



Transfer function of a double notch filter is:

$$G_F(s) = \frac{(1/\omega_r)^2 s^2 + (2\xi_r/\omega_r)s + 1}{(1/\omega_a)^2 s^2 + (2\xi_a/\omega_a)s + 1} \quad (3)$$

where resonant frequency and resonance damping are:

$$\omega_r = \sqrt{d_{12} \frac{J_M + J_L}{J_M J_L}}, \text{ resp. } \xi_r = \frac{b_{12}}{2} \sqrt{\frac{J_M + J_L}{d_{12} J_M J_L}},$$

and antiresonant frequency and antiresonance damping are:

$$\omega_a = \sqrt{\frac{d_{12}}{J_L}}, \text{ resp. } \xi_a = \frac{b_{12}}{2\sqrt{d_{12} J_L}}.$$

### III. CONTROL SYSTEM

Different methods for controlling the system design were applied [1], [2], [4]-[7]. The control system design was based on idealised conditions with infinitely rigid connection being considered instead of flexible connection between the actuator and the load.

#### A. Feedback Control of Angular Velocity

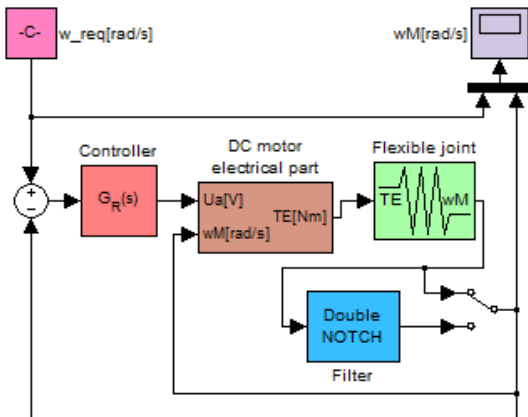


Fig. 4 Model of angular velocity feedback control

Transfer function of system depicted in Fig. 4 is:

$$G_1(s) = \frac{G_R(s)G_{DCM\_FJ}(s)}{1 + G_R(s)G_{DCM\_FJ}(s)} \quad (4)$$

where:  $G_R(s)$  is controller (6), (7), (9) and transfer function of a DCM with flexible joint with/without filter is:

$$G_{DCM\_FJ}(s) = \frac{G_{arm}(s)c\Phi.G_{FJ}(s)[G_F(s)]}{1 + G_{arm}(s)c\Phi^2.G_{FJ}(s)[G_F(s)]} \quad (5)$$

#### 1) PI Controller – Naslin Method

Coefficients for controller of PI type in form (6) calculated based on solution of inequalities derived from system closed control loop are in Table I.

$$G_R(s) = r_0 + \frac{r_{-1}}{s} \quad (6)$$

TABLE I  
PI (NASLIN) CONTROLLER PARAMETERS

Parameter	Value
$r_0$	0.1667
$r_{-1}$	0.0233

#### 2) PID Controller – Modulus Optimum Method

Coefficients for controller of PID type in form (7) calculated based on solution of equations system in matrix form derived from open control loop frequency response are in Table II.

$$G_R(s) = r_0 + \frac{r_{-1}}{s} + r_1 s \quad (7)$$

TABLE II  
PID (MOM) CONTROLLER PARAMETERS

Parameter	Value
$r_0$	0.1602
$r_{-1}$	0.0199
$r_1$	-0.0323

#### 3) PID Controller – Method of Inverse Dynamics

For possibility to calculate controller coefficients by Method of Inverse Dynamics was the transfer function of an actuator rigidly connected with a load modified to the form:

$$G_{DCM}(s) = \frac{K}{T_0^2 s^2 + 2\xi T_0 s + 1} \quad (8)$$

The coefficients of PID controller in form (9) calculated based on this method are in Table III.

$$G_R(s) = P(1 + \frac{T_i}{s} + T_d s) \quad (9)$$

TABLE III  
PID (MID) CONTROLLER PARAMETERS

Parameter	Value
P	46.6713
$T_i$	15.5556
$T_d$	5

#### B. Cascade Control of the Mechatronic System

Transfer function of system depicted in Fig. 5 is:

$$G_2(s) = \frac{G_{R2}(s)G_{CLI}(s)c\Phi.G_{FJ}(s)[G_F(s)]}{1 + G_{R2}(s)G_{CLI}(s)c\Phi.G_{FJ}(s)[G_F(s)]} \quad (10)$$

where:  $G_{R2}(s)$  is speed controller (13), (14), transfer function of a current closed control loop is:

$$G_{CLI}(s) = \frac{G_{R1}(s)G_{arm}(s)}{1 + G_{R1}(s)G_{arm}(s)} \quad (11)$$

$G_{R1}(s)$  is current controller (12).

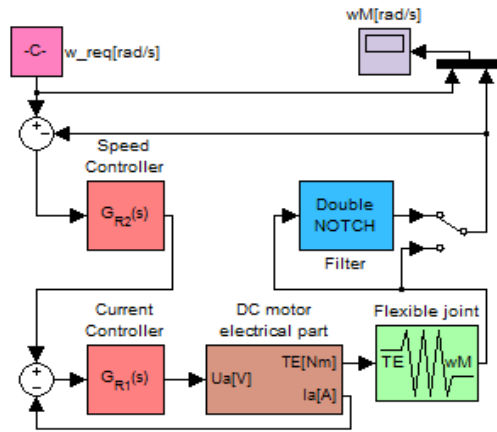


Fig. 5 Model of DCM cascade control

### 1) Current Controller

The tuning method based on the Modulus Optimum optimization criterion was used for current controller design. According to this criterion was specified a PI controller in form:

$$R_{PI} = k_{PI} \frac{Ti_{PI}s + 1}{Ti_{PI}s} \quad (12)$$

The coefficients of cascade control subordinate PI current controller designed by Modulus Optimum Method are in Table IV.

TABLE IV  
PI (MOM) CURRENT CONTROLLER PARAMETERS

Parameter	Value
$k_{PI}$	0.025
$Ti_{PI}$	5

### 2) Speed Controller I

The tuning method based on the Symmetric Optimum Criterion was used for first version of speed controller design. According to this method was specified a PID controller in form:

$$R_{PID} = k_{PID} + \frac{Ti_{PID}}{s} + Td_{PID}s \quad (13)$$

The coefficients of cascade control master PID speed controller designed by Symmetric Optimum Criterion are in Table V.

TABLE V  
PID (SOC) CURRENT CONTROLLER PARAMETERS

Parameter	Value
$k_{PID}$	568.75
$Ti_{PID}$	7.2917
$Td_{PID}$	4083.3

### 3) Speed Controller II

The tuning method based on the Modulus Optimum

criterion was used for second version of speed controller design. According to this method was specified a PD controller in form:

$$R_{PD} = k_{PD}(1 + Td_{PD}s) \quad (14)$$

The coefficients of cascade control master PD speed controller designed by Modulus Optimum Method are in Table VI.

TABLE VI  
PD (MOM) CURRENT CONTROLLER PARAMETERS

Parameter	Value
$k_{PD}$	58.3333
$Td_{PD}$	4

## IV. SYSTEM SENSITIVITY

The behaviour of the system is described by its characteristic functions, i.e. characteristics, the system sensitivity analysis is based on a sensitivity analysis of its characteristic functions.[3], [8], [9]

If a system is given by the input  $u(\xi)$  and the response  $y(\xi)$ , it can be expressed by:

$$y(\xi) = F\{\xi, \alpha, u_0, u(\xi)\} \quad (15)$$

where  $F\{\xi, \alpha\}$  is a system function with  $\xi$  argument depending on the system element parameters  $\alpha = \{\alpha_k\}$  for  $k = 1, 2, \dots, n$ . Then the absolute differential sensitivity of the system to changes in the parameter is expressed as:

$$S_k(\xi, \alpha_k) = \frac{\partial}{\partial \alpha_k} F\{\xi, \alpha\} \quad (16)$$

and the relative differential sensitivity expressing the percentage change in the system function for a 1% change in the value of the parameter is given by:

$$S_r(\xi, \alpha_k) = \frac{\partial \ln F\{\xi, \alpha\}}{\partial \ln \alpha_k} = \frac{\alpha_k}{F\{\xi, \alpha\}} S_k(\xi, \alpha_k) \quad (17)$$

The sensitivity of the system response was analysed based on the transfer function of closed-loop control system according to (4) and (10).

The sensitivity of the frequency-response characteristic was analysed based on the frequency response of closed-loop control system representing the system function (18).

$$G(j\omega) = \{G(s)\}_{s=j\omega} = G(\omega)e^{j\Phi(\omega)} \quad (18)$$

## V. SYSTEM TOLERANCE

The system tolerance expresses a change in system parameters, which does not alter the behavior of the system beyond the permitted respectively the desired behavior. [3], [8], [9] If the system behaviour is described by (15) and change of system response for defined input  $u_0, u(\xi)$  with the

change of the  $k^{\text{th}}$  parameter  $\alpha_k$  is expressed as:

$$\Delta y(\xi, \alpha_k) = \frac{\partial F(\xi, \alpha)}{\partial \alpha_k} \Delta \alpha_k \quad (19)$$

If the permitted change is denoted as  $\Delta y(\xi, \alpha_k)$ , then permitted change of parameter  $\alpha_k$  tolerance is possible to express (19) as:

$$\Delta \alpha_k = \frac{1}{S(\xi, \alpha_k)} \Delta y(\xi, \alpha_k) \quad (20)$$

where  $S(\xi, \alpha_k)$  according to (16) is absolute differential sensitivity of the system to changes in the parameter  $\alpha_k$ .

The tolerance of the system response was analysed based on the transfer function of closed-loop control system according to (4) and (10).

The tolerance of the frequency-response characteristic was analysed based on the frequency response of closed-loop control system representing the system function (18).

## VI. GRAPHICAL USER INTERFACE FOR SENSITIVITY AND TOLERANCE ANALYSIS

The tool created in Matlab GUI [10] environment allows user to:

- define values of system basic physical parameters
  - $R_a, L_a, C\phi, J_M, J_L, d_{12}, b_{12}$
- select type of control system with modification of proposed controller parameters
  - PI (Naslin), PID (MOM), PID (MID), PI+PID (MOM, SOC), PI+PD (MOM, MOM)
- select the using of parasitic frequencies filter
- choose sensitivity or tolerance analysis
- select system parameters for sensitivity or tolerance analysis
- choose a system characteristic for influence of selected parameters analyse
  - step response, frequency response
- display 3D static or 2D interactive graph of influence of chosen parameter to characteristic shape

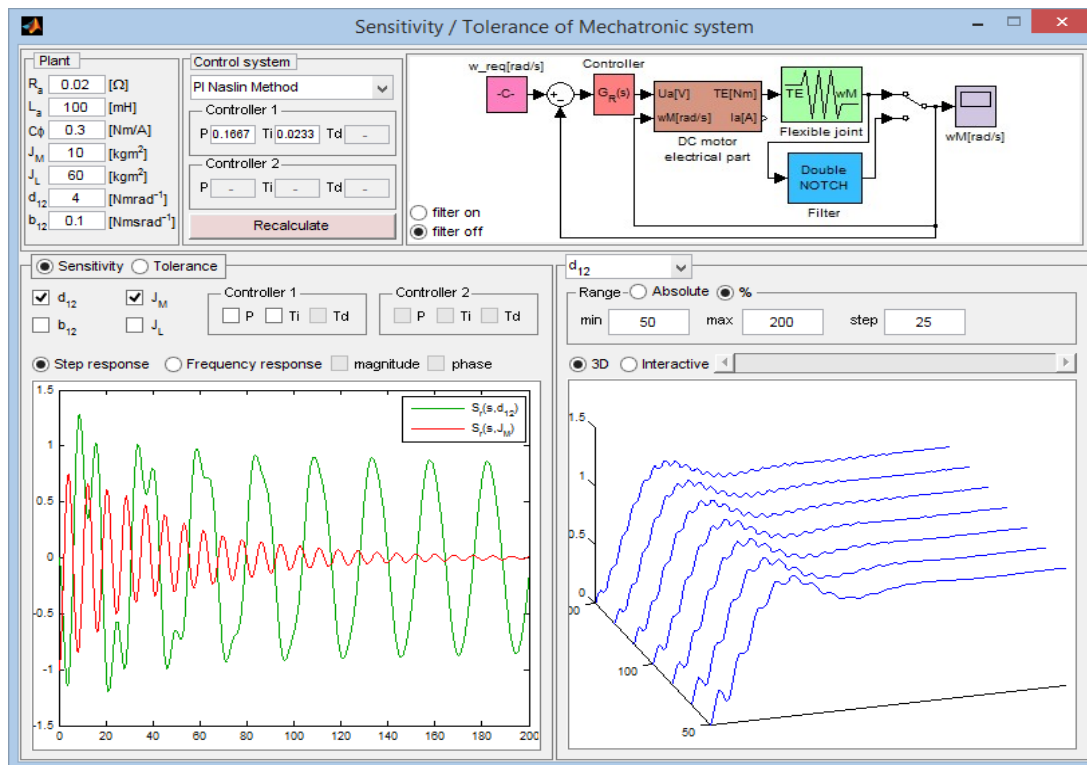


Fig. 6 GUI of the tool for an analysis of the mechatronic system sensitivity and tolerance

The Symbolic Math Toolbox [11] was used for expression of system function for sensitivity and tolerance analysis. In designed tool were utilised this toolbox functions:

- *syms* for symbolic variables definition
- *poly2sym* for transform polynomial coefficient vector to symbolic polynomial
- *subs* for symbolic variables substitution
- *diff* for expression derivation

- *eval* for expression evaluation
- *sym2poly* for symbolic-to-numeric polynomial conversion
- *numden* for numerator and denominator extraction of expression

## VII. CONCLUSION

The result of this work is the tool in Matlab GUI form that is designed to analyse a mechatronic system sensitivity and

tolerance. Designed tool allows an analysis of influence of torsion stiffness, torsion damping, inertia moment of the motor, inertia moment of the load and controllers parameters (for different methods of control system design) in terms of the impact of these parameter change on the response in the form of system step response and system frequency-response logarithmic characteristics.

#### ACKNOWLEDGMENT

This publication was written with financial support of the VEGA agency in the frame of the project 1/0463/13 „Study of flexible mechatronics system variable parameters influence on its control“.

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