

Two-Dimensional Symmetric Half-Plane Recursive Doubly Complementary Digital Lattice Filters

Ju-Hong Lee, Chong-Jia Ciou, Yuan-Hau Yang

Abstract—This paper deals with the problem of two-dimensional (2-D) recursive doubly complementary (DC) digital filter design. We present a structure of 2-D recursive DC filters by using 2-D symmetric half-plane (SHP) recursive digital all-pass lattice filters (DALFs). The novelty of using 2-D SHP recursive DALFs to construct a 2-D recursive DC digital lattice filter is that the resulting 2-D SHP recursive DC digital lattice filter provides better performance than the existing 2-D SHP recursive DC digital filter. Moreover, the proposed structure possesses a favorable 2-D DC half-band (DC-HB) property that allows about half of the 2-D SHP recursive DALF's coefficients to be zero. This leads to considerable savings in computational burden for implementation. To ensure the stability of a designed 2-D SHP recursive DC digital lattice filter, some necessary constraints on the phase of the 2-D SHP recursive DALF during the design process are presented. Design of a 2-D diamond-shape decimation/interpolation filter is presented for illustration and comparison.

Keywords—All-pass digital filter, doubly complementary, lattice structure, symmetric half-plane digital filter, sampling rate conversion.

I. INTRODUCTION

TWO digital filters become a complementary pair if the passbands of one matches the stopbands of the other and vice versa. Therefore, complementary digital filters are widely used in many signal processing systems where different frequency bands are to be processed separately to measure signal strengths at each band or to achieve for example, data compression or noise reduction. Furthermore, a complementary filter pair is used to split the input signal in two adjacent bands, and also is of importance for constructing complex multirate systems and filter banks. A pair of one-dimensional (1-D) complementary digital filters $H(z)$ and $G(z)$ can be designed to exhibit (1) all-pass complementary property: $H(z) + G(z) = A(z)$, where $A(z)$ is an all-pass function; and (2) power complementary property: $H(z)H(z^{-1}) + G(z)G(z^{-1}) = 1$. This pair of $H(z)$ and $G(z)$ is called doubly complementary (DC) filters. Various techniques have been presented for the design of 1-D DC filters based on a parallel structure of two 1-D all-pass building blocks [1]-[4].

Many research achievements associated with the design and implementation of 1-D digital lattice filters have been reported in the literature [5]-[7]. 1-D digital lattice filter structure

exhibits the attractive advantages of low passband sensitivity and robustness to quantization error. Additionally, 1-D digital lattice filter structure requires lower computational cost than 1-D direct-form digital filter with similar design specifications. The above advantages of 1-D digital lattice filter structure over conventional direct-form 1-D digital filter structure are also possessed by 2-D digital lattice filters. The minimal realization of a 2-D system [8] has been widely concerned because it results in the least hardware requirement and less computational complexity. The minimal delay realization for a 2-D digital lattice structure has been presented in [9]. The cascaded 2-D digital lattice filter has the minimal number of delay elements $2n$ when n basic lattice sections are employed. The resulting 2-D transfer function exhibits all-pass characteristics with quarter-plane (QP) support region.

Due to the advent of digital video systems and the rapidly increasing use of digital signal processors, it is worth exploring the properties of two-dimensional (2-D) DC digital filters and their design methods. In this paper, we present a 2-D SHP recursive DC digital filter composed of a pure delay section and a 2-D SHP recursive DAF with a lattice structure. The proposed 2-D SHP recursive DC digital filter possesses the favorable 2-D DC half-band (DC-HB) characteristics allowing about half of the 2-D SHP recursive DALF's coefficients to be zero. This leads to considerable savings in computational burden. During the design process, we impose some necessary constraints on the phase of the 2-D SHP recursive DAF to ensure the stability of a designed 2-D SHP recursive DC digital lattice filter. A design example of a diamond-shape decimation/interpolation filter is provided to show the effectiveness of the proposed 2-D SHP recursive DC digital lattice filter.

This paper is organized as follows. Section II presents the theory of 2-D SHP recursive digital all-pass lattice filters (DALFs). In Section III, we propose a 2-D SHP recursive doubly complementary digital filters based on the 2-D SHP recursive DALFs. We also describe the 2-D DC-HB characteristics possessed by the proposed 2-D SHP recursive DC digital lattice filter pair and formulate the least-squares design problem in Section III. Section IV presents an iterative technique for designing the 2-D SHP recursive DC digital lattice filters. The phase constraints for ensuring the stability of the design result are also presented. An example of designing a 2-D diamond-shaped filter is provided in Section V for comparison. Finally, we conclude the paper in Section VI.

II. 2-D SHP RECURSIVE DIGITAL ALL-PASS FILTERS

A. Conventional Direct-Form 2-D SHP Recursive DAFs

Consider a 2-D recursive direct-form DAF with order $M \times N$

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with its transfer function given by [10]:

$$A(z_1, z_2) = z_1^{-M} z_2^{-N} \frac{D(z_1^{-1}, z_2^{-1})}{D(z_1, z_2)}. \quad (1)$$

The all-pass filter $A(z_1, z_2)$ is completely characterized by the denominator polynomial $D(z_1, z_2)$. Let the phase response of $A(z_1, z_2)$ and that of $D(z_1, z_2)$ be $\theta(\omega_1, \omega_2)$ and $\phi(\omega_1, \omega_2)$, respectively. We can obtain from (1) that:

$$\phi(\omega_1, \omega_2) = -\frac{[M\omega_1 + N\omega_2 + \theta(\omega_1, \omega_2)]}{2} \quad (2)$$

The denominator polynomial $D(z_1, z_2)$ has the symmetric half-plane (SHP) support region for its coefficients and is given by:

$$D(z_1, z_2) = d(0,0) + \sum_{m=-M}^M \sum_{n=1}^N d(m,n) z_1^{-m} z_2^{-n} \quad (3)$$

B. 2-D SHP Recursive DAFs with a Lattice Structure

Let the coefficients $d(-m,n) = d(m,n)$, we rewrite (3) as:

$$\begin{aligned} D(z_1, z_2) &= d(0,0) + \sum_{n=1}^N \left[d(0,n) + \sum_{m=1}^M d(m,n) (z_1^{-m} + z_1^m) \right] z_2^{-n} \\ &= \sum_{n=0}^N c(n, z_1) z_2^{-n} = D(z_1^{-1}, z_2), \end{aligned} \quad (4)$$

where $c(n, z_2)$ is given by:

$$c(n, z_1) = d(0,0) + \sum_{m=1}^M d(m,n) (z_1^{-m} + z_1^m) \quad (5)$$

for $n = 1, 2, \dots, N$. Based on (5), we present a lattice structure as shown in Fig. 1 for realizing a 2-D SHP recursive DAF. The input/output relationship of the p th lattice section in Fig. 1 is expressed by:

$$\begin{bmatrix} Q_p(z_1, z_2) \\ R_p(z_1, z_2) \end{bmatrix} = \begin{bmatrix} 1 & k_p(z_1) z_2^{-1} \\ k_p(z_1) & z_2^{-1} \end{bmatrix} \begin{bmatrix} Q_{p-1}(z_1, z_2) \\ R_{p-1}(z_1, z_2) \end{bmatrix}, \quad (6)$$

for $p = 1, 2, \dots, N$, where

$$k_p(z_1) = r_p(0) + \sum_{m=1}^M r_p(m) (z_1^{-m} + z_1^m), \quad (7)$$

for $p = 1, 2, \dots, N$. By setting $Q_0(z_1, z_2) = R_0(z_1, z_2) = U(z_1, z_2)$ and using the forward recursion given by (6), we can derive

$$\frac{Q_N(z_1, z_2)}{U(z_1, z_2)} = L_N(z_1, z_2) \quad (8)$$

and

$$\frac{R_N(z_1, z_2)}{U(z_1, z_2)} = z_2^{-N} L_N(z_1^{-1}, z_2^{-1}) \quad (9)$$

As a result, the overall transfer function of Fig. 1 is given by:

$$\begin{aligned} \frac{Y(z_1, z_2)}{X(z_1, z_2)} &= z_1^{-M} \frac{R_N(z_1, z_2)}{U(z_1, z_2)} \times \frac{U(z_1, z_2)}{Q_N(z_1, z_2)} \\ &= z_1^{-M} \frac{z_2^{-N} L_N(z_1^{-1}, z_2^{-1})}{L_N(z_1, z_2)} = A(z_1, z_2) \end{aligned} \quad (10)$$

We note from (10) that Fig. 1 generates a lattice-form 2-D SHP recursive DAF $A(z_1, z_2)$ with order $M \times N$. As the 1-D lattice filters, if $|k_p(z_1)| < 1$, for $p = 1, 2, \dots, N$, then $L_N(z_1, z_2)$ will be a minimum-phase polynomial [11].

C. Transformation from Direct Form to Lattice Form

Suppose that the direct-form 2-D SHP recursive DAF (1) is designed by utilizing the existing techniques, it can be transformed into the proposed lattice form. The reflection coefficient functions $k_p(z_1)$ in Fig. 1 can be calculated by inverting the recursion (6) as follows:

$$\begin{bmatrix} Q_{p-1}(z_1, z_2) \\ R_{p-1}(z_1, z_2) \end{bmatrix} = \frac{1}{1 - k_p^2(z_1)} \begin{bmatrix} 1 & -k_p(z_1) \\ -k_p(z_1) z_2 & z_2 \end{bmatrix} \begin{bmatrix} Q_p(z_1, z_2) \\ R_p(z_1, z_2) \end{bmatrix}. \quad (11)$$

Initially, for $p = N$, we set:

$$Q_p(z_1, z_2) = D_N(z_1, z_2) \quad (12)$$

and

$$R_p(z_1, z_2) = z_2^{-N} D_N(z_1, z_2^{-1}). \quad (13)$$

The reflection coefficient function $k_N(z_1)$ is chosen so that it equals the 1-D coefficient function corresponding to the z_2^{-N} term in $D_N(z_1, z_2)$, i.e.

$$k_N(z_1) = c(N, z_1) = d(0, N) + \sum_{m=1}^M d(m, N) (z_1^{-m} + z_1^m) \quad (14)$$

For $p = N-1, N-2, \dots, 1$, we can calculate the reflection coefficient function $k_p(z_1)$ that is the 1-D coefficient function corresponding to the z_2^{-p} term in $Q_p(z_1, z_2)$. Then, $Q_{p-1}(z_1, z_2)$ and $R_{p-1}(z_1, z_2)$ can be recursively derived by calculating (11).

III. 2-D SHP RECURSIVE DOUBLY COMPLEMENTARY FILTERS

A. 2-D SHP Recursive DC Digital Lattice Filter Pairs

Consider the following two 2-D digital filters with frequency responses given by:

$$G(e^{j\omega_1}, e^{j\omega_2}) = \frac{e^{-jM\omega_1} e^{-jN\omega_2} + A(e^{j\omega_1}, e^{j\omega_2})}{2} \quad (15)$$

and

$$H(e^{j\omega_1}, e^{j\omega_2}) = \frac{e^{-jM\omega_1} e^{-jN\omega_2} - A(e^{j\omega_1}, e^{j\omega_2})}{2} \quad (16)$$

where $A(e^{j\omega_1}, e^{j\omega_2})$ is a 2-D SHP recursive DALF having a transfer function given by (10) with order $M_2 \times N_2$. Substituting (2) into (15) and (16) yields:

$$G(e^{j\omega_1}, e^{j\omega_2}) = \frac{e^{-jM_1\omega_1} e^{-jN_1\omega_2} + e^{j(-M_2\omega_1 - N_2\omega_2 - 2\phi(\omega_1, \omega_2))}}{2} \quad (17)$$

$$= \cos\{\theta_m(\omega_1, \omega_2)\} \exp\{j\theta_p(\omega_1, \omega_2)\}$$

and

$$H(e^{j\omega_1}, e^{j\omega_2}) = \frac{e^{-jM_1\omega_1} e^{-jN_1\omega_2} - e^{j(-M_2\omega_1 - N_2\omega_2 - 2\phi(\omega_1, \omega_2))}}{2} \quad (18)$$

$$= \sin\{\theta_m(\omega_1, \omega_2)\} \exp\{j(\theta_p(\omega_1, \omega_2) + \frac{\pi}{2})\}$$

where

$$\theta_m(\omega_1, \omega_2) = \frac{-(M_1 - M_2)\omega_1 + (N_1 - N_2)\omega_2}{2} + \phi(\omega_1, \omega_2) \quad (19)$$

and

$$\theta_p(\omega_1, \omega_2) = \frac{[(M_1 + M_2)\omega_1 + (N_1 + N_2)\omega_2]}{2} - \phi(\omega_1, \omega_2) \quad (20)$$

Based on (17) and (18), two properties of the proposed 2-D SHP recursive digital lattice filters can be easily obtained as:

I. The **all-pass-complementary** property:

$$|G(e^{j\omega_1}, e^{j\omega_2}) + H(e^{j\omega_1}, e^{j\omega_2})| = 1, \text{ for } \forall (\omega_1, \omega_2). \quad (21)$$

II. The **power-complementary** property:

$$|G(e^{j\omega_1}, e^{j\omega_2})|^2 + |H(e^{j\omega_1}, e^{j\omega_2})|^2 = 1, \text{ for } \forall (\omega_1, \omega_2). \quad (22)$$

Therefore, the construction for $G(z_1, z_2)$ and $H(z_1, z_2)$ shown in Fig. 2 forms a 2-D SHP recursive DC digital lattice filter pair. Moreover, we have from (19) and (20) that the phase response $\phi(\omega_1, \omega_2)$ of $L_{N_2}(z_1, z_2)$ is given by:

$$\phi(\omega_1, \omega_2) = \frac{-(M_2\omega_1 + N_2\omega_2) - (\theta_p(\omega_1, \omega_2) - \theta_m(\omega_1, \omega_2))}{2} \quad (23)$$

We observe from (23) that the problem for designing the 2-D SHP recursive DC digital lattice filters $G(z_1, z_2)$ and $H(z_1, z_2)$ can be formulated as a problem of finding the phase $\phi(\omega_1, \omega_2)$ which approximates to some desired phase response $\phi_d(\omega_1, \omega_2)$ of $L_{N_2}(z_1, z_2)$ in some optimal sense.

As a result, the design problem in least-squares sense can be formulated as:

$$\text{Minimize } \|W(\omega_1, \omega_2)(\arg\{L_{N_2}(e^{j\omega_1}, e^{j\omega_2})\} - \phi_d(\omega_1, \omega_2))\|^2 \quad (24)$$

where $\|x\|^2$ is the squared norm of x , $W(\omega_1, \omega_2)$ is the preset frequency weighting function, and $\arg\{L_{N_2}(e^{j\omega_1}, e^{j\omega_2})\}$ is the

phase response of $L_{N_2}(z_1, z_2)$.

B. The Half-Band Property of 2-D SHP DC Lattice Filters

The 2-D DC half-band (DC-HB) property obtained from the proposed 2-D SHP recursive DC digital lattice filters is explored. From (15), we have:

$$G(e^{j(\omega_1 - \pi)}, e^{j(\omega_2 - \pi)}) = \frac{1}{2} [e^{-jM_1(\omega_1 - \pi)} e^{-jN_1(\omega_2 - \pi)} + A(e^{j(\omega_1 - \pi)}, e^{j(\omega_2 - \pi)})]$$

$$= \frac{1}{2} [e^{-jM_1\omega_1} e^{-jN_1\omega_2} + A(-e^{j\omega_1}, -e^{j\omega_2})] \quad (25)$$

Based on (16) and (25), we further obtain:

$$G(e^{j(\omega_1 - \pi)}, e^{j(\omega_2 - \pi)}) = \begin{cases} H(e^{j\omega_1}, e^{j\omega_2}), & \text{for } M_1 + N_1: \text{even and } M_2 + N_2: \text{odd} \\ -H(e^{j\omega_1}, e^{j\omega_2}), & \text{for } M_1 + N_1: \text{odd and } M_2 + N_2: \text{even} \end{cases} \quad (26)$$

with the parameters $r_p(m) = 0$ in (7), for $m + p = \text{an odd number}$. Therefore, $G(e^{j\omega_1}, e^{j\omega_2})$ and its shifted version $G(e^{j(\omega_1 - \pi)}, e^{j(\omega_2 - \pi)})$ possess the DC properties, i.e.

$$|G(e^{j\omega_1}, e^{j\omega_2}) + G(e^{j(\omega_1 - \pi)}, e^{j(\omega_2 - \pi)})| = 1 \quad (27)$$

and

$$|G(e^{j\omega_1}, e^{j\omega_2})|^2 + |G(e^{j(\omega_1 - \pi)}, e^{j(\omega_2 - \pi)})|^2 = 1 \quad (28)$$

These properties indicate that the frequency response of $G(z_1, z_2)$ possesses the DC symmetry with respect to the half-band frequency $(\omega_1, \omega_2) = (\pi/2, \pi/2)$ in the first quarter of the frequency plane, i.e., the DC-HB property. The 2-D DC-HB characteristics in the first quarter plane obtained from the proposed 2-D SHP recursive DC digital lattice filters are depicted in Fig. 3. The passband Ω_p and stopband Ω_s of are symmetric with respect to the half-band frequency. If we set $\omega_{1p} + \omega_{1s} = \omega_{2p} + \omega_{2s} = \pi$, the passband of the shifted version of $G(e^{j\omega_1}, e^{j\omega_2})$, i.e., $G(e^{j(\omega_1 - \pi)}, e^{j(\omega_2 - \pi)})$, will cover the passband of $H(e^{j\omega_1}, e^{j\omega_2})$. This means that if $|G(e^{j\omega_1}, e^{j\omega_2})| = 0$ in the stopband, then the magnitude response of $G(z_1, z_2)$ becomes 1 in the passband. Hence, we only need to approximate the passband or stopband response during the design of $G(z_1, z_2)$. Moreover, the DC-HB property reveals that about half of the parameters $r_p(m) = 0$ in (7), for $m + p = \text{an odd number}$. Both advantages lead to significant savings in computational burden during the design process.

IV. THE DESIGN TECHNIQUE

In this section, we present a design technique for solving the minimization problem (24). This is through a frequency sampling and iterative approximation scheme to find the optimal reflection coefficient functions $k_p(z_1)$ for $p = 1, 2, \dots, N_2$, for the proposed 2-D recursive SHP DALF.

A. Stability of the 2-D Recursive SHP Lattice Filter

The stability issue is crucial for the proposed 2-D SHP recursive digital lattice filter. The stability of an $(M \times N)$ th order 2-D recursive DAF $A(e^{j\omega_1}, e^{j\omega_2})$ is guaranteed when the phase response $\arg\{A(e^{j\omega_1}, e^{j\omega_2})\}$ satisfies the following conditions [12]: (I) $\arg\{A(e^{j\omega_1}, e^{j\omega_2})\}$ is monotonically decreasing along ω_1 axis and $\arg\{A(e^{j\pi}, e^{j\omega_2})\} = \arg\{A(e^{j0}, e^{j\omega_2})\} - M\pi$ for $-\pi \leq \omega_2 \leq \pi$; (II) $\arg\{A(e^{j\omega_1}, e^{j\omega_2})\}$ is monotonically decreasing along ω_2 axis and $\arg\{A(e^{j\omega_1}, e^{j\pi})\} = \arg\{A(e^{j\omega_1}, e^{j0})\} - N\pi$ for $-\pi \leq \omega_1 \leq \pi$. Hence, we only need to focus on the minimization problem (24) and the stability of the designed 2-D SHP recursive digital lattice filters will be simultaneously guaranteed if the desired phase response $\phi_d(\omega_1, \omega_2)$ satisfies the above two constraints.

B. The Iterative Design Procedure

As shown in (24), the design problem for finding the optimal parameters $r_p(m)$, for $m = 0, 1, 2, \dots, M_2$ and $p = 1, 2, \dots, N_2$ for the reflection coefficient functions $k_p(z_1)$ is a highly nonlinear optimization problem. However, it is appropriate to employ the trust-region method [13], [14] to iteratively solve this problem. In the following, we summarize the iterative design procedure step by step.

- Step 1:** Specify the ideal phase response $\phi_d(\omega_1, \omega_2)$ which satisfies the stability criterion as presented in Section IV. A for ensuring the stability of the designed 2-D SHP recursive digital lattice filter.
- Step 2:** At the initial iteration, we set the iteration number $l = 0$ and the parameters ${}^{(0)}r_p(m) = 0$, for $m = 0, 1, 2, \dots, M_2$ and $p = 1, 2, \dots, N_2$.
- Step 3:** At the k th iteration, we compute the reflection coefficient function $k_p(z_1)$ for $p = 1, 2, \dots, N_2$, from (7) and its corresponding lattice-form polynomial $L_{N_2}(z_1, z_2)$ by calculating the forward recursion as presented in Section II. B.
- Step 4:** Compute the difference between $\arg\{L_{N_2}(e^{j\omega_1}, e^{j\omega_2})\}$ and $\phi_d(\omega_1, \omega_2)$ over a finite set of discrete frequencies $(\omega_{1i}, \omega_{2j})$, where $(\omega_{1i}, \omega_{2j})$ denotes the (i, j) th discrete frequency point taken on the 2-D (ω_1, ω_2) plane.
- Step 5:** Utilize the trust-region optimization method of [13], [14] to compute the adjustment in ${}^{(l)}r_p(m) = 0$, for $m = 0, 1, 2, \dots, M_2$ and $p = 1, 2, \dots, N_2$.
- Step 6:** Repeat Steps 3 – 5 and increase the iteration number by one per iteration until a satisfactory design result is achieved.

V. COMPUTER SIMULATION RESULTS

A. Design Example for Sampling Structure Conversion

The conversion between different periodic sampling structures is important for applications related to

multidimensional signal processing. It is known that there are numerous choices for periodicity matrix $\mathbf{P}$ and the sampling matrix $\mathbf{S}$ [11], [15]. Here, we consider two widely used sampling structures—rectangular sampling structure and hexagonal sampling structure as follows: For a rectangular sampling, the sampling matrix $\mathbf{S}_R$ and periodicity matrix $\mathbf{P}_R$ are given by:

$$\mathbf{S}_R = \begin{bmatrix} T_1 & 0 \\ 0 & T_2 \end{bmatrix}$$

and

$$\mathbf{P}_R = (\mathbf{S}_R^{-1})^T = \begin{bmatrix} \frac{1}{T_1} & 0 \\ 0 & \frac{1}{T_2} \end{bmatrix}, \quad (29)$$

where T_1 and T_2 represent the horizontal and vertical sampling periods for a spatial sampling pattern, whereas for spatiotemporal conversions, T_1 represents the vertical period and T_2 represents the frame period, respectively. In the case of (L, K) hexagonal sampling matrix, these matrices are defined by:

$$\mathbf{S}_{L,K} = \begin{bmatrix} LT_1 & LT_1 \\ KT_2 & -KT_2 \end{bmatrix}$$

and

$$\mathbf{P}_{L,K} = (\mathbf{S}_{L,K}^{-1})^T = \begin{bmatrix} \frac{1}{2LT_1} & \frac{1}{2LT_1} \\ \frac{1}{2KT_2} & -\frac{1}{2KT_2} \end{bmatrix}, \quad (30)$$

where L and K are strictly positive integer parameters of the hexagonal sampling structure. It is shown that the diamond-shaped decimation/interpolation filters are good candidates for conversion processing between rectangular and hexagonal sampling structures because they allow a maximum resolution in the horizontal and vertical directions. If T_1 and T_2 are normalized to 1, the ideal diamond-shaped filter, which possesses a quadrantal symmetry, is characterized in the first quarter plane with $0 \leq \omega_1 \leq \pi$ and $0 \leq \omega_2 \leq \pi$ by:

$$I(\omega_1, \omega_2) = \begin{cases} Ge^{-j(g_{\omega_1}\omega_1 + g_{\omega_2}\omega_2)}, & \text{for } \frac{|\omega_1|}{\omega_{1p}} + \frac{|\omega_2|}{\omega_{2p}} \leq 1 \\ 0, & \text{otherwise} \end{cases} \quad (31)$$

where $\omega_{1p} = \pi / L$ and $\omega_{2p} = \pi / K$. The filter gain G is equal to 1 for a decimation filter, whereas it is equal to $2LK$ for interpolation processing. Here, we present the design of 2-D diamond-shaped decimation/interpolation filters, which are widely used for sampling structure conversion, using the proposed 2-D SHP recursive DC digital lattice filter.

B. The Design Specifications

This example is similar to that considered by [16], [17]. We use the following specifications for the design example: The $(L, K) = (1, 1)$ 2-D diamond-shaped filter which is widely used

for sampling rate conversion by a factor of two, with the passband edge frequencies $\omega_{1p} = \pi$, $\omega_{2p} = 0.8\pi$ and the stopband edge frequencies $\omega_{1s} = 0.25\pi$, $\omega_{2s} = 0.4\pi$, i.e., the desired magnitude response of the filter is given by:

$$|G_d(\omega_1, \omega_2)| = \begin{cases} 1 & \text{for } \frac{|\omega_1|}{\pi} + \frac{|\omega_2|}{0.8\pi} \leq 1 \\ 0 & \text{for } \frac{|\omega_1|}{1.5\pi} + \frac{|\omega_2|}{1.2\pi} \geq 1 \end{cases} \quad (32)$$

Clearly, the magnitude characteristics do not possess the 2-D HB property. Only uniformly sampled passband frequency grid points are taken during the design process. Based on the proposed 2-D SHP recursive DC digital lattice filter, the 2-D diamond-shaped filter is designed by setting $M_1 = M_2 = 4$ and $N_1 + 1 = N_2 = 7$. Thus, the number of independent parameters is $(M_2 + 1)N_2 = 35$ which is the same as that of [17]. According to (19) and (20), we set the desired responses for $\theta_m(\omega_1, \omega_2)$ and $\theta_p(\omega_1, \omega_2)$ as follows:

$$\theta_{m,d}(\omega_1, \omega_2) = \begin{cases} 0, & \text{for } (\omega_1, \omega_2) \in \Omega_p \\ 0, & \text{for } (\omega_1, \omega_2) \in \Omega_s \end{cases}$$

and

$$\theta_{p,d}(\omega_1, \omega_2) = \begin{cases} -M_1\omega_1 - N_1\omega_2, & \text{for } (\omega_1, \omega_2) \in \Omega_p \\ -M_1\omega_1 - N_1\omega_2 - \pi, & \text{for } (\omega_1, \omega_2) \in \Omega_s \end{cases} \quad (33)$$

As a result, the desired phase response $\phi_d(\omega_1, \omega_2)$ of $L_{N_2}(z_1, z_2)$ is given by:

$$\phi_d(\omega_1, \omega_2) = \begin{cases} \frac{M_1 - M_2}{2}\omega_1 + \frac{N_1 - N_2}{2}\omega_2, & \text{for } (\omega_1, \omega_2) \in \Omega_p \\ \frac{M_1 - M_2}{2}\omega_1 + \frac{N_1 - N_2}{2}\omega_2 + \frac{\pi}{2}, & \text{for } (\omega_1, \omega_2) \in \Omega_s \end{cases} \quad (34)$$

Accordingly, (34) becomes:

$$\phi_d(\omega_1, \omega_2) = \begin{cases} -\frac{1}{2}\omega_2, & \text{for } (\omega_1, \omega_2) \in \Omega_p \\ -\frac{1}{2}\omega_2 + \frac{\pi}{2}, & \text{for } (\omega_1, \omega_2) \in \Omega_s \end{cases} \quad (35)$$

for this design case of $M_1 = M_2 = 4$ and $N_1 + 1 = N_2 = 7$. We note from (35) that $\phi_d(\omega_1, \omega_2)$ satisfies the phase stability conditions described in Section IV.A. We would expect that the stability of the designed 2-D SHP recursive DC digital lattice filter is ensured. The 2-D fast Fourier transform used during this design is 70×70 . The frequency weighting function $W(\omega_1, \omega_2)$ is set to 1 for the entire frequency plane. Table I lists the comparison of the significant design results in terms of the following performance parameters:

Passband Magnitude Mean-Squared Errors (PMSE)

$$\text{PMSE} = \frac{\sum_{(\omega_1, \omega_2) \in \Omega_p} \left(|G(e^{j\omega_1}, e^{j\omega_2})| - |G_d(\omega_1, \omega_2)| \right)^2}{\text{number of grid points in the passband}}$$

Stopband Magnitude Mean-Squared Errors (SMSE)

$$\text{SMSE} = \frac{\sum_{(\omega_1, \omega_2) \in \Omega_s} \left(|G(e^{j\omega_1}, e^{j\omega_2})| - |G_d(\omega_1, \omega_2)| \right)^2}{\text{number of grid points in the stopband}}$$

Peak Stopband Attenuation (PSA)

$$\text{PSA} = - \max_{(\omega_1, \omega_2) \in \Omega_s} 20 \log_{10} \left(|G(e^{j\omega_1}, e^{j\omega_2})| \right) \text{ (dB)}$$

Passband Phase Mean-Squared Error (PPMSE)

$$\text{PPMSE} = \frac{\sum_{(\omega_1, \omega_2) \in \Omega_p} \left[\arg \{ L_{N_2}(e^{j\omega_1}, e^{j\omega_2}) \} - \phi_d(\omega_1, \omega_2) \right]^2}{\text{number of grid points in the passband}} \text{ (radian}^2\text{)}$$

Peak Passband Phase Error (PPPE)

$$\text{PPPE} = \max_{(\omega_1, \omega_2) \in \Omega_p} \left| \arg \{ L_{N_2}(e^{j\omega_1}, e^{j\omega_2}) \} - \phi_d(\omega_1, \omega_2) \right| \text{ (radian)}$$

For the design of using the proposed 2-D SHP recursive DC digital lattice filter, Fig. 4 shows the magnitude response of the designed $G(e^{j\omega_1}, e^{j\omega_2})$. Fig. 5 depicts the phase response of the designed denominator $L_{N_2}(z_1, z_2)$, whereas Fig. 6 plots the absolute phase error $|\arg \{ L_{N_2}(e^{j\omega_1}, e^{j\omega_2}) \} - \phi_d(\omega_1, \omega_2)|$. From the simulation results, we observe that the design method using the proposed 2-D SHP recursive DC digital lattice filter can provide better results than the existing conventional direct-form design [17].

VI. CONCLUSION

This paper has presented a lattice structure of 2-D recursive doubly complementary (DC) digital filters using 2-D symmetric half-plane (SHP) recursive digital all-pass filters (DAFs). The proposed 2-D SHP recursive DC digital lattice filter possesses a favorable 2-D DC half-band (DC-HB) property that allows about half of the 2-D SHP recursive DALF's coefficients to be zero. The computer simulation results of a 2-D diamond-shape decimation/interpolation filter design show the effectiveness of the proposed 2-D SHP recursive DC digital lattice filters.

TABLE I
SIGNIFICANT DESIGN RESULTS FOR THE DESIGN EXAMPLE.

	Conventional Direct-Form Design [17]	The Proposed Lattice-Form Design
PMSE	5.1715×10^{-10}	8.1617×10^{-11}
SMSE	1.3422×10^{-6}	8.5503×10^{-7}
PSA	34.9551	39.8172
PPMSE	4.2567×10^{-6}	3.1589×10^{-6}
PPPE	4.9311×10^{-2}	3.4069×10^{-2}
Number of iterations	0	8

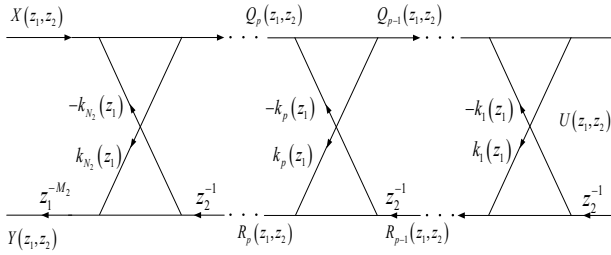


Fig. 1 The proposed 2-D SHP recursive DALF

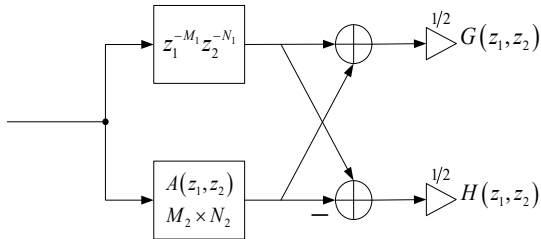


Fig. 2 Structure of the proposed 2-D doubly complementary digital lattice filter pair

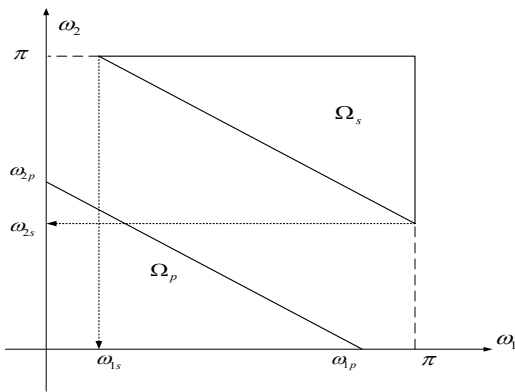
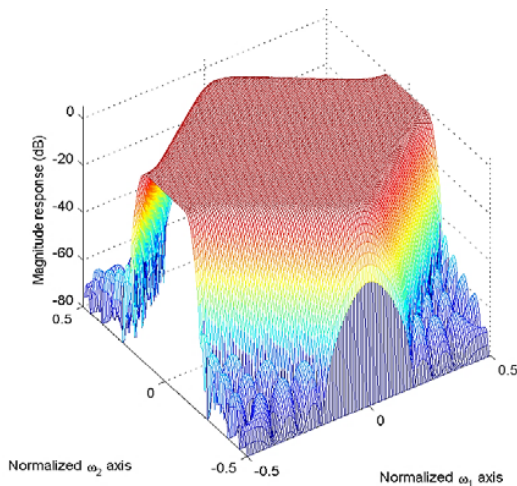
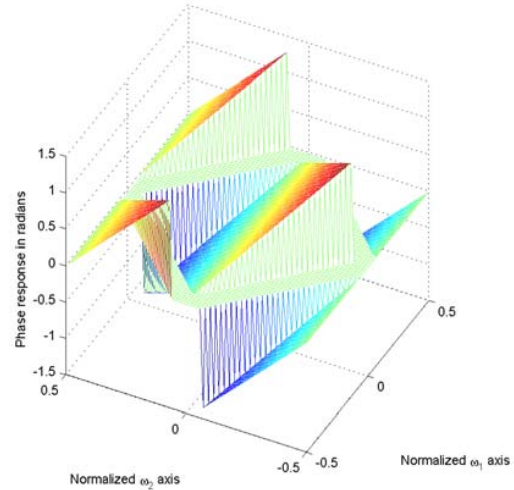
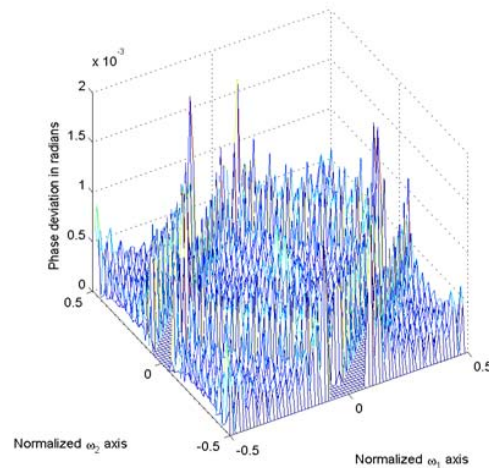


Fig. 3 The passband and stopband regions in the first quarter plane


 Fig. 4 The magnitude response of the designed 2-D SHP recursive DC digital lattice filter $G(z_1, z_2)$

 Fig. 5 The phase response of the designed denominator $L_{N_2}(z_1, z_2)$

 Fig. 6 The absolute phase error $|\arg\{L_{N_2}(e^{j\omega_1}, e^{j\omega_2})\} - \phi_d(\omega_1, \omega_2)|$

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