012, Hong Kong ISBN: 978-988-19251-9-0.

[6] Iheagwara, I., Opara, J., Esemokumo, A., and Lebechi I. (2014). Application of Linear Programming Problem on Niger Mills Company PLC Calabar. International Journal of Innovation and Research in Educational Sciences Volume 1, Issue 2, ISSN (Online): 2349–5219.

[7] Al-kuhali K, Zain M., & Hussein M. (2012). Production Planning of LCDs: Optimal Linear Programming and Sensitivity Analysis. ISSN 2224-6096 (Paper) ISSN 2225-0581 (online) Vol 2, No.9, 2012.

[8] Rob A. Zuidwijk. (2005). Linear Parametric Sensitivity Analysis of the Constraint Coefficient Matrix in Linear Programs. ERIM Report Series reference number ERS-2005-055-LIS, Publication October 2005.

[9] Cohon, J. L. (1979) Multi-objective programming and planning, Academic Press New York, 1979.