

# Ultimate Shear Resistance of Plate Girders Part 2- Höglund Theory

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**Abstract**—Ultimate shear resistance (USR) of slender plate girders can be predicted theoretically using Cardiff theory or Höglund theory. This paper will be concerned with predicting the USR using Höglund theory and EC3. Two main factors can affect the USR, the panel width “ $b$ ” and the web depth “ $d$ ”, consequently, the panel aspect ratio ( $b/d$ ) has to be identified by limits. In most of the previous study, there is no limit for panel aspect ratio indicated. In this paper theoretical analysis has been conducted to study the effect of ( $b/d$ ) on the USR. The analysis based on ninety six test results of steel plate girders subjected to shear executed and collected by others. New formula proposed to predict the percentage of the distance between the plastic hinges form in the flanges “ $c$ ” to panel width “ $b$ ”. Conservative limits of ( $c/b$ ) have been suggested to get a consistent value of USR.

**Keywords**—Ultimate shear resistance, Plate Girder, Höglund’s theory, EC3.

## I. INTRODUCTION

FOR a plate girder subjected to a small shear load, bending theory can be used to determine how the internal forces are carried by the web and the flanges. When the applied load is increased, the failure mode of a plate girder will depend largely on the panel aspect ratio ( $b/d$ ) and the web slenderness ratio ( $d/t$ ). The ultimate shear resistance of steel plate girders has been studied extensively, both experimentally and theoretically [1]-[6]. Experimental studies of the ultimate shear resistance of steel plate girders have indicated that at failure, the girders exhibit the characteristic diagonal shear buckling of the web and developed the plastic hinges in the flanges. Höglund’s theory is based on a system of perpendicular bars in compression and tension, which is assumed to represent the web panel [7]. The procedures incorporated in EC3 are divided in two methods of design; the first method is the simple post critical design, which is applicable to either stiffened or unstiffened girders; where the second design method called tension-field method is applicable to stiffened girder [8]. The rotating- stress-field theory developed by Höglund forms is the basis of the first method of design stated in Euro code 3. The second design procedure in EC3 based on Cardiff tension-field theory and it is applied to girders with intermediate transverse stiffeners. This method is intended to produce more economical designs for a limited range of girder configurations. Theoretical predictions of the ultimate shear resistance of plate girders

based on EC3 simple post-critical design procedure appear inconsistent and unduly conservative when compared with the available test data [1]. Theoretical predictions based on EC3 tension-field design procedure (*the second method*), taking into account the limited range of web panel aspect ratios, are less conservative. Analysis of EC3 design methods made by Nethercot and Field [1] indicated that, the existing procedures do not achieve the specified reliability by predicting the ultimate shear resistance of plate girders, the partial safety factor  $\gamma_m$  would have to be increased from 1.05 to 1.35, which may reduce the competitiveness of many aspects of steel construction. In this paper theoretical analysis using Höglund theory and EC3 2<sup>nd</sup> design method will be conducted to determine the limits of panel aspect ratio which can be applicable in each case.

## II. SHEAR STRENGTH OF PLATE GIRDER WEB PANEL USING HÖGLUND THEORY

Höglund rotating- stress-field theory is based on a system of perpendicular bars in compression and tension, which are assumed to represent the web panel. Originally it was developed for girders with web stiffeners at support only. For webs in shear, there is a substantial post-buckling strength provided after buckling by the anchoring system with the surrounding flanges and transverse stiffeners [7]. In pure shear the absolute value of the principal membrane stresses  $\sigma_1$  and  $\sigma_2$  are the same as long as no buckling occurred ( $\tau < \tau_{cr}$ ). After buckling load reached ( $V_{cr} = d * t_w * \tau_{cr}$ ), the web plate would buckle and redistribution of stresses would start. Any increase in the applied load will increase the tensile stress  $\sigma_1$  associated with slight increase in compressive stress  $\sigma_2$  may be occurred as shown in Fig. 1.

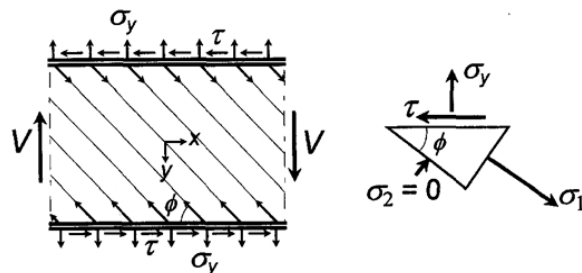


Fig. 1 State of stress in web

From Fig. 1;

$$\tau = \sigma_1 * \sin \phi * \cos \phi = 0.50 * \sigma_1 * \sin 2\phi \quad (1)$$

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where the direction of the tensile stresses chose to give  $\tau$  maximum. When  $\sigma_1$  is equal to the yield strength of the web,  $f_{yw}$ , then

$$\frac{\tau_u}{f_v} = \frac{0.5 * f_{yw}}{f_v} = \frac{\sqrt{3}}{2} \quad \text{for } \phi = 45^\circ \quad (2)$$

where

$$f_v = \frac{f_{yw}}{\sqrt{3}} \quad (3)$$

This theory is called ideal tension field theory, and is valid only if the flanges are prevented from moving towards each other by an external structure [7].

Figs. 2 (a) and (b) show the total longitudinal force and shear forces in the web panel due to the externally applied load, where 2 (c) and (d) show the stress distribution through the beam cross section corresponding to these forces. In long beam, with transverse stiffeners at the end only, the web prevents the flanges to move towards each other, that is why the membrane stresses in the transverse direction are zero. From the triangle shown in Fig. 2 (g) gives:

$$\sigma_1 = \frac{\tau}{\tan \phi} \quad (4)$$

$$\sigma_2 = -\tau * \tan \phi \quad (5)$$

where  $\phi$  constitutes the direction of the principal stress. This state of stress has a stress component  $\sigma_h$  in the longitudinal direction. This component can be expressed as follow:

$$\sigma_h = \tau * \left( \frac{1}{\tan \phi} - \tan \phi \right) = \sigma_1 + \sigma_2 \quad (6)$$

The total longitudinal force in the web is less than

$$N_h = \sigma_h * d * t_w \quad (7)$$

This force has to be anchored at the ends of the beam by a transverse short beam called rigid end post, in order to fully develop the rotated stress field as shown in Fig. 2. The ultimate shear strength of the beam can be derived using the Von Mises yield theory criterion as follow:

$$\sigma_1^2 - \sigma_1 * \sigma_2 + \sigma_2^2 = f_{yw}^2 \quad (8)$$

Assuming that the compression stress remain equal to the shear buckling stress after buckling, and acting in a smaller angle than  $45^\circ$ :

$$\sigma_2 = -\tau_{cr} \quad (9)$$

Furthermore, the slenderness parameter  $\lambda_w$  is introduced

$$\lambda_w = \sqrt{\frac{f_v}{\tau_{cr}}}$$

where

$$\tau_{cr} = K \left[ \frac{\pi^2 E}{12(1-\mu^2)} \right] * \left( \frac{t}{d} \right)^2 \leq \tau_{yw} \quad [9] \quad (10)$$

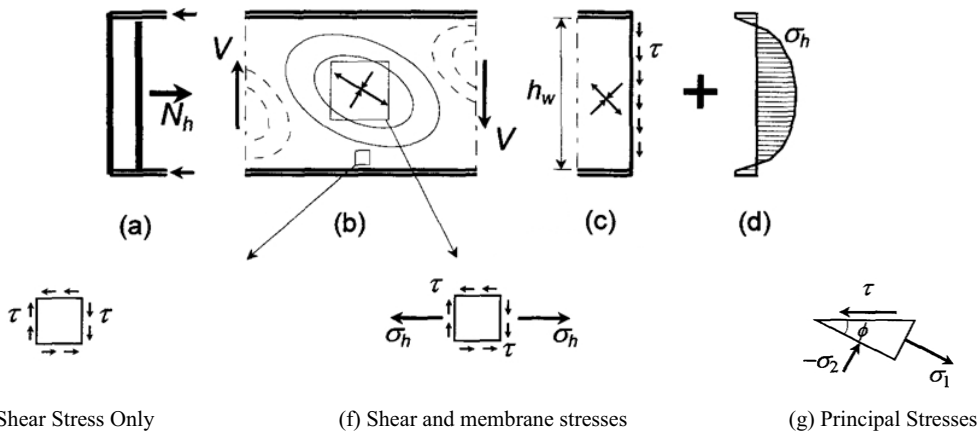


Fig. 2 State of stress in web of a beam with transverse stiffeners at the ends only

From (3)-(5), (8)-(10) the ultimate strength  $\tau_u = \tau$  can be derived as a function of  $\lambda_w$  as follow:

$$\frac{\tau_u}{f_v} = \frac{\sqrt{3}}{\lambda_w} * \sqrt{\sqrt{1 - \frac{1}{4\lambda_w^4}} - \frac{1}{2\sqrt{3}\lambda_w^2}} \quad \text{for } \lambda_w \geq 1.00 \quad (11)$$

The square-root in (11) is close to 1.00 if  $\lambda_w \geq 2.5$ ; this leads to

$$\frac{\tau_u}{f_v} \cong \frac{1.32}{\lambda_w} \quad (12)$$

The inclination of the tension stress  $\sigma_1$ , defined by the angle  $\phi$ , is decreased when the ratio  $\frac{\tau_u}{\tau_{cr}}$  is increased. For this reason

the theory is called “*Rotated Stress Field Theory*”.

The shear buckling capacity can be obtained from

$$V_w = \rho_v * f_{yw} * d * t_w \quad (13)$$

where  $\rho_v$  can be defined as shear buckling reduction factor which is given in Table I:

TABLE I  
REDUCTION FACTOR  $\rho_v$  FOR SHEAR BUCKLING

$\lambda_w$	Rigid end Post (Steel)	Non-Rigid end Post (Steel)
$\lambda_w < 0.48/\eta$	$\eta$	$\eta$
$0.48/\eta \leq \lambda_w < 1.08$	$\frac{0.48}{\lambda_w}$	$\frac{0.48}{\lambda_w}$
$\lambda_w \geq 1.08$	$\frac{0.79}{(0.7 + \lambda_w)}$	$\frac{0.48}{\lambda_w}$

The value of  $\rho_v$  is a reduced value related to scatter in test results such as initial imperfections [7]. For small slenderness ratios,  $\lambda_w < 0.48/\eta$ , strain hardening in shear can take place, this produce larger strength than the corresponding to initial yielding  $\eta = \frac{1}{\sqrt{3}} \approx 0.58$ .

#### A. Transversely Stiffened Web

Transverse stiffeners welded to the web have two main effects on the behavior and strength of a girder in shear; first, increase the elastic buckling strength by preventing the web from out-of-plane deflection, and second, they prevent the flanges from coming closer to each other. The shear buckling coefficient  $K$ , can be expressed as follow [9]:

$$K = 5.35 + 4 * \left(\frac{d}{b}\right)^2, \quad \text{for } \frac{b}{d} > 1 \quad (14)$$

$$K = 5.35 * \left(\frac{d}{b}\right)^2 + 4, \quad \frac{b}{d} < 1$$

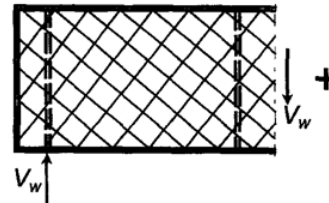
In failure stage, four hinges denoted E, H, G, and K, form in the top and bottom flanges respectively as shown in Fig. 3 (b).

A tension stress field, EHGK, develops in the web as illustrated in Fig. 3 (b). The ultimate shear force,  $V_f$ , which is transmitted by the tension stress field is obtained from the equation of equilibrium of the flanges portion  $c$  as follow:

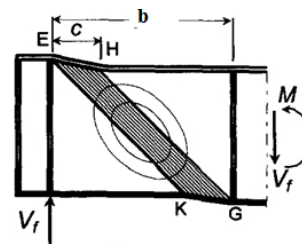
$$V_f = \frac{4 * Z * f_{yf}}{c} \quad (15)$$

However, it is assumed that the shear resistance of the web,  $V_w$  is not changed by the formation of the tension field between flanges. Then the shear resistance of the girder,  $V_u$ , is the sum of the shear resistance of the web,  $V_w$ , and the shear resistance contributed by the flanges  $V_f$ .

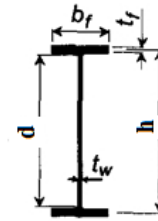
$$V_u = V_w + V_f \quad (16)$$



(a) Shear force carried by web



(b) Shear force carried by truss action



(c) Cross Section

Fig. 3 Model of web in post buckling range

The distance  $c$  is estimated for steel plate girders as follow:

$$c = \left( 0.25 + \frac{1.6 * b_f * t_f^2 * f_{yf}}{t_w * d^2 * f_{yw}} \right) * b \quad (17)$$

### III. EUROCODE 3

Two methods of design are proposed in EC3, the first is the simple post critical design where the second is the tension field design method. The simple post critical design procedure incorporated in EC3 is based on theory proposed by Höglund and is applicable to either stiffened or unstiffened girders. In this method of design the ultimate design shear resistance of a plate girders referred to it by  $V_{ba,Rd}$  and is given as:

$$V_{ba,Rd} = \frac{d * t * \tau_{ba}}{\gamma_m} \quad \text{for } (M \leq M_f) \quad (18)$$

where  $\gamma_m$  is the partial material safety factor and  $\tau_{ba}$  is the simple post-critical shear stress, which depends on the web slenderness parameter  $\lambda_w$ .

For webs with transverse stiffeners at the supports and intermediate transverse stiffeners  $\tau_{cr}$  is calculated using  $K$  as stated in (14). For webs with transverse stiffeners at the supports but without intermediate transverse stiffeners,  $\tau_{cr}$  is calculated assuming  $K = 5.3$  [4].

For stocky webs ( $\lambda_w \leq 0.8$ )

$$\tau_{ba} = \tau_{yw} \quad (19)$$

For webs of intermediate slenderness ( $0.8 \leq \lambda_w \leq 1.2$ )

$$\tau_{ba} = \tau_{yw} * [1 - 0.625 * (\lambda_w - 0.80)] \quad (20)$$

For slender webs ( $\lambda_w \geq 1.2$ )

$$\tau_{ba} = \tau_{yw} * \left( \frac{0.9}{\lambda_w} \right) \quad (21)$$

where the second method is known as tension-field design method. EC3 tension-field design shear resistance  $V_{bb,Rd}$  is based on the Cardiff tension-field theory and is expressed as:

$$V_{bb,Rd} = \frac{\tau_{bb} * d * t + 0.9 * g * t * \sigma_{bb} * \sin \theta}{\gamma_m} \quad \text{for } (M \leq M_f) \quad (22)$$

where  $\tau_{bb}$  is the shear buckling stress,  $\sigma_{bb}$  is the tension-field stress,  $g$  is the width of the tension field and  $\theta$  is the inclination of the tension field. for stocky webs ( $\lambda_w \leq 0.8$ )

$$\tau_{bb} = \tau_{yw} \quad (23)$$

For webs of intermediate slenderness ( $0.8 < \lambda_w < 1.25$ )

$$\tau_{bb} = \tau_{yw} * [1 - 0.8 * (\lambda_w - 0.80)] \quad (24)$$

For slender webs ( $\lambda_w \geq 1.25$ )

$$\tau_{bb} = \tau_{yw} * \left( \frac{1}{\lambda_w^2} \right) \quad (25)$$

where the tension-field stress  $\sigma_{bb}$  is given by

$$\sigma_{bb} = \sqrt{(\sigma_{yw}^2 - 3 * \tau_{bb}^2 + \psi^2)} - \psi \quad (26)$$

in which

$$\psi = 1.5 * \tau_{bb} * \sin 2\theta \quad (27)$$

The width of the tension-field  $g$  is given by

$$g = d * \cos \theta - (b - S_c - S_t) * \sin \theta \quad (28)$$

where  $S_c$  and  $S_t$  are distances at which plastic hinges form in the compression and tension flanges, respectively, given by

$$S = \frac{2}{\sin \theta} * \sqrt{\left( \frac{M_{Nf,Rk}}{\sigma_{bb} * t} \right)} \quad (29)$$

In which  $M_{Nf,Rk}$  is the reduced plastic moment of the flange allowing for a longitudinal force  $N_{F,sd}$  in the flange and can be expressed as:

$$M_{Nf,Rk} = 0.25 * \sigma_{yf} * b_f * t_f^2 * \left( 1 - \left( \frac{N_{F,sd}}{\sigma_{yf} * b_f * t_f / \gamma_m} \right)^2 \right) \quad (30)$$

The angle  $\theta$  can be either determined by iteration to give the maximum value of  $V_{bb,Rd}$  or approximated as

$$\theta = \frac{2}{3} * \tan^{-1} \left( \frac{d}{b} \right) \quad (31)$$

#### IV. EXPERIMENTAL RESULTS

Extensive Experimental studies have been conducted in Cardiff on the ultimate shear resistance of steel plate girders, summary of which have been presented by Höglund, Nethercot and Newark [1] presented in Table II. However, ninety six test results collected by Höglund, Nethercot and Byfield and listed by Davies et al [1] will be used in this study. A summary of the test results (girder dimensions, material properties and failure loads) is presented in Table II.

TABLE II  
DETAILS OF TEST GIRDERS AND TEST RESULTS

Girder reference	$b$	$d$	$t_w$	$b_f$	$t_f$	$E$	$f_{yw}$	$f_{yt}$	$V_u$	$V_{exp./VS}$ Höglund	$V_{exp./VS}$ EC3
<b>C4</b>	254	356	1.47	41	6.4	210000	258	287	41	0.86	0.76
<b>G6-T1</b>	1905	1270	4.9	308	19.8	210000	253	261	516	1.27	1.27
<b>G6-T2</b>	953	1270	4.9	308	19.8	210000	253	261	662	1.21	1.05
<b>G6-T3</b>	635	1270	4.9	308	19.8	210000	253	261	787	1.12	0.99
<b>G7-T1</b>	1270	1270	4.98	310	19.5	210000	253	259	623	1.30	1.17
<b>G7-T2</b>	1270	1270	4.98	310	19.5	210000	253	259	645	1.35	1.21
<b>G8-T1</b>	3810	1270	5.08	305	19.1	210000	263	284	375	0.96	1.33
<b>G8-T2</b>	1905	1270	5.08	305	19.1	210000	263	284	445	1.01	1.02
<b>G8-T3</b>	1905	1270	5.08	305	19.1	210000	263	284	516	1.17	1.18
<b>G9-T1</b>	3810	1270	3.33	305	19.1	210000	307	288	213	1.04	1.22
<b>G9-T2</b>	1905	1270	3.33	305	19.1	210000	307	288	334	1.38	1.12
<b>G9-T3</b>	1905	1270	3.33	305	19.1	210000	307	288	352	1.45	1.18
<b>HIT2</b>	1905	1270	9.98	459	24.8	210000	745	703	3421	1.24	1.27
<b>H2T1</b>	1270	1270	9.91	459	51.2	210000	760	750	4079	1.00	0.91
<b>C-AC2</b>	2490	457.2	3.1	102	9.7	210000	215	755	120	1.09	1.38
<b>C-AC4</b>	2515	457.2	4.3	127	16.3	210000	236	783	245	1.18	1.19
<b>C-AC5</b>	2515	457.2	4.3	127	19.1	210000	236	790	232	1.11	1.10
<b>B</b>	1200	1200	4.5	240	12	210000	490	491	760	1.38	1.13
<b>S-2</b>	581	319	3.2	100	10.5	210000	352	273	161	1.03	0.98
<b>S-3</b>	577	477	3.2	101	10.5	210000	317	272	198	1.12	1.09
<b>2.2</b>	1440	600	2	175	6	210000	255	255	75	1.21	1.52
<b>TG3</b>	1000	1000	2.5	200	16.4	210000	200	281	190	1.41	1.11
<b>TG3.1</b>	1000	1000	2.5	200	16.4	210000	200	281	190	1.41	1.11
<b>TG4</b>	1000	1000	2.5	200	20.2	210000	200	281	219	1.48	1.17
<b>TG4.1</b>	1000	1000	2.5	200	20.1	210000	200	281	207	1.40	1.11
<b>TG5</b>	1000	1000	2.5	200	29.7	210000	200	281	308	1.70	1.35
<b>TG5.1</b>	1000	1000	2.5	200	29.7	210000	200	281	300	1.65	1.31
<b>US2/5</b>	788	359	3.17	97	12	210000	230	422	135	1.10	1.04
<b>US3/5</b>	788	359	2.7	96	12	210000	257	422	90	0.87	0.89
<b>TG14</b>	305	305	0.97	76	3.12	210000	219	305	25	1.46	1.27
<b>TG15</b>	305	305	0.97	76	5	210000	219	286	29	1.50	1.27
<b>TG16</b>	305	305	0.97	76	6.45	210000	219	337	32	1.44	1.22
<b>TG17</b>	305	305	0.97	76	9.32	210000	219	308	39	1.50	1.26
<b>TG18</b>	305	305	0.97	76	13	210000	219	304	51	1.68	1.36
<b>TG19</b>	305	305	0.97	76	15.5	210000	219	268	55	1.73	1.37
<b>TG22</b>	305	305	2.03	76	6.5	210000	229	337	79	1.23	1.13
<b>TG23</b>	305	305	2.03	76	9.2	210000	229	308	81	1.17	1.08
<b>TG24</b>	305	305	2.03	76	13	210000	229	304	96	1.24	1.17
<b>TG25</b>	305	305	2.03	76	15.5	210000	229	268	104	1.29	1.22
<b>STG1</b>	551	279	2	127	7.9	210000	255	275	60	1.03	1.05
<b>STG2</b>	502	253	1.6	127	6.4	210000	272	275	40	0.99	1.04
<b>STG4</b>	498	251	1.25	102	6.4	210000	246	275	35	1.36	1.44
<b>RTG1</b>	305	305	1.27	76	4.5	210000	244	275	40	1.35	1.23
<b>RTG2</b>	305	305	1.27	76	4.7	210000	244	275	41	1.37	1.25
<b>RTG4</b>	254	254	0.95	76	4.7	210000	259	275	24	1.22	1.06
<b>TS1/3</b>	700	813	4.06	209	12	210000	265	429	312	0.94	0.85
<b>TS1/4</b>	700	813	4.06	212	12	210000	265	429	387	1.16	1.06
<b>MSO</b>	947	608	2.01	102	10.1	210000	261	269	94	1.32	1.27
<b>SD1</b>	594	594	2	250	12	210000	276	212	129	1.28	1.06
<b>SD3</b>	594	594	2	250	12	210000	276	212	156	1.54	1.28
<b>TGV1-1</b>	1200	600	2.07	200	10	210000	211	247	83	1.25	1.33
<b>TGV1-2</b>	600	600	2.07	200	10	210000	211	247	111	1.31	1.14
<b>TGV2-2</b>	600	600	2.08	200	10	210000	211	247	115	1.34	1.17
<b>TGV3-2</b>	600	600	2.01	200	10	210000	211	247	113	1.39	1.20
<b>TGV4</b>	597	598	1.97	201	10.1	210000	224	255	102	1.23	1.05
<b>TGV5</b>	595	598	1.98	201	10	210000	232	252	105	1.24	1.06
<b>TGV6</b>	595	598	1.97	201	10.1	210000	228	254	102	1.22	1.04

TGV7-2	596	599	1.98	201	10.1	210000	221	250	106	1.28	1.10
TGV10-1	595	599	1.91	200	10	210000	219	284	102	1.28	1.08
TGV10-2	595	599	1.91	200	10	210000	219	284	106	1.33	1.12
TGV11-2	597	599	1.91	200	10	210000	220	211	102	1.35	1.15
S3/1	300	300	1.03	35	3.2	200000	169	295	19	1.26	1.18
S4/1	345	351	1.07	40	3.2	200000	169	295	21	1.26	1.15
S5/1	400	399	1.09	39	3.2	200000	169	295	23	1.32	1.19
S2/1.5	375	249	1.05	40	3.2	200000	169	295	16	1.19	1.26
S3/1.5	450	301	1.03	39	3.2	200000	169	295	16	1.19	1.26
S4/1.5	522	352	1.1	39	3.3	200000	169	295	13	0.84	0.89
IS1-BA	942	608	2.1	100	10	191000	183	269	76	1.25	1.24
LS3-BA	947	608	2.46	100	10.1	197000	201	283	103	1.22	1.27
MCS1-PB3	732	1000	4.4	300	15.1	210000	169.7	226.6	388	1.16	1.05
PA1	600	800	1	249	12	210000	216	206	81	1.75	1.09
PA2	600	800	1	249	12	210000	216	206	84	1.82	1.14
PA3	600	800	1	249	12	210000	216	206	85	1.84	1.15
PB1	500	800	1	249	12	210000	216	206	90	1.67	1.11
PB2	500	800	1	249	12	210000	216	206	91	1.69	1.13
PC1	1000	800	1	250	10	210000	216	262	54	1.77	1.03
PC2	1000	800	1	250	10	210000	216	262	54	1.77	1.03
PD1	750	800	1	250	10	210000	216	262	65	1.76	1.03
PD2	750	800	1	250	10	210000	216	262	65	1.76	1.03
PD3	750	800	1	250	10	210000	216	262	75	2.03	1.19
PC3	750	800	1	250	10	210000	216	262	79	2.14	1.25
PB3	732	1000	4.4	300	15.1	205000	169.7	226.6	388	1.17	1.06
PB4	732	1000	4.4	300	15.1	205000	169.7	226.6	388	1.17	1.06
B1	9000	600	2.86	226	9.9	210000	419	294	146	1.00	2.73
B4	9000	600	2	151	6.1	210000	280	304	71	1.19	3.64
K1	6000	600	2.86	226	9.9	210000	419	294	158	1.07	2.57
1A	8100	600	2.96	225	10	210000	243	251	146	1.30	2.75
1B	8100	600	2.97	225	10	210000	243	251	132	1.17	2.46
2A	8100	600	3	225	10	210000	243	251	132	1.15	2.40
2B	8100	600	2.94	225	10	210000	243	251	127	1.14	2.43
3A	8100	600	2	150	6	210000	292	286	59	0.96	2.90
3B	8100	600	2	150	6	210000	292	286	61	1.00	2.99
4A	8100	600	2.01	150	6	210000	292	286	71	1.15	3.45
4B	8100	600	2.03	150	6	210000	292	286	68	1.08	3.23
CPI/1	747	500	2.04	100	8	210000	246	256	88	1.31	1.34
RCPI/1	710	718	2.01	100	8.1	210000	271	288	127	1.52	1.23

V. RELATION BETWEEN PANEL ASPECT RATIO ( $B/D$ ) AND ULTIMATE SHEAR BY HÖGLUND OR EC3

Based on the extensive experimental study conducted by Höglund, Nethercot and Byfield and collected by Davies et al, [1] the following theoretical analysis is performed. For each test from the ninety six girders, two different values of the ultimate shear were calculated theoretically, the first using Höglund theory, where the second using the second method of design stated in EC3. Comparison between the shear values obtained from both cases with different ranges of  $b/d$  are shown in Figs. from 4 to 7; and listed in Tables from III to VI.

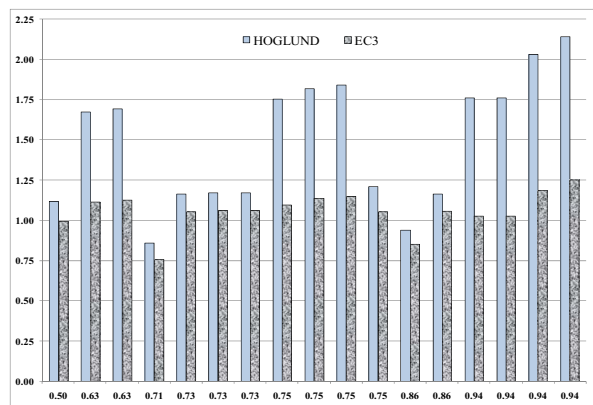


Fig. 4 Comparison between  $V_{exp}/V_S$  predicted using Höglund

Fig. 4 illustrated the relation between  $V_{exp}/V_S$  with panel aspect ratio  $b/d$ , varies from 0.5 to 0.94, these results related to 17 samples out of 96 samples. Table III also summarized the

mean, standard deviation and coefficient of variation for the results of the ratio between ultimate shear resistance obtained experimentally compared with the value of ultimate shear resistance obtained using Höglund theory or EC3 (2<sup>nd</sup> method). From Fig. 4 and Table III, it can be summarized that the results obtained for ultimate shear resistance of plate girder using EC3 (2<sup>nd</sup> method) more consistent than Höglund theory in this range of  $b/d$ .

Fig. 5 shows the relation between  $V_{exp} / V_S$  with panel aspect ratio  $b/d$ , varies from 0.98 to 1, these results related to 39 samples out of 96 samples. Table IV also summarized the mean, standard deviation and coefficient of variation for the results of the ratio between ultimate shear resistance obtained experimentally compared with the value of ultimate shear resistance obtained using Höglund theory or EC3 (2<sup>nd</sup> method). From Fig. 5 and Table IV, it can be summarized that the results obtained for ultimate shear resistance of plate girder using it can be summarized that the results obtained for ultimate shear resistance of plate girder using EC3 (2<sup>nd</sup> method) more consistent than Höglund theory in this range of  $b/d$ .

Fig. 6 shows the relation between  $V_{exp} / V_S$  with panel aspect ratio  $b/d$ , varies from 1.2 to 3, these results related to 26 samples out of 96 samples. Table V also summarized the mean, standard deviation and coefficient of variation for the results of the ratio between ultimate shear resistance obtained experimentally compared with the value of ultimate shear resistance obtained using Höglund theory or EC3 (2<sup>nd</sup> method). From Fig. 6 and Table V, it can be summarized that the results obtained for ultimate shear resistance of plate girder using it can be summarized that the results obtained for ultimate shear resistance of plate girder using EC3 (2<sup>nd</sup> method) more consistent than Höglund theory in this range of  $b/d$ .

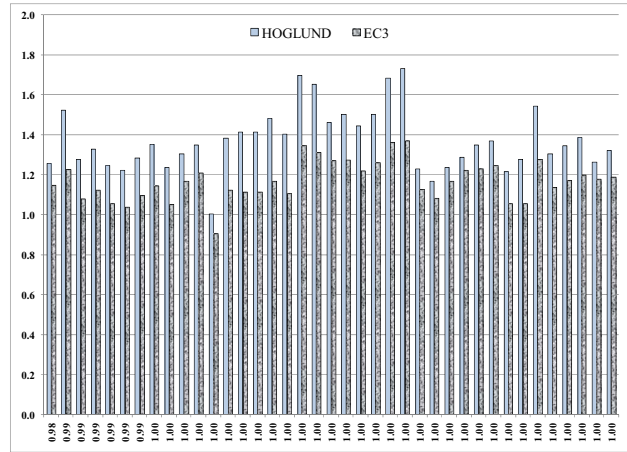


Fig. 5 Comparison between  $V_{exp}/V_S$  predicted using Höglund and EC3 for  $0.98 \leq b/d \leq 1$

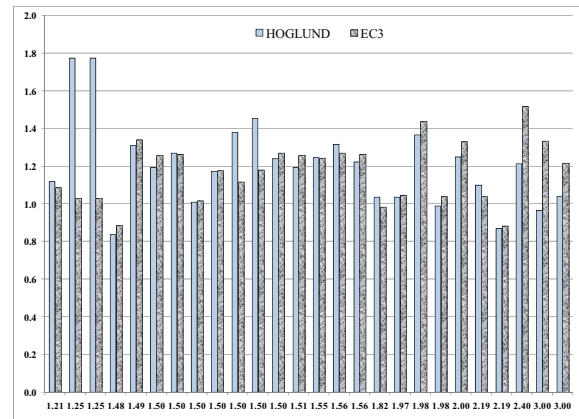


Fig. 6 Comparison between  $V_{exp}/V_S$  predicted using Höglund

TABLE III  
MEAN, STANDARD DEVIATION AND COEFFICIENT OF VARIATION FOR 17  
SAMPLES WITH  $0.5 \leq b/d < 1$

Höglund and EC3 Data		Sample Size (S.Size) = 17		
From $b/d \geq$	To $b/d <$	Mean	Standard deviation	Coefficient of variation
0.5	1			
	Höglund	<b>1.49</b>	<b>0.39</b>	<b>0.26</b>
	EC3	<b>1.06</b>	<b>0.11</b>	<b>0.11</b>

Fig. 7 shows the relation between  $V_{exp} / V_S$  with panel aspect ratio  $b/d$ , varies from 3 to 15, these results related to 14 samples out of 96 samples. Table VI also summarized the mean, standard deviation and coefficient of variation for the results of the ratio between ultimate shear resistance obtained experimentally compared with the value of ultimate shear resistance obtained using Höglund theory or EC3 (2<sup>nd</sup> method). From Fig. 7 and Table VI, it can be summarized that the results obtained for ultimate shear resistance of plate girder using it can be summarized that the results obtained for ultimate shear resistance of plate girder using Höglund theory more consistent than EC3 (2<sup>nd</sup> method) in this range of  $b/d$ .

TABLE IV  
MEAN, STANDARD DEVIATION AND COEFFICIENT OF VARIATION FOR 39  
SAMPLES WITH  $0.98 \leq b/d \leq 1$

Höglund and EC3 Data		Sample Size (S.Size) = 26		
From $b/d \geq$	To $b/d \leq$	Mean	Standard deviation	Coefficient of variation
1.2	3			
	Höglund	<b>1.21</b>	<b>0.22</b>	<b>0.18</b>
	EC3	<b>1.17</b>	<b>0.16</b>	<b>0.13</b>

TABLE V  
MEAN, STANDARD DEVIATION AND COEFFICIENT OF VARIATION FOR 26  
SAMPLES WITH  $1.2 \leq b/d \leq 1.6$

Höglund and EC3 Data		Sample Size (S.Size) = 39		
From $b/d \geq$	To $b/d \leq$	Mean	Standard deviation	Coefficient of variation
0.98	1			
	Höglund	<b>1.37</b>	<b>0.15</b>	<b>0.11</b>
	EC3	<b>1.17</b>	<b>0.10</b>	<b>0.08</b>

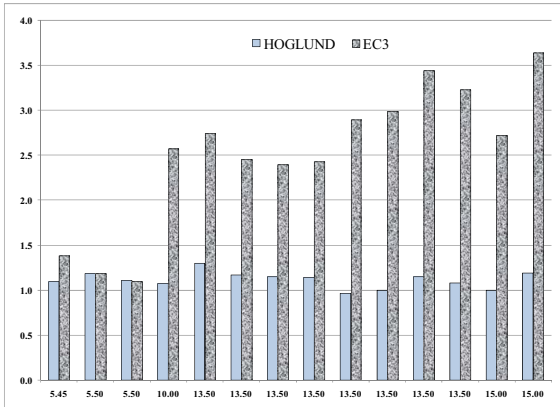


Fig. 7 Comparison between  $V_{exp}/V_S$  predicted using Höglund

TABLE VI  
MEAN, STANDARD DEVIATION AND COEFFICIENT OF VARIATION FOR 14  
SAMPLES WITH  $3 \leq b/D \leq 15$

Höglund and EC3 Data		Sample Size (S.Size) = 14		
From b/d	To b/d	Mean	Standard deviation	Coefficient of variation
More than 3 up to 15				
Höglund		1.11	0.09	0.08
EC3		2.52	0.77	0.30

VI. RELATION BETWEEN PANEL ASPECT RATIO (B/D) AND (C/B)

Table VII shows the relation between panel aspect ratio “(b/d)” and the corresponding value of the ratio (c/b); where “c” is the distance between the plastic hinges form in the flanges and “b” is the panel width.

TABLE VII  
RELATION BETWEEN B/D VALUE AND C/B

b/d	c/b Höglund	c/b EC3	Notes	b/d	c/b Höglund	c/b EC3	Notes
0.50	0.27	0.50		1.70	0.27	0.21	
0.60	0.27	0.39		1.80	0.27	0.21	
0.75	0.27	0.32		1.90	0.27	0.21	
1.00	0.27	0.26	Höglund value is Constant	2.00	0.27	0.21	
1.10	0.27	0.25		2.20	0.27	0.20	
1.20	0.27	0.24		2.50	0.27	0.20	
1.30	0.27	0.23		2.70	0.27	0.20	Constant in both Höglund and EC3
1.40	0.27	0.23		2.80	0.27	0.20	
1.50	0.27	0.22		2.90	0.27	0.20	
1.60	0.27	0.22		3.00	0.27	0.19	

The effect of the distance between the plastic hinges “c” on ultimate shear resistance value has been studied using the corresponding equations No. (17) and (29). This parametric study based on the data of G8-T2 girder which has percentage of experimental ultimate shear resistance to theoretical 1.01 and 1.02 as obtained using Höglund and EC3 (2<sup>nd</sup> design method) respectively. The only variable will be apply is the panel width b where the rest of the data will be constant (web thickness, flange thickness, web depth, .....). By comparing the results obtained from (b/d) equal 0.5 to (b/d) equal 3 in both cases of design methods with the relevant value of c/b; it can be summarized that; the convenient value of ultimate shear resistance can be obtained in case of (c/b) ranged between 0.2 and 0.50. Using these limits one chart plotted to represent the relationship between b/d and c/b in case of EC3 where the results obtained from Höglund theory were constant for all different value of b/d as shown in Fig. 8.

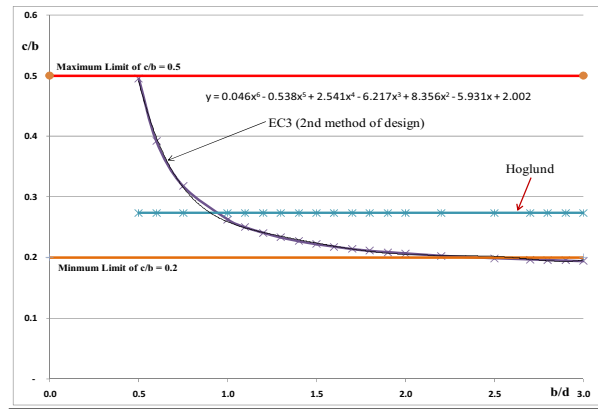


Fig. 8 Relation between c/b and panel aspect ratio b/d

From Fig. 8, a formula proposed to predict the value of c/b using the assumed panel aspect ratio b/d. This formula as follow:

In case of EC3 (2<sup>nd</sup> method of design - tension-field design method)

$$\frac{c}{b} = 0.046 * \left(\frac{b}{d}\right)^6 - 0.538 * \left(\frac{b}{d}\right)^5 + 2.514 * \left(\frac{b}{d}\right)^4 - 6.217 * \left(\frac{b}{d}\right)^3 + 8.356 * \left(\frac{b}{d}\right)^2 - 5.93 * \left(\frac{b}{d}\right) + 2$$



## VII. CONCLUSION

- Höglund theory can be used to predict the ultimate shear resistance of plate girder having intermediate transverse stiffeners in case of web panel aspect ratios  $b/d$  (width of web panel/depth of web panel) more than 3.
- In case of  $b/d$  varies from 0.5 to 3, the ultimate shear resistance of plate girder using EC3 (2<sup>nd</sup> method of design- *tension-field design method*) more consistent than Höglund theory.
- With panel aspect ratio  $b/d$ , more than 3, the ultimate shear resistance of plate girder using Höglund theory more consistent than EC3 (2<sup>nd</sup> method of design- *tension-field design method*).
- New formula based on EC3 (2<sup>nd</sup> method of design- *tension-field design method*) proposed to predict the value of  $c/b$  in the begging of design by using the assumed distance between transverse stiffeners and web depth.
- The consistent values of  $(c/b)$  ranged between 0.2 and 0.50 to predict a convenient value of ultimate shear resistance using EC3 (2<sup>nd</sup> method of design- *tension-field design method*).
- The panel aspect ratio ( $b/d$ ) on Höglund theory is not the main factor where the web slenderness and flange rigidity are the main factors which affect theory results.

## REFERENCES

- [1] A.W. Davies, D. S. Griffith "Shear Strength of steel plate girder" Prog. Instn Civ. Engrs Structs & Bldgs, 1999, 134 May, PP 147-157.
- [2] M. Sulyok, T.V. Galambos "Evaluation of web buckling test results on welded beams and plate girders subjected to shear" J Engineering Structures June 1996, pp. 459-464.
- [3] S.C. Lee, J.S. Davidson and C.H. Yoo, Shear buckling coefficients of plate girder web panels, Computers and Structures 59 (5) (1996), pp. 789-795.
- [4] S.C. Lee and C.H. Yoo, Strength of plate girder web panels under pure shear, Journal of Structural Engineering ASCE 124 (2) (1998), pp. 184-194.
- [5] F. Shahabian and T.M. Roberts "Combined Shear and Patch Loading of plate girders" Journal of Structural engineering, March 2000.
- [6] M.A. Bradford, Improved shear strength of webs designed in accordance with the LRFD specification, Engineering Journal 33 (3) (1996), pp. 95-100.
- [7] Torsten Höglund, "Shear Buckling Resistance of Steel and Aluminium Plate Girders", Journal of Thin Walled Structures Vol. 29 1997 pp. 13-30.
- [8] ENV 1993-1-3, Eurocode 3: Design of steel structures: Part 1.1. General rules and rules for buildings, 1992, and Amendment A2 of Eurocode 3: Annex N 'Openings in webs'. British Standards Institution, 1998.
- [9] S.P. Timoshenko and J.M. Gere (Int. Student edn, 2nd ed.), Theory of elastic stability vol. 541, McGraw-Hill (1985).