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Transient Heat Transfer of a Spiral Fin

Sen-Yung Lee, Li-Kuo Chou, Chao-Kuang Chen

Abstract**—**In this study, the problem of temperature transient response of a spiral fin, with its end insulated, is analyzed with base end subjected to a variation of fluid temperature. The hybrid method of Laplace transforms/Adomian decomposed method-Padé, is applied to the temperature transient response of the fin, the result of the temperature distribution and the heat flux at the base of the spiral fin are obtained, show a good agreement in the physical phenomenon.

Keywords**—**Laplace transforms/Adomian decomposed method-Padé, transient response, heat transfer.

I. Introduction

THE spiral fins can be widely used in typical industries, such as generators, power plants, mold injection machines, turbines, and drills. The problem of heat dissipation in the fins has been of interest for many engineers and researchers.

The transient response of two-dimensional straight fins and circular fins, one-dimensional annular fins and the composite straight fins were presented by Chu et al. [1], [2] using the Fourier series inverse technique. Mokheimer [3] studied the performance of annual fins with differential profiles subjected to variable heat transfer coefficients. Wang et al. [4] investigated transient response of a spiral fin with its base subjected to the variating heat flux. The temperature distribution and the heat flux at the base of the fin are were obtained. Malekzadeh et al. [5] studied the two-dimensional nonlinear transient heat transfer of variable section pin fins by using the incremental differential quadrature method (IDQM). The results agree well with those in the existing literatures. Recursive formulations on thermal analysis of an annular fin with variable thermal properties were presented by Yuan Lai et al. [6].

Adomian decomposition method (ADM) has been developed to solve many nonlinear and random vibration problems [7]. Wazwaz proposed a combined ADM and Padé approximation technique to study different nonlinear systems [8]. Hsu et al. [9] used the modified Adomian decomposition method to study the free vibration of non-uniform Euler-Bernoulli beams with general elastically end constraints. The method which combined the Adomian decomposition method and Laplace transform had been proposed to study to the heat dissipation of a spiral fin [10]. However, the method will encounter numerical difficulty while taking large number of terms. In this paper, a hybrid method which combined the Adomian decomposition method, Laplace transform and the Padé approximation technique is employed to overcome the difficulty. The numerical results are compared with those in the existing literature.

Sen-Yung Lee is with the Mechanical Engineering Department, National Cheng Kung University, Tainan, Taiwan, Republic of China (phone: +886-6-2757575 ext.62150; e-mail: sylee@mail.ncku.edu.tw).

Li-Kuo Chou and Chao-Kuang Chen are with the Mechanical Engineering Department, National Cheng Kung University, Tainan, Taiwan, Republic of China (e-mail: n18981131@mail.ncku.edu.tw, ckchen@mail.ncku.edu.tw).

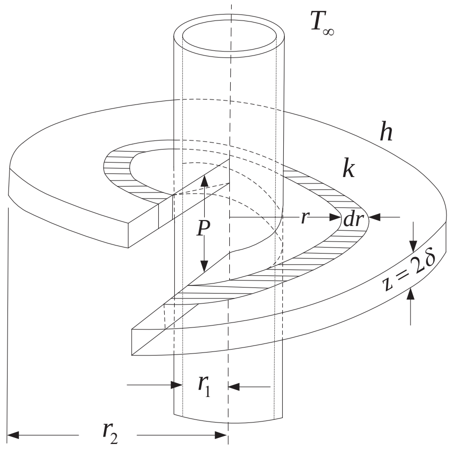

Fig. 1 Physical system of a spiral fin

II. Mathematical Model

Consider a spiral fin of uniform thickness 2δ as shown in Fig. 1. It is assumed that one-dimensional theory is valid in the present analysis. At the inner edge, $r=r_1$, the spiral fin dissipates heat by convection to the environment with convective heat transfer coefficient h. The thermal resistance and capacity of the material in the inner wall of the tube is assumed to be negligible. At the outer edge, $r=r_2$, the fin is assumed to be perfectly insulated. The surrounding temperature is T_∞. The initial temperature of the fin is T_f.

Outer radius r_2, pitch P and thermal conductivity k is shown as Fig. 1. The end of the fin, i.e., $r=r_2$ is assumed to be perfectly insulated is valid. Initial, the fin is in thermal equilibrium with the surrounding fluid temperature T_∞. At time t=0, the base temperature is suddenly raised to T_f or subjected to heat flux q_0^* and from then on, the spiral fin dissipated heat by convection to the environment through a convective heat transfer coefficient h, h_f and the properties k, ρ, c of the material of the fin are all assumed to be constant. At the other boundary condition, i.e., at $r=r_1$ the convective boundary by ignoring the thermal resistance and capacity of the material in inner wall tube is assumed.

Based on the conservation of energy, the governing differential equation of the temperature of the spiral fin can be derived as [4]:

$$\frac{\partial}{\partial r}\{\sqrt{(\frac{P}{2\pi})^2+r^2}\frac{\partial T}{\partial r}\}-\frac{h}{k\delta}\sqrt{(\frac{P}{2\pi})^2+r^2}(T-T_\infty)$$
$$=\frac{1}{\alpha}\sqrt{(\frac{P}{2\pi})^2+r^2}\frac{\partial T}{\partial t},\quad t>0,\ r_1<r<r_2 \tag{1}$$

where T is the temperature field, r is the radius, P is the pitch and k is the heat conduction coefficient of the spiral fin. ρ and c is the mass density and the heat capacity of the fin, respectively. $\alpha=k/\rho c$ is the thermal diffusivity. The Biot number is $B_i=h_f r_1/k_f$ and $\alpha=k/\rho c$ is the thermal diffusivity. h_f is base convective heat transfer coefficient, k_f is thermal conductivity of fluid for the fin base and r_1 is the inner radius of the fin.

After introducing the non-dimensional variables,

$$\xi=\frac{r}{r_1},N=\frac{hr_1^2}{k\delta},\theta=\frac{T-T_\infty}{T_f-T_\infty},\tau=\frac{\alpha t}{r_1^2},P_i=\frac{P}{2\pi r_1},R=\frac{r_2}{r_1} \tag{2}$$

The dimensionless governing equation is:

$$\frac{\partial}{\partial\xi}\{\sqrt{P_i^2+\xi^2}\frac{\partial\theta}{\partial\xi}\}-N\sqrt{P_i^2+\xi^2}\theta=\sqrt{P_i^2+\xi^2}\frac{\partial\theta}{\partial\tau},$$
$$\tau>0,\ 1<\xi<R \tag{3}$$

The dimensionless initial and boundary:

$$\theta(\xi,0)=0,\ 1<\xi<R \tag{4}$$

$$-\frac{\partial\theta}{\partial\xi}\Big|_{\xi=1,\tau}+B_i\theta=B_i,\ \xi=1,\ \tau>0 \tag{5}$$

$$\frac{\partial\theta}{\partial\xi}(R,\tau)=0,\ \tau>0 \tag{6}$$

where B_i is the Biot number ($B_i=h_f r_1/k_f$). Taking the Laplace transformation with respect to τ for (2), (4)-(6), we have the differential equation:

$$\frac{d}{d\xi}\{\sqrt{P_i^2+\xi^2}\frac{d\bar\theta}{d\xi}\}-N\sqrt{P_i^2+\xi^2}\bar\theta=\sqrt{P_i^2+\xi^2}\bar\theta s, \tag{7}$$

Define:

$$L[\theta(\xi,\tau)]=\int_0^\infty\theta(\xi,\tau)e^{-s\tau}d\tau=\bar\theta(\xi,s)$$

and the boundary conditions of (5) and (6) can be express as:

$$-\frac{\partial\bar\theta}{\partial\xi}\Big|_{\xi=1,\tau}+B_i\bar\theta=\frac{B_i}{s}, \tag{8}$$

$$\frac{\partial\bar\theta}{\partial\xi}(R,s)=0, \tag{9}$$

III. Fin Temperature Distribution

The dimensionless energy equation (7) can be simplified as:

$$\frac{d^2\bar\theta}{d\xi^2}=\bar\theta s+N\bar\theta-\frac{\xi}{P_i^2+\xi^2}\frac{d\bar\theta}{d\xi}, \tag{10}$$

The linear operator in the Adomain decomposition analysis is defined as:

$$L_{\xi\xi}=\frac{d^2}{d\xi^2} \tag{11}$$

Consequently, (10) becomes:

$$L_{\xi\xi}\bar\theta=\bar\theta s+N\bar\theta-\frac{\xi}{P_i^2+\xi^2}\frac{d\bar\theta}{d\xi}, \tag{12}$$

Operating on both sides of (12) with the inverse the operator L^{-1}, we can obtain:

$$\bar\theta=\bar\theta^*+\bar\theta^{*\prime}(\xi-1)+L_{\xi\xi}^{-1}\{(\bar\theta^* s)+(N\bar\theta^*)-\frac{\xi}{P_i^2+\xi^2}\frac{d\bar\theta^*}{d\xi}\} \tag{13}$$

where, $\bar\theta^*=\bar\theta(1,s)$.

From the boundary condition of (9), we can get the value $\bar\theta^{*\prime}=B_i\theta-B_i/s$, and (13) become to:

$$\bar\theta=\bar\theta^*+B_i(\bar\theta^*-\frac{1}{s})(\xi-1)+L_{\xi\xi}^{-1}\{(\bar\theta^* s)+(N\bar\theta^*)-\frac{\xi}{P_i^2+\xi^2}\frac{d\bar\theta^*}{d\xi}\} \tag{14}$$

The linear terms in (14):

$$A(\xi)=(\bar\theta^* s)+(N\bar\theta^*)-(\frac{\xi}{P_i}-\frac{\xi^3}{P_i^4}+\frac{\xi^5}{P_i^6}-\frac{\xi^7}{P_i^8}+\frac{\xi^9}{P_i^{10}}+O(\xi)^{10})\frac{d\bar\theta^*}{d\xi} \tag{15}$$

Adomian decomposition method is introduced in an infinite series as:

$$\bar\theta=\sum_{N=0}^{\infty}\bar\theta_n(\xi)=\bar\theta^*+B_i(\bar\theta^*-\frac{1}{s})(\xi-1)+L_{\xi\xi}^{-1}\sum_{n=0}^{\infty}A_n(\xi) \tag{16}$$

$\bar\theta_0(\xi)$ was defined as:

$$\bar\theta_0(\xi)=\bar\theta^*+B_i(\bar\theta^*-\frac{1}{s})(\xi-1) \tag{17}$$

$$\bar\theta_1(\xi)=L_{\xi\xi}^{-1}A_0(\xi) \tag{18}$$

The components of $\bar\theta(\xi)$ are determined from the following recursive relationship:

$$\bar\theta_{n+1}(\xi)=\bar\theta^*+B_i(\bar\theta^*-\frac{1}{s})(\xi-1)+L_{\xi\xi}^{-1}A_n(\xi),\ \ n=1,2,... \tag{19}$$

Adomian polynomials A_n can be presented as the following few terms:

$$A_0 = \bar{\theta}_0 = \bar{\theta}^* + B_i(\bar{\theta}^* - \frac{1}{s})(\xi - 1) \tag{20}$$

$$A_1 = \bar{\theta}_1 = L_{\xi\xi}^{-1} A_0(\xi) = \frac{N^2\bar{\theta}^*\xi^4}{2} + \frac{s^2\bar{\theta}^*\xi^2}{2} + \frac{\xi^2 B_i}{2} + \frac{N\xi^2 B_i}{2} - \frac{s\bar{\theta}^*\xi^2 B_i}{2} - \frac{\xi^3 B_i}{6} - \dots \tag{21}$$

$$A_2 = \bar{\theta}_2 = L_{\xi\xi}^{-1} A_1(\xi) = \frac{N^2\bar{\theta}^*\xi^4}{24} + \frac{Ns\bar{\theta}^*\xi^4}{12} + \frac{s^2\bar{\theta}^*\xi^4}{24} + \frac{N\xi^4 B_i}{12} + \frac{N^2\xi^4 B_i}{24} + \frac{s\xi^4 B_i}{24} - \dots \tag{22}$$

$$A_3 = \bar{\theta}_3 = L_{\xi\xi}^{-1} A_2(\xi) = \frac{N^2\bar{\theta}^*\xi^6}{720} + \frac{N^2 s\bar{\theta}^*\xi^6}{240} + \frac{Ns^2\bar{\theta}^*\xi^6}{240} + \frac{s^3\bar{\theta}^*\xi^6}{720} + \frac{N^2\xi^6 B_i}{240} + \frac{Ns\xi^6 B_i}{240} - \dots \tag{23}$$

$$A_4 = \bar{\theta}_4 = L_{\xi\xi}^{-1} A_3(\xi) = \frac{N^4\bar{\theta}^*\xi^8}{40320} + \frac{N^3 s\bar{\theta}^*\xi^8}{10080} + \frac{N^2 s^2\bar{\theta}^*\xi^8}{6720} + \frac{Ns^3\bar{\theta}^*\xi^8}{10080} + \frac{N^3\xi^8 B_i}{40320} + \frac{N^2 s\xi^8 B_i}{6720} - \dots \tag{24}$$

$$\bar{\theta}(1) = \theta(1,\tau)$$

Taking inverse Laplace transformation with respect to s for (20)-(24), we can express the following recursive relationship

$$\theta_0 = \theta(1)\delta(\tau) + B_i - \theta(1)\delta(\tau)B_i - \xi B_i + \theta(1)\delta(\tau)\xi B_i \tag{25}$$

$$\theta_1 = \frac{N^2\theta(1)\delta(\tau)\xi^4}{2} + \frac{N\xi^2 B_i}{2} - \frac{N\xi^3 B_i}{6} + \frac{\xi\delta(\tau)B_i}{2} - \frac{N\theta(1)\delta(\tau)\xi^2 B_i}{2} - \frac{\xi^3\delta(\tau)B_i}{6} + \dots \tag{26}$$

$$\theta_2 = \frac{N^2\theta(1)\delta(\tau)\xi^4}{24} + \frac{N^2\xi^4 B_i}{24} - \frac{N^2\xi^5 B_i}{120} + \frac{N\xi^4\delta(\tau)B_i}{12} - \frac{N^2\xi^4\theta(1)\delta(\tau)B_i}{24} - \frac{N\xi^5\delta(\tau)B_i}{60} - \dots \tag{27}$$

$$\theta_3 = \frac{N^3\theta(1)\delta(\tau)\xi^6}{720} + \frac{N^3\xi^6 B_i}{720} - \frac{N^3\xi^7 B_i}{5040} + \frac{N^2\xi^6\delta(\tau)B_i}{240} - \frac{N^3\theta(1)\xi^6}{720} + \dots \tag{28}$$

$$\theta_4 = \frac{N^4\theta(1)\delta(\tau)\xi^8}{40320} + \frac{N^4\xi^8 B_i}{40320} - \frac{N^4\xi^9}{362880} + \frac{N^3\xi^8\delta(\tau)B_i}{10080} - \frac{N^4\xi^8 B_i\theta(1)\delta(\tau)}{40320} - \frac{N^3\xi^9\delta(\tau)B_i}{90720} - \dots \tag{29}$$

Define:

$$L^{-1}[1] = \delta(\tau)$$

by obtaining the components $\theta_n(\tau)$, for n = 0,1,2,3,…, the approximate analytic solution for the four iteration step is

$$\theta(\tau) = \sum_{n=0}^{l} \theta_n(\tau) = \theta_0 + \theta_1 + \theta_2 + \theta_3 + \dots + \theta_n + \dots \tag{30}$$

Defining the non-dimensional heat flux at the fin base as:

$$q_0^* = q_b r_1 / 4k[\delta\pi r_1][T_f - T_\infty] \tag{31}$$

After employing the non-dimensional parameters, the heat flux can be expresses as:

$$q_0^* = -\sqrt{P_i^2 + 1}\,\frac{\partial\theta(1,\tau)}{\partial\xi}, \quad \tau > 0 \tag{32}$$

After applying the temperature distribution θ(τ) in (30), the non-dimensional heat flux at the base of spiral fin is given by,

$$q_0^* = -\sqrt{P_i^2 + 1}\,\frac{\partial\theta_{n+1}(1,\tau)}{\partial\xi} \tag{33}$$

IV. The LADM-Padé Approximation

Combining the obtained series solutions by the LADM in the previous section with the Padé approximation is the main part of this section. To the end, Then, we apply this process for obtaining some high accuracy computational results for problem (7) with boundary conditions (9). Then, we transform the power series obtained by the Laplace Adomian Decomposition Method (33) into a rational function as follow:

$$[M/N] = \sum_{j=0}^{M} a_j\xi^j \Big/ \sum_{j=0}^{N} b_j\xi^j \tag{34}$$

We know that if $N \geq M$ then the limit at infinity in the boundary conditions (8) and (9) has a correct behavior. So the rational function (34) has M+N+1 coefficient that we can select them. If $[M/N](\tau)$ is exactly a Padé approximation then $\theta(\tau) - [M/N](\tau) = O(\xi^{M+N+1})$. Then, we can obtain the coefficient a_j and b_j by the following relations:

$$\sum_{i=0}^{j} b_j\theta_{j-i} = a_j, \quad j = 0,\dots,M, \tag{35}$$

$$\sum_{i=0}^{j} b_j\theta_{j-i} = 0, \quad j = M+1,\dots,M+N, \tag{36}$$

where $a_j - b_j = 0$ if $j > N$, From (35) and (36), we can obtain the values of $a_i(O \leq i \leq M)$ and $a_j(1 \leq j \leq N)$. We know that if the function $\theta(\tau)$ is bounded i.e. for all $\tau > 0$, we have $\theta(\tau) < M$ and the $\lim_{\tau\to 0}\theta(\tau) = \theta(0)$ be exist, then $\lim_{s\to 0}\bar{\theta}(s) = \theta(0)$, $\bar{\theta}(s) = L(\theta(\tau))$, the Laplace transform of the function $\theta(\tau)$.

Which obtained by the LADM-Padé is shown in Figs. 2-4, The accuracy of proposed method can be understand from these plots.

V. Results and discussion

The LADM-Padé method provides an analytical solution in the form of an infinite power series. In study, take the first five terms (n = 4) from (30). The temperature distribution of the spiral fin θ is plotted in Figs. 2-4 for different values of τ, *Bi*, *N*, *Pi* and *R*.

Figs. 2, 3 and 4 show that the temperature distribution θ increases as time τ elapsed. Also, the absolute slope of temperature distribution θ of the spiral fin base has a trend of decreasing with an increase of time τ which implies that the output heat flux in base of fin will be decreased with an increase of time τ.

Fig. 2 presents the temperature distribution θ alone the radius of the spiral fin for the dimensionless variable *Pi* and τ at Bi = 1.0, R = 2, τ and N = 1. Also, due to the increase of internal temperature of the spiral fin when time τ is increasing. At the same time, for the greater values *Pi* of spiral fin.

Fig. 3 presents the temperature distribution θ alone the radius of the spiral fin for the dimensionless variable *Pi* and τ at Bi = 10, R = 2 and N = 1. Also, due to the increase of internal temperature of the spiral fin when time τ is increasing. At the same time, the temperature distribution θ is a function of Bi become of the heat flux at fin base transferred form convection is larger value of Bi =10.

Fig. 4 presents the temperature distribution θ alone the radius of the spiral fin for the dimensionless variable *Pi* and τ at Bi = 10, R = 2 and N = 5. Also, due to the increase of internal temperature of the spiral fin when time τ is increasing. At the same time, for the greater values N = 5 of spiral fin.

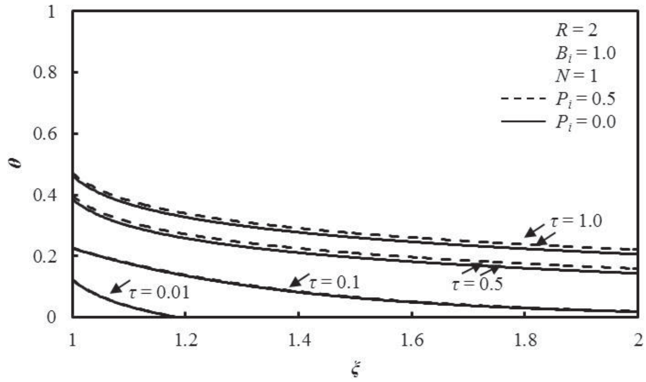

Fig. 2 Temperature distribution for R = 2, N = 1, B_i = 1.0

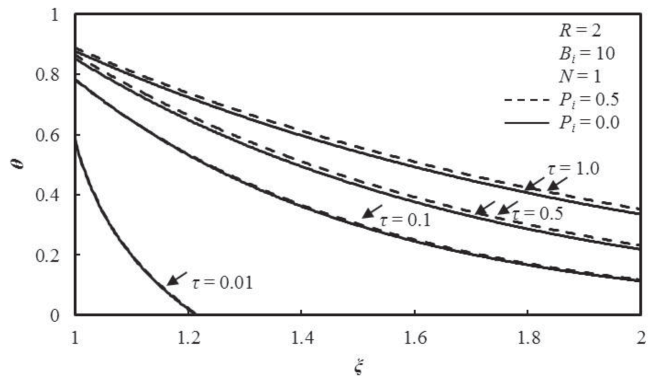

Fig. 3 Temperature distribution for R = 2, N = 1, B_i = 10

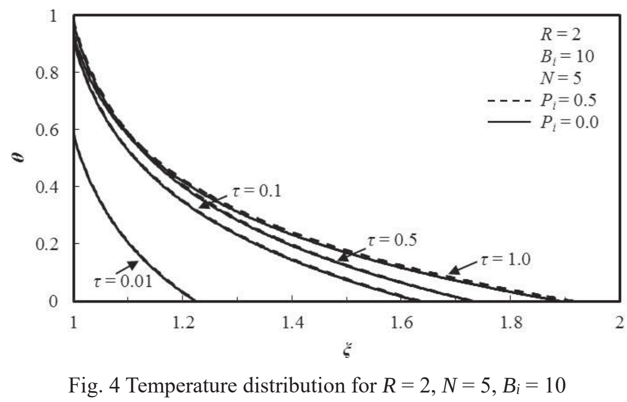

Fig. 4 Temperature distribution for R = 2, N = 5, B_i = 10

VI. Conclusions

Many problems are linear in the engineering applications, and LADM-Padé approximant is a good way quickly to find the approximate solution. In this study, we apply to the transient response. Both of a unit step change and a sinusoidal temperature change are analyzed. The result show that the temperature variation is affected by the parameters N, Pi, R, As N becomes large, more heat transform forms the spiral fin by the temperature distribution also become large. The Pi of a spiral fin has almost no effect for the temperature distribution. For the heat flux distribution at the spiral fin base q_0^*, the parameter N and Pi are the important factors, and the parameter Pi affect the q_0^* more as the parameter N increases.

References

[1] Chu, H.S., Chen, C.K. and Weng, C.I., "Applications of Fourier series technique to transient heat transfer problem," J. Chem. Eng. Commun., vol. 16, 1982, pp. 215– 225.
[2] Chu, H.S., Chen, C.K. and Weng, C.I., "Transient response of circular pins," ASME. J. Heat Transfer, vol. 105, 1983, pp. 205– 208.
[3] Mokheimer, E.M.A., "Performance of annual fins with differential profiles subject to variable heat transfer coefficient," Int. J. heat Mass Transfer, vol. 45, 2002, pp. 3631– 3642.
[4] Wang, J.S., Luo, W.J. and Hsu, S.P., "Transient Response of a Spiral Fin with its Base Subjected to the Variation of Heat Flux," Journal of Applied Sciences, vol. 8, 2008, pp. 1798– 1811.
[5] Malekzadeh, P. and Rahideh, H., "Two-dimensional nonlinear transient heat transfer analysis of variable section pin fins," Energy Conversion and Management, vol. 50, 2009, pp. 916– 922.
[6] Lai, C.Y., Kou, H.S. and Lee, J.J., "Recursive formulation on thermal analysis of an annular fin with variable thermal properties," Applied Thermal Engineering, vol. 29, 2009, pp. 779– 786.
[7] Adomian, G., "Solution of the coupled nonlinear partial differential equations by decomposition," Computers & Mathematics with Applications, vol. 31, 1996, pp. 117– 120.
[8] Wazwaz, A.M., "The modified decomposition method and Padé approximants for solving the Thomas–Fermi equation," Applied Mathematics and Computation, vol. 105, 1999, pp. 11–19.
[9] Hsu, J.C., Lai, H.Y., and Chen, C.K., "Free vibration of non-uniform Euler-Bernoulli beams with general elastically end constraints using Adomian modified decomposition method," Journal of Sound and Vibration, vol.318, 2008, pp. 965–981.
[10] Chiu, C.H. and Chen, C.K., "A decomposition method for solving the convective longitudinal fins with variable thermal conductivity," International of Heat and Mass Transfer, vol. 45, 2002, pp. 2067–2075.

Sen Yung Lee is a Distinguished Professor of Department of Mechanical Engineering, National Cheng Kung University, Taiwan, R. O. C. He received his Ph.D. from State University of New York at Buffalo, USA in 1984. His research interests are solid mechanics, heat conduction, and creative design with more than 80 patients.