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Uniformly Strong Persistence for a Predator-Prey Model with Modified Leslie-Gower and Holling-Type II Schemes

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Abstract—In this paper, a asymptotically periodic predator-prey model with Modified Leslie-Gower and Holling-Type II schemes is investigated. Some sufficient conditions for the uniformly strong persistence of the system are established. Our result is an important complementarity to the earlier results.

Keywords—Predator-prey model, uniformly strong persistence, asymptotically periodic, Holling-type II.

I. Introduction

It is well known that the dynamical behavior of predator-prey systems is a form of very common biological inter action in the natural word. This topic has attracted a lot of attention and many good results have already been reported. For example, Chen and Chen [1] studied the linear stability of trivial periodic solution and semi-trivial periodic solutions of a periodic predator-prey system with distributed time delays and impulsive effect. Mukherjee [2] made a discussion on the uniform persistence in a generalized prey-predator system with parasitic infection. Chen [3] gave a theoretical study on the almost periodic solution of the non-autonomous two-species competitive model with stage structure. Sen et al. [4] analyzed the bifurcation behavior of a ratio-dependent prey-predator model with the Allee effect. Agiza et al. [5] investigated the chaotic phenomena of a discrete prey-predator model with Holling type II. Aggelis et al. [6] considered the coexistence of both prey and predator populations of a prey-predator model. Nindjin and Aziz-Alaoui [7] focused on the persistence and global stability in a delayed Leslie-Gower type three species food chain. Ko and Ryu[8] discussed the coexistence states of a nonlinear Lotka-Volterra type predator-prey model with cross-diffusion. Fazly and Hesaaraki [9] dealt with periodic solutions of a predator-prey system with monotone functional responses. One can see [10-52] etc. For more related stud ies. However, the research work on asymptotically periodic predator-prey model is very few at present.

The so-called asymptotically periodic function is the func tion $\bar{a}(t)$ which can be expressed by the form $\bar{a}(t) = a(t) + \tilde{a}(t)$, where $a(t)$ is a periodic function and $\tilde{a}(t)$ satisfies $\lim_{t\to+\infty} \tilde{a}(t) = 0$.

In 2003, Aziz-Alaoui and Okiye [53] investigated the stab ility and bifurcation of the following predator-prey model with time delay

$$\begin{cases} \frac{dx}{dt} = x(t)\left[a - bx(t) - \frac{cy}{x(t)+k_1}\right], \\ \frac{dy}{dt} = y(t)\left[d - \frac{ey(t)}{x(t)+k_2}\right], \end{cases} \tag{1}$$

with initial conditions $x(0) \geq 0, y(0) \geq 0$, where $x(t)$ denotes the densities of prey, at time t; $y(t)$ denotes the densities of the predator at time t; a, b, c, e, k_1, k_2 are all positive constants. In details, one can see [53].

We must point out that all biological and environment parameters in model (1) are constants in time. However, any biological or environmental parameters are naturally subject to fluctuation in time. Thus the effect of a periodically vary ing environment is important for evolutionary theory as the selective forces on systems in a fluctuating environment differ from those in a stable environment. Therefore, the assumptions of periodicity of the parameters are a way of incorporating the periodicity the environment(such as seasonal effects of weather, food supplies, mating habits and so on). Stimulated by above discussion and considering the asymptotically peri odic function, in this paper, we will modify system (1) as the form

$$\begin{cases} \frac{dx}{dt} = x(t)\Big[a(t) + \tilde{a}(t) - (b(t) + \tilde{b}(t))x(t) \\ \qquad - \frac{(c(t)+\tilde{c}(t))y}{x(t)+k_1(t)+\tilde{k}_1(t)}\Big], \\ \frac{dy}{dt} = y(t)\left[d(t) + \tilde{d}(t) - \frac{(e(t)+\tilde{e}(t))y(t)}{x(t)+k_2(t)+\tilde{k}_2(t)}\right], \end{cases} \tag{2}$$

with initial conditions $x(0) \geq 0, y(0) \geq 0$.

The principle object of this article is to investigate the uniformly strong persistence of system (2). Only very few papers which deal with this topic, see [10,54].

In this paper, we always assume that system (2) satisfies

(H) $a(t), b(t), c(t), d(t), k_1(t), k_2(t)$ are continuous, non-negative periodic functions; $\tilde{a}(t), \tilde{b}(t), \tilde{c}(t), \tilde{d}(t),\ \tilde{k}_1(t), \tilde{k}_2(t)$ are continuous, nonnegative asymptotically items of asymp totically periodic functions.

II. Uniformly Strong Persistence

In this section, we shall present some result about the uniformly strong persistence of system (2). For convenience and simplicity in the following discussion, we introduce the notations, definitions and Lemmas. Let

$$0 < f^l = \lim_{t\to+\infty} \inf f(t) \leq \lim_{t\to+\infty} \sup f(t) = f^u < +\infty.$$

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In view of the definitions of lower limit and upper limit, it follows that for any $\varepsilon > 0$, there exists $T > 0$ such that

$$f^l - \varepsilon \le f(t) \le f^u + \varepsilon, \text{ for } t \ge T. \tag{3}$$

Definition 1. *The system (2) is said to be strong persistence, if every solution $x(t)$ of system (2) satisfies*

$$0 < \lim_{t\to+\infty} \inf x(t) \le \lim_{t\to+\infty} \sup x(t) \le \delta < +\infty.$$

Lemma 1. *Both the positive and nonnegative cones of R^2 are invariant with respect to system (2).*

It follows from Lemma 1 that any solution of system (2) with a nonnegative initial condition remains nonnegative.

Lemma 2.[10] *If $a > 0, b > 0$ and $\dot{x}(t) \ge (\le) x(t)(b - ax^{\alpha}(t))$, where α is a positive constant, when $t \ge 0$ and $x(0) > 0$, we have*

$$x(t) \ge (\le) \left(\frac{b}{a}\right)^{\frac{1}{\alpha}} \left[1 + \left(\frac{bx^{-\alpha}(0)}{a} - 1\right) e^{-b\alpha t}\right]^{-\frac{1}{\alpha}}.$$

In the following, we will ready to state our result.

Theorem 1. *Let θ_2 be defined by (9). Assume that the condition (H) and $a^l k_1^l > c^u \theta_2$ hold, then system (2) is uniformly strong persistence.*

Proof It follows from (3) that for any $\varepsilon > 0$, there exists $T_1 > 0$ such that for $t \ge T_1$,

$$\begin{cases} a^l - \varepsilon \le a(t) \le a^u + \varepsilon, -\varepsilon < \tilde{a}(t) < \varepsilon, \\ b^l - \varepsilon \le b(t) \le b^u + \varepsilon, -\varepsilon < \tilde{b}(t) < \varepsilon, \\ c^l - \varepsilon \le c(t) \le c^u + \varepsilon, -\varepsilon < \tilde{c}(t) < \varepsilon, \\ d^l - \varepsilon \le d(t) \le a_2^u + \varepsilon, -\varepsilon < \tilde{d}(t) < \varepsilon, \\ k_1^l - \varepsilon \le k_1(t) \le k_1^u + \varepsilon, -\varepsilon < \tilde{k}_1(t) < \varepsilon, \\ k_2^l - \varepsilon \le k_2(t) \le k_2^u + \varepsilon, -\varepsilon < \tilde{k}_2(t) < \varepsilon. \end{cases} \tag{4}$$

Substituting (4) into the first equation of system (2), we have

$$\begin{aligned} \frac{dx}{dt} &= x(t)\Big[a(t) + \tilde{a}(t) - (b(t) + \tilde{b}(t))x(t) \\ &\quad - \frac{(c(t) + \tilde{c}(t))y}{x(t) + k_1(t) + \tilde{k}_1(t)}\Big] \\ &\le x(t)\Big[a(t) + \tilde{a}(t) - (b(t) + \tilde{b}(t))x(t)\Big] \\ &\le x(t)\left[(a^u + 2\varepsilon) - (b^l - 2\varepsilon)x(t)\right]. \end{aligned} \tag{5}$$

By Lemma 2, we get

$$\lim_{t\to+\infty} \sup x(t) \le \frac{a^u}{b^l} := \theta_1. \tag{6}$$

Then for any $\varepsilon > 0$, there exists $T_2 > T_1 > 0$ such that

$$x(t) \le \theta_1 + \varepsilon, \ t \ge T_2. \tag{7}$$

Similarly, from (3) and the second equation of system (2), we obtain that for any $\varepsilon > 0$, there exists $T_3 > T_2 > 0$ such that

$$\begin{aligned} \dot{y}(t) &= y(t)\left[d(t) + \tilde{d}(t) - \frac{(e(t) + \tilde{e}(t))y(t)}{x(t) + k_2(t) + \tilde{k}_2(t)}\right] \\ &\le y(t)\left[(d^u - 2\varepsilon) - \frac{(e^l - 2\varepsilon)y(t)}{\theta_1 + \varepsilon + k_2^u + 2\varepsilon}\right]. \end{aligned} \tag{8}$$

In view of Lemma 2, we derive

$$\lim_{t\to+\infty} \sup y(t) \le \frac{d^u(\theta_1 + k_2^u)}{e^l} := \theta_2. \tag{9}$$

Then for any $\varepsilon > 0$, there exists $T_4 > T_3 > 0$ such that

$$y(t) \le \theta_2 + \varepsilon, \ t \ge T_4. \tag{10}$$

By (7), (10) and the first equation of system (2), we obtain that for any $\varepsilon > 0$, there exists $T_5 > T_4 > 0$ such that

$$\begin{aligned} \frac{dx}{dt} &= x(t)\Big[a(t) + \tilde{a}(t) - (b(t) + \tilde{b}(t))x(t) \\ &\quad - \frac{(c(t) + \tilde{c}(t))y}{x(t) + k_1(t) + \tilde{k}_1(t)}\Big] \\ &\ge x(t)\Big[(a^l - 2\varepsilon) - (b^u + 2\varepsilon)x(t) \\ &\quad - \frac{(c^u + 2\varepsilon)(\theta_2 + \varepsilon)}{k_1^l - 2\varepsilon}\Big]. \end{aligned} \tag{11}$$

Using Lemma 2 again, we have

$$\lim_{t\to+\infty} \inf x(t) \ge \frac{a^l k_1^l - c^u \theta_2}{k_1^l b^u} := \delta_1. \tag{12}$$

Thus for any $\varepsilon > 0$, there exists $T_6 > T_5 > 0$ such that

$$x(t) \ge \delta_1 - \varepsilon. \tag{13}$$

According (7), (10) and the second equation of system (2), we obtain that for any $\varepsilon > 0$, there exists $T_7 > T_6 > 0$ such that

$$\begin{aligned} \dot{y}(t) &= y(t)\left[d(t) + \tilde{d}(t) - \frac{(e(t) + \tilde{e}(t))y(t)}{x(t) + k_2(t) + \tilde{k}_2(t)}\right] \\ &\ge y(t)\left[d^l + 2\varepsilon - \frac{e^u + 2\varepsilon}{k_2^l - 2\varepsilon} y(t)\right]. \end{aligned} \tag{14}$$

Using Lemma 2 again, we have

$$\lim_{t\to+\infty} \inf y(t) \ge \frac{d_1^l e^u}{k_2^l} := \delta_2. \tag{15}$$

Thus we complete the proof of Theorem 1.

III. Conclusions

In this paper, we have investigated a asymptotically pe riodic predator-prey model with modified Leslie-gower and Holling-type II schemes. A set of sufficient conditions for the uniformly strong persistence of the system are derived. It is shown that under some suitable conditions, the asymptotically periodic predator-prey model with modified Leslie-Gower and Holling-type II schemes is uniformly strong persistence. Our results obtained in this paper complement the earlier results.

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