

# Traffic Density Estimation for Multiple Segment Freeways

Karandeep Singh and Baibing Li

**Abstract**—Traffic density, an indicator of traffic conditions, is one of the most critical characteristics to Intelligent Transport Systems (ITS). This paper investigates recursive traffic density estimation using the information provided from inductive loop detectors. On the basis of the phenomenological relationship between speed and density, the existing studies incorporate a state space model and update the density estimate using vehicular speed observations via the extended Kalman filter, where an approximation is made because of the linearization of the nonlinear observation equation. In practice, this may lead to substantial estimation errors. This paper incorporates a suitable transformation to deal with the nonlinear observation equation so that the approximation is avoided when using Kalman filter to estimate the traffic density. A numerical study is conducted. It is shown that the developed method outperforms the existing methods for traffic density estimation.

**Keywords**—Density estimation, Kalman filter, speed-density relationship, Traffic surveillance.

## I. INTRODUCTION

**I**N both developing and developed countries, major cities continue to struggle with increasing traffic congestion and related problems. Often complex Traffic Management Systems (TMS) are employed to help manage the flow of traffic. These TMS's fuse traffic surveillance related information from multiple supporting systems.

A TMS including Intelligent Transport Systems (ITS) requires real-time information to make effective control decisions and to deliver trustworthy information to users. Transportation planning and analysis need a detailed study of transport variables such as traffic flow, vehicle speed, travel time, congestion level etc. One of the most commonly used pieces of information for transportation planning is the traffic flow within a highway segment. Traffic flow (counts) of vehicles and their speed measurements can be available from surveillance sensors such as magnetic loop detectors. However, the problem of estimating the number of vehicles traversing a highway segment (traffic link) presents an

important issue which has attracted a great deal of research in the recent decades.

Generally, it is believed that traffic density estimation or prediction is difficult. The reason behind this is due to the type of sensors employed. Inductive loop detectors are commonly used which are point sensors while density is a range concept. Location, weather, vehicle types, etc. are some of the disturbances to density estimation. As a matter of fact, traffic density is very difficult to be measured/estimated accurately in real-time although they are required in traffic control in practice, such as coordinated ramp metering and variable speed limit control [1].

The Kalman filter is an efficient recursive technique to update the estimates of the state vectors of linear dynamic systems from a series of noisy measurements. It is widely used in the traffic studies (see, e.g., [2] and [3]). The speed-density relationship is usually serves as the observation equation of state space models for the density estimation problem. As the speed-density relationship in the Drake's model [4] is nonlinear, the existing methods utilize the extended Kalman filter to rapidly update the traffic state vectors for online traffic surveillance. As noted in [5], however, the extended Kalman filter involves an approximation when the nonlinear system is linearised. This may sometimes cause substantial estimation errors.

In this research, the approach presented in [2] have been followed and speed-density relationship is used to estimate vehicle counts recursively by employing the Kalman filtering technique. To avoid the linearization in the extended Kalman filter used in the existing methods, some suitable transformation have been employed and it has been shown that the accuracy of traffic density estimation is greatly improved. vehicle density estimation

Vehicular density plays a vital role for managing and controlling traffic operations in urban networks. As an instantaneous and range concept, traffic density is primarily defined by considering into a snapshot photo of the traffic by an aerial camera along a stretch of freeway [6], [7]. The density here is simply the number of vehicles divided by the length of the selected road segment. Average density over lanes is naturally deduced by further dividing the number of lanes. Note that this concept can be described as continuous in space but discrete in time. However, practical traffic network systems, particularly freeway networks, do not have aerial cameras to continuously monitor the traffic in real-time. Although dense point sensor systems (inductive or magnetic loop detectors) could approximate continuous measurement in

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space, the cost is prohibitive in general. In practice, point sensors such as inductive loop detectors are popularly used for traffic detection, which could continuously count vehicle numbers in traffic streams in real-time at the sensor locations [1]. The vehicular density information may be derived from raw counts obtained from inductive loop detectors or other detection devices. However, these counts are subject to errors, which can degrade the density estimates substantially.

To improve count estimates, the study in [8] carried out in the early 70s used a filtering technique, an extended version of Kalman filter. Their algorithm assumes a discrete-time control system which re-linearises the dynamics of each new estimate as it becomes available. Since then the Kalman filtering has been employed quite frequently in the literature. Different versions of Kalman filter including extended Kalman filter ([2], [3]), mixture Kalman filter [9], and linear Kalman filter [10] have been used to estimate traffic densities over highways and roadways. This work is based on the model in [2] where the extended Kalman filter was applied to obtain density estimates. Note that to employ the extended Kalman filter in [2], a Taylor expansion of the original nonlinear systems was used. However, this linearization can sometimes lead to biased results and therefore can distort the actual traffic flow model.

II. METHODOLOGY

Consider a typical freeway section as schematically shown in Fig 1. Consider that there are number of sections of a freeway with embedded inductive loop detectors. Traffic flow over the freeway must be regulated for various segments of the freeway to avoid traffic congestion. Clearly, effective real time control of freeway traffic relies on information regarding the number of vehicles in different sections of the freeway. Using the traffic speed-flow relationship and traffic information from inductive loop detectors, the Kalman filter can be employed to improve the vehicle count estimates.

This paper considers a multi-section roadway with  $N$  different sections in tandem. The idea behind selecting such a situation is that the traffic density estimate can be improved by considering the fact that the counting error for vehicles leaving a given section is the same as the error for the vehicles entering the very next section.

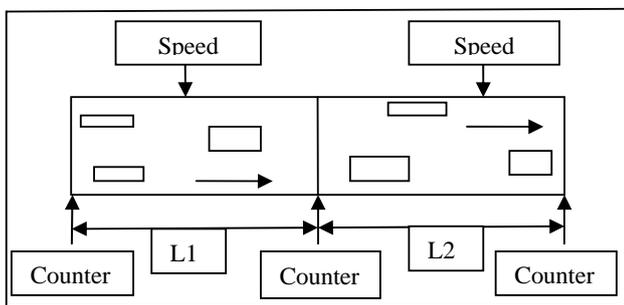


Fig. 1 A schematic plot showing two sections of a freeway

A. State Space Model for Density Estimation

The problem of the estimation of traffic density is

formulated as follows. The number of vehicles  $y_{k,j}$  in each section  $j$  of a multi-section roadway at the  $k$ th time step ( $k=1, \dots, K$ ) is considered as a state variable. For ease of exposition, a two section problem is considered but the results can be extended to the general situation where the roadway is split into  $N$  section. The state equations for the state variables are given by

$$y_{k+1,j} = y_{k,j} + u_{k,j} - u_{k,j+1} + \varepsilon_{k,j} - \varepsilon_{k,j+1} \quad (j=1,2), \quad (1)$$

where quantities  $u_{k,j}$  and  $u_{k,j+1}$  are the numbers of vehicles entering and leaving section  $j$  at the  $k$ th time step respectively. The  $\varepsilon_{k,j}$  is the counting error for the quantity  $u_{k,j}$ . Following [2], assume  $\varepsilon_{k,j}$  has a normal distribution with zero mean and variance  $\sigma^2$ . The variance of detectors is assumed to be the same for all detectors, i.e.  $\text{var}(\varepsilon_{k,j}) = \sigma^2$ . It is straightforward to obtain

$$\text{var}(y_{k,j}) = 2\sigma^2,$$

and

$$\text{cov}(y_{k,j}, y_{k,j+1}) = -\sigma^2.$$

Let  $w_{k,j} = u_{k,j} - u_{k,j+1}$  and  $\xi_{k,j} = \varepsilon_{k,j} - \varepsilon_{k,j+1}$ . Then the state equation can be written in a matrix form:

$$\mathbf{y}_{k+1} = \mathbf{A}\mathbf{y}_k + \mathbf{w}_k + \xi_k, \quad (2)$$

where  $\mathbf{A}$  is a 2-dimensional identity matrix.  $\mathbf{y}_k = [y_{k,1}, y_{k,2}]^T$ .  $\mathbf{w}_k$  and  $\xi_k$  are similarly defined.  $\xi_k \sim N(0, \mathbf{Q})$ , with

$$\mathbf{Q} = \sigma^2 \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}.$$

Next, the observation equation is considered. Following [2] and [3], observations of traffic density are obtained from the Drake's model [4] which is a phenomenological relationship between speed and density:

$$v_{k,j} = v_{f,j} \exp[-(y_{k,j} / (n_{0,j} L_j))^2 / 2] \quad (j=1,2), \quad (3)$$

where  $v_{k,j}$  is the speed measurement in section  $j$  at the  $k$ th time step and  $v_{f,j}$  represents the free flow speed.  $L_j$  is the length of section  $j$ .  $n_{0,j}$  is the density corresponding to the maximum flow in section  $j$ .

The Kalman filter is an efficient recursive filter that estimates the state of linear dynamic system from a series of noisy measurements. To apply the Kalman filtering technique, the above observation equation was linearised in [2]. Unfortunately the linearization can sometimes produce unreliable results where the estimates of the state variables were totally off the track.

In order to overcome this problem, this paper proposes

some suitable transformation to the observation equation to yield a linear form so that the approximation is completely avoided. As shown later in this paper, this approach is more robust and accurate.

Specifically a transformed speed observation is defined:

$$z_{k,j} = \{\log(v_{f,j} / v_{k,j})\}^{1/2}. \quad (4)$$

From equation (2), we obtain  $z_{k,j} = [1/(\sqrt{2}n_{0,j}L_j)]y_{k,j}$ . By taking into account measurement errors, the transformed observation equation can be written:

$$z_{k,j} = [1/(\sqrt{2}n_{0,j}L_j)]y_{k,j} + \eta_{k,j},$$

where the errors  $\eta_{kj}$  are assumed to approximately follow a normal distribution, i.e.  $\eta_{k,j} \sim N(0, \tau^2)$ . The free speed  $v_{fj}$  required in the transformation can be estimated prior to the Kalman filter is applied. Alternatively, following [2] these can be treated as tuning parameters in practice.

Define  $\mathbf{R} = \tau^2 \mathbf{I}$  and  $\mathbf{H} = \text{diag}\{n_{0,1}L_1/\sqrt{2}, n_{0,2}L_2/\sqrt{2}\}$ , where  $\mathbf{I}$  is a two-dimensional identity matrix. The observation equation can thus be written in a matrix form:

$$\mathbf{z}_k = \mathbf{H}\mathbf{y}_k + \boldsymbol{\eta}_k, \quad (5)$$

where  $\mathbf{z}_k = [z_{k,1}, z_{k,2}]^T$  and  $\boldsymbol{\eta}_k$  is similarly defined.  $\boldsymbol{\eta}_k \sim N(0, \mathbf{R})$ .

### B. Density Estimation – A Kalman Filter Approach

In this subsection the Kalman filtering technique is applied to update the estimate of traffic density.

Let  $\hat{\mathbf{y}}_{k,k}$  and  $\hat{\mathbf{y}}_{k+1,k}$  denote the updated estimate and one-step forecast of the state vector at time step  $k$ , and  $\mathbf{P}_{k,k}$  and  $\mathbf{P}_{k+1,k}$  denote the corresponding covariance matrices. The estimation of a multi-section roadway vehicle density is carried out as follows:

Step 1. Initialization:

$$\hat{\mathbf{y}}_{0,0} = \hat{\mathbf{y}}_0, \mathbf{P}_{0,0} = \mathbf{P}_0 \text{ and } k=0. \quad (6)$$

Step 2. One-step forecast of the state vector:

$$\hat{\mathbf{y}}_{k+1,k} = \mathbf{A}\hat{\mathbf{y}}_{k,k} + \mathbf{w}_k, \quad (7)$$

$$\mathbf{P}_{k+1,k} = \mathbf{A}\mathbf{P}_{k,k}\mathbf{A}^T + \mathbf{Q}. \quad (8)$$

Step 3. Computing the Kalman gain matrix

$$\mathbf{M} = \mathbf{P}_{k+1,k} \mathbf{H}(\mathbf{H}\mathbf{P}_{k+1,k} \mathbf{H}^T + \mathbf{R})^{-1}. \quad (9)$$

Step 4. Updating the state vector and its covariance matrix

$$\hat{\mathbf{y}}_{k+1,k+1} = \hat{\mathbf{y}}_{k+1,k} + \mathbf{M}(\mathbf{z}_k - \mathbf{H}\hat{\mathbf{y}}_{k+1,k}), \quad (10)$$

$$\mathbf{P}_{k+1,k+1} = \mathbf{P}_{k+1,k} - \mathbf{M}\mathbf{H}\mathbf{P}_{k+1,k}. \quad (11)$$

Step 5. Let  $k=k+1$  and return to Step 2.

It can be seen that the main difference between the developed method and the method in [2] is that instead of applying the extended Kalman filter and carrying out the linearization of the speed-density relationship, a suitable transformation is applied to avoid the approximation caused by linearization.

## III. SIMULATION

The developed method will be tested in this section via microscopic simulation. One major advantage of carrying out simulation studies is that ‘true’ values of traffic densities are known *a priori* so that it is straightforward to assess the performance of a method in terms of accuracy [5].

### A. Simulation Description

A self developed microscopic simulator was used to simulate traffic in a single lane having multiple segments. Two sections of roadway were considered with 400 and 500 meters long respectively. The simulation scenario was with duration of 2 hours and the estimation time step was 20s. Following [2], the density corresponding to the maximum flow in both sections,  $n_{0,j}$ , were set equal to 32. The initial guess of the numbers of vehicles in the two sections was set equal to  $L_j * n_{0,j} / 2$ , i.e. 50% of the maximum traffic counts. Free speeds for two sections were set to be 104.76 km/h [2].

The real time traffic counts entering section  $j$ ,  $u_{k,j}$ , were simulated using Poisson variates with a mean of  $\lambda$  and the error terms  $\varepsilon_{k,j}$  for traffic counts and  $\eta_{k,j}$  for speed measurements were simulated as normal variates with zero means of covariance matrices  $\mathbf{Q}$  and  $\mathbf{R}$  respectively.

The parameters in the simulation varied from experiment to experiment to reflect different scenarios. Each experiment was repeated 100 times. The evaluation of each method was based on the following Root Mean Square Error criterion:

$$RMSE = \{K^{-1} \sum_{k=1}^K [\hat{N}(k) - N(k)]^2\}^{1/2},$$

where  $N(k)$  and  $\hat{N}(k)$  are simulated and estimated vehicle counts in a section.

### B. Simulation Results

Fig 2 and Fig 3 display simulated vehicle counts (broken

line) and the estimated vehicle counts (real line) in an experiment for the two sections with two different scenarios where the parameters of the error terms in the state equation and observation equation differ, i.e.  $\sigma=3$  and  $\tau=1$  for Fig 2, and  $\sigma=2$  and  $\tau=2$  for Fig 3. Also note that Fig 2 represents the scenario that the traffic is light (the average number of vehicles arrival is 3 per 20s). In contrast, in Fig 3 the average number of vehicles arrival is increased to 8 per 20s. It is clear that the developed method performed well: the estimated traffic density was close to the simulated traffic density.

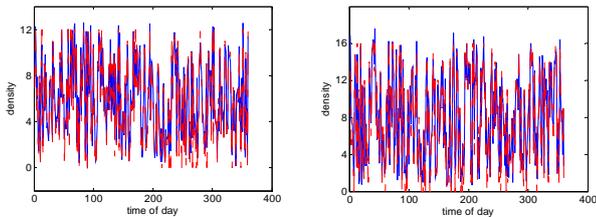


Fig. 2 Simulated traffic counts (broken line) and the corresponding estimated traffic counts (real line) using the developed method for section 1 (left) and section 2 (right) with  $\sigma=3$ ,  $\tau=1$  and  $\lambda=3$

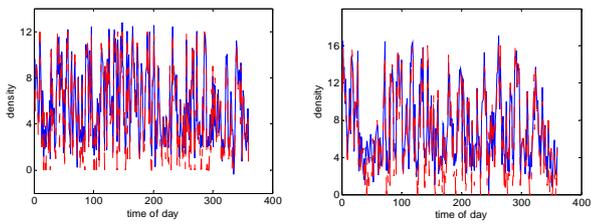


Fig. 3 Simulated traffic counts (broken line) and the corresponding estimated traffic counts (real line) using the developed method for section 1 (left) and section 2 (right) with  $\sigma=2$ ,  $\tau=2$  and  $\lambda=8$

It is also of interest to investigate the impact of the error parameters  $\sigma$  and  $\tau$  on the accuracy of estimation. TABLE I displays RMSE values for different scenarios using the developed method. It can be seen that overall the estimated traffic counts are quite accurate. When the parameters  $\sigma$  and  $\tau$  are small, the average estimation errors are about one vehicle per 20s. When the two parameters become larger, the average estimation errors increase but are still at a low level.

TABLE I  
AVERAGE RMSES FOR TWO ROADWAY SECTIONS OVER 100 SIMULATION RUNS USING THE DEVELOPED METHOD WITH  $\lambda=10$

	$\tau=1$	$\tau=2$	$\tau=3$
$\sigma=1$	0.6410; 0.7826	0.6148; 0.7439	0.6104; 0.7309
$\sigma=2$	0.9740; 1.1959	0.9777; 1.1704	0.9663; 1.1639
$\sigma=3$	1.2684; 1.5394	1.2805; 1.5597	1.2717; 1.5495
$\sigma=5$	1.7792; 2.1775	1.8221; 2.2382	1.8289; 2.2420
$\sigma=8$	2.4677; 3.0314	2.5343; 3.0887	2.5344; 3.0996

C. Comparison with the Existing Method

Next the performance of the developed method with that of the method in [2] is compared where the density was estimated using the extended Kalman filter. TABLE II displays the RMSE values for different scenarios using the method in [2].

It can be seen that the developed method has a better performance than that in [2]. This is not surprising as pointed out earlier the method in [2] is an approximate method. It is not unusual in the simulation experiments that this method completely broke down (i.e. greatly deviated from the trajectory of actual vehicular density) even if all the conditions were favorable.

TABLE II  
AVERAGE RMSES FOR THE TWO ROADWAY SECTIONS OVER 100 SIMULATION RUNS USING THE METHOD IN [2] WITH  $\lambda=10$

	$\tau=1$	$\tau=2$	$\tau=3$
$\sigma=1$	0.9273; 0.9842	4.8887; 4.3649	11.1270; 10.0727
$\sigma=2$	1.1069; 1.3109	3.4771; 3.5431	7.4076; 8.0536
$\sigma=3$	1.3905; 1.6450	3.4038; 3.1777	6.5289; 7.0746
$\sigma=5$	1.8530; 2.3127	3.2032; 3.4549	6.2301; 6.7294
$\sigma=8$	2.5397; 3.2299	3.6254; 3.8918	5.9113; 6.6841

To have a closer look at the performance of the method in [2], the density estimation using the method in [2] is considered for the same data in Fig 2 and Fig 3. The results are displayed in Fig 4 and Fig 5 respectively. It can be seen that the estimates obtained using the method in [2] have a poor performance for this particular data. This is due to the approximation made by the linearization in the extended Kalman filter.

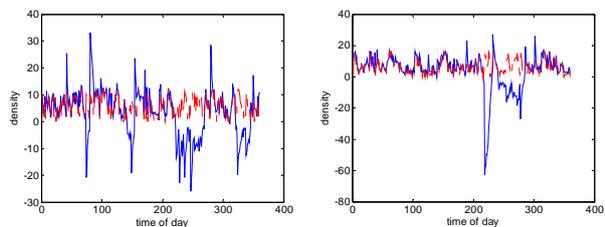


Fig. 4 Simulated traffic counts (broken line) and the corresponding estimated traffic counts (real line) using the method in [2] for section 1 (left) and section 2 (right) with  $\sigma=3$ ,  $\tau=1$  and  $\lambda=3$

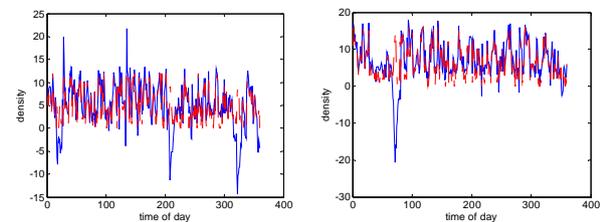


Fig. 5 Simulated traffic counts (broken line) and the corresponding estimated traffic counts (real line) using the method in [2] for section 1 (left) and section 2 (right) with  $\sigma=2$ ,  $\tau=2$  and  $\lambda=8$

#### D. Light Traffic Scenario

Finally the above results are compared where the congestion level was medium with the light traffic scenario where the average number of vehicles is 3.

For the following simulation experiments with  $\lambda=3$  and the specified values of  $\sigma$  and  $\tau$ , each experiment was repeated times. The resulting RMSEs averaged over the 100 runs using the developed method and the method used in [2] are displayed in TABLE III and TABLE IV respectively.

It can be seen that, compared with the results in TABLE I and TABLE II, the estimation errors are slightly larger for both methods. It is also clear that the developed method outperformed the method in [2]. This shows that the developed method is robust: its accuracy is better than the existing method under different road traffic conditions.

TABLE III  
AVERAGE RMSES FOR THE TWO ROADWAY SECTIONS OVER 100 SIMULATION RUNS USING THE DEVELOPED METHOD FOR  $\lambda=3$

	$\tau=1$	$\tau=2$	$\tau=3$
$\sigma=1$	0.7732; 1.0101	2.6739; 3.3629	2.5701; 3.1999
$\sigma=2$	1.1497; 1.4981	1.0701; 1.3357	1.0069; 1.2217
$\sigma=3$	1.4915; 1.9521	1.3765; 1.7362	1.3245; 1.6160
$\sigma=5$	2.0804; 2.7015	1.9414; 2.4607	1.8726; 2.2930
$\sigma=8$	2.8170; 3.6798	2.6766; 3.3546	2.6074; 3.2090

TABLE IV  
AVERAGE RMSES FOR THE TWO ROADWAY SECTIONS OVER 100 SIMULATION RUNS USING THE METHOD IN [2] FOR  $\lambda=3$

	$\tau=1$	$\tau=2$	$\tau=3$
$\sigma=1$	1.2804; 1.2807	3.8987; 4.1185	6.1428; 6.5683
$\sigma=2$	1.4714; 1.7192	3.9889; 4.1615	7.5427; 8.2706
$\sigma=3$	1.7161; 2.1688	4.0599; 4.5865	7.0270; 7.2755
$\sigma=5$	2.2637; 3.0050	3.9614; 4.4617	6.2814; 6.4598
$\sigma=8$	3.0616; 4.1422	4.0412; 4.7097	5.8344; 6.7040

#### IV. CONCLUSIONS

In this paper, the recursive estimation of traffic density is investigated using vehicular speed-density relationship. The proposed recursive estimation method uses a suitable transformation so that the state space system become linear to avoid the linearization of observation equation used in the existing method. Hence it is not surprising that the developed method has a better performance than the method in [2]. The simulation study suggests that the developed method is not affected by road traffic conditions and performs satisfactorily in different scenarios.

For ease of exposition, the developed method is presented for two sections in tandem. However this approach can be extended to estimate traffic density for multi-section freeways. Provided enough vehicle detectors installed in the network, the developed method can effectively identify traffic density on a real time basis. The estimated traffic density can facilitate effective traffic management of networks, and also provide inputs for planning and controlling both short term and long term urban transport.

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