

Timetabling Communities' Demands for an Effective Examination Timetabling Using Integer Linear Programming

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Abstract—This paper explains the educational timetabling problem, a type of scheduling problem that is considered as one of the most challenging problem in optimization and operational research. The university examination timetabling problem (UETP), which involves assigning a set number of exams into a set number of timeslots whilst fulfilling all required conditions, has been widely investigated. The limitation of available timeslots and resources with the increasing number of examinations are the main reasons in the difficulty of solving this problem. Dynamical change in the examination scheduling system adds up the complication particularly in coping up with the demand and new requirements by the communities. Our objective is to investigate these demands and requirements with subjects taken from Universiti Malaysia Terengganu (UMT), through questionnaires. Integer linear programming model which reflects the preferences obtained to produce an effective examination timetabling was formed.

Keywords—Demands, educational timetabling, integer linear programming, scheduling, university examination timetabling problem.

I. INTRODUCTION

SIMILAR to the other scheduling problem faced by many institutions, UETP is one of the tough assignment problems that resolve around educational sector. It differs from University Course Timetabling Problem (UCTP) [2] as UETP is scheduled to take place at final of each semester and all exam subjects are scheduled simultaneously in the limited period [9]. Additionally, each exam is only allowed to be scheduled in a limited number of available rooms prepared by the institutions. This restriction added on the complexity of finding the best feasible solution to the problem. With many conditions and restrictions applied during the scheduling process, solving it manually is almost impossible since it often time consuming and difficult tedious. It is already difficult to find the feasible solution considering the necessary constraint and condition set by institutions, if the other constraints associated with the desire of the community were to be included, the feasible solution will be certainly hard to find [17]. Due to this complexity, numerous researchers are still yet

attempting to analyze and develop the best solution to solve the problem.

In searching for the solution to this problem, scheduler tends to ignore the most important features that certify the quality of the schedule. Soft constraints, especially the constraints related to the human needs such as preferring exams in the hall over smaller classroom and requiring taking only one exam in a day are the guidelines for the construction of a high quality examination timetable. These types of preferences are often seen as insignificant requirements due to difficulty in solving it without getting infeasible solution as the result. Thus, it is understandable why the scheduler ignored these constraints. However, trying to solve as many soft constraints as possible is much better option in order to produce the best quality solution for this problem [1].

This paper presents a real world examination timetabling problem faced by UMT communities. In the next section, we describe the examination timetabling problem and the literature behind the problem. In Section III, survey method and modeling of general integer programming model that is used in this paper are presented. Section IV describes the result of the survey and newly designed model. In the last section, we summarize the contribution and conclude our research.

II. LITERATURE REVIEW

A. Examination Timetabling Problem

Generally, in educational timetabling problem, the problem is classified into three main classes; school, course and examination timetabling. School timetabling problem can be similar to the course timetabling problem as it is assigned weekly with purpose of preventing overlapping of teacher or lecturer from having conflicted schedule [20]. In this paper, the only focus is on examination timetabling problem. Examination timetabling problem is defined as follows: "The assigning of examinations to a limited number of available times periods in such a way that there are no conflicts or clashes" [12].

Solving timetabling problem is even more complicated in any institution of higher education due to the number of students, courses, and subjects at the institution especially when planning the examination timetable for all students at every end of the semester. Being different from course timetable assignment, the exams timetable is assigned with the purpose to avoid the student from having exam at consecutive

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period or having to take exams twice in a day [15].

Most institutions face the similar problem when assigning timetable for their student's examination. Some constraints must be solved without any violation and some constraints need to be solved as long as they give feasible solution to the problem. These constraints are known as hard and soft constraint. Hard constraint cannot be violated and the timetable is considered feasible when all hard constraints are completely solved. An example of hard constraint is that all exams must be scheduled to a timeslot (completeness). While soft constraint is a constraint that is not necessary to be solved, if scheduler wishes to construct the best quality timetable, it is better to solve the maximum number of soft constraint. One can say that the soft constraint is the medium to evaluate the quality of the solution for the timetable [17]. Soft constraints are the preference of communities such as desired gap or rest day between each exam. More examples on hard and soft constraint can be found in [16], [3], [8].

B. Related Work

1. Survey

Among the researches done in the literature, only few have came out with a survey research that discusses on general constraints that can be used to produce a solution or model which can be applied to solve this problem as a whole. In the other words, no survey has been conducted to identify the constraints that can be used to build one model which is applicable and compatible to all educational institutions. This is due to the difference in preference and other factors such as religions which have prevented researcher to find one fix solution to the problem. Thus, researcher has conducted a research to solve this difference by surveying the related communities to know the different constraint that needed to be satisfied as to find the feasible solution at the studied institution.

Reference [3] has conducted a survey questionnaire on the preference of University Malaysia Terengganu communities on their examination timetable. The survey inquires the communities about their preferred requirements and opinions on the criteria of a good examination timetable. The survey was distributed to the mathematics students and lectures from School of Informatics and Applied Mathematics (SIAM) in UMT. As a result, we managed to identify eight factors that affect the scheduling of examination timetable based on the communities' preference. We have also listed down the criteria of a good timetable based on the communities' preference which should be included in developing the best model to solve the examination timetabling problem in UMT. Other researchers that have conducted a survey on examination timetabling problem can be found in [11], [13], [14].

2. Modelling

Many mathematical programming and heuristic-based approaches have been proposed for solving a variety of timetabling problem. The same goes to the examination timetabling problem. However, there is still yet a model or

solution that can solve this problem generally due to the individual specification and requirements by the institutions. The need or constraint varies from one institution to another as different institution has different requirement and number of enrolment as well as different type of course offered. Therefore, the best way to construct examination timetable for an institution is by considering the needs and requirement of the individual institutes. For more information, [19] discuss on the variant of the problem.

Integer Linear Programming (LP) method is a widely used mathematical programming by the previous researchers in solving the timetabling problem. Reference [6] did a research using a mixed integer programming approach to a class timetabling problem, reference [10] has done a research on Sudoku problem using IP model, and reference [21] uses a formulation of binary integer programming for scheduling in market-driven foundries. In UETP problem, researchers used this method to generate a conflict-free exams timetable and in a decision making problem to produce the best quality timetable.

Reference [18] reports the research on the maximization of paper spread in predefined examination timetable with the purpose to increase the amount of study time between exams for each student using the integer programming model. Constraint used in this model is the constraint and requirement that can be found in most academic institutions. As example the conflict type constraint such as two exams in the same timeslot, two exams in the same day and two exams in two consecutive timeslots. The approach is then applied on the real world examination timetabling problem. Other example can be found in [7].

There are papers which focus on the general model that can be applied to solve the general timetabling problem in various areas. Reference [8] wrote an article about the general model for timetabling problem using integer programming approach. It is well known that there is still yet a model that can be used to solve timetabling problem in general. Therefore, they analyzed the basic model that is generally used in various fields that are related to each other as the base to design the model. Using the basic model, they develop a general integer programming model which is applicable in the field that they have studied and used a set of random data to test the model. The model developed is modifiable to all types of scheduling problems which have the same basic constraints. Another example of research on the same topic is [4].

III. METHODOLOGY

A. Survey Method

The present research investigates the timetabling communities in UMT. Students from eight schools in UMT are chosen randomly from various program and year of study. As the major user of the timetable, they were given the chance to express their opinions regarding to the scheduling of examinations at UMT. We will find out their views and opinions regarding to the current examination timetable at UMT using a survey questionnaire which is distributed by

hand to all respondents. Following this step was to create a timetable according to their standards and to fulfill the requirements of the university at the same time. We used the results obtained from the survey as our guidelines to understand their preferences, and thus we made a model that fits the preferences.

Questions in the survey were design by referring to the previous references mainly [11] and [1], also from interviews and own observations. Constraints listed by those references and information are reconstructed in form of question. Since this survey is conducted prior to development of a new model, only constraints that are compatible to the nature of exams in UMT chosen as question in the survey. For example, UMT has a major number of Muslim students enrolled for almost all exam subjects, thus they need to include a restriction of timeslot for those students in order to refrain from having to take exam during Jumaat prayers time which is from 12.00 p.m. to 2.00 p.m. in every Friday. This is because this time can be said as forbidden time for any unrelated activity for all Muslims and this must be respected. Thus, to avoid the conflict, a new constraint is added for UMT schedule so that no exam is scheduled during the prayers time.

B. Modelling Method

We want to develop a new mathematical model for UMT communities so that we can build a new examination timetable that is closer to the communities' preferences. Thus, we will use Binary Integer Programming Method (BIP) to develop the model and solve the problem by using the latest developed software in optimization and operational research, AIMMS software.

Binary Integer Programming (BIP) or 0-1 integer programming method is commonly used in modelling of optimization and operational research problem, especially in solving timetabling problem such as [4] and [5]. We formulate a basic model for the problem with the constraints from the survey analysis result, and the constraints will be converted into the form of equation, relationship, and mathematical formula. The following expressions are the standard formulations of IP problem:

Maximize

$$\sum_{j=1}^n c_j x_j$$

Subject to:

$$\sum_{j=1}^n c_{ij} x_j = b_i \quad (i = 1, 2, \dots, m)$$

$$x_j \geq 0 \quad (j = 1, 2, \dots, n)$$

$$x_j \text{ integer (for some or all } j = 1, 2, \dots, n)$$

where: c_j = cost coefficient, x_j = variable, c_{ij} = technological coefficient, b_i = constraints limit, m = number of constraints, n = number of variables.

From the problem above, all variables are restricted to 0 and 1 as we have decided to use binary integer programming to solve our problem. The difference between BIP and other IP

method is that if some of the variables are restricted to be integer values and some are real values, it can be a mixed integer programming or if all variables are integer, it is known as pure integer.

IV. RESULT

A. Survey Result

The result in this section shows the frequency result of respondent preference scale for the question. 36 questions were asked to the respondent, and the question is designed based on the constraints found in previous literature and they were listed based on the observation on the nature of examination in UMT and from interview with the timetabling communities. The result is described on the total number of respondent by choosing the scale that they preferred.

Based on the statistical results in Table I, we have included the constraints that are basic to the construction of UMT examination timetable. For the modelling section of this paper, we will only use the basic constraint to build a new mathematical model by using the present method.

B. Basic Constraint in UMT Systems

From our interview with the scheduler in UMT, we have identified five constraints used for designing the examination timetable in UMT. The five constraints are used as the basic constraint that is assigned first before the other changes due to the requests made by the communities. The following items are the basic constraints used by UMT's scheduler to design the examination timetable in UMT:

- 1) All exams must be scheduled and are scheduled once in the timetable (Completeness)
- 2) No students are assigning to two exams at the same time. (Conflict)
- 3) Exam with the most students must be scheduled at the earliest timeslots. (This constraint is purposed to give lecturers that teach subject with a large number of students a longer time to mark the paper so that they can finish grading before the given due date.)
- 4) Exam A must be scheduled before Exam B. (Precedence), (This constraint allows exams such as structural exam or essay to be scheduled before objective exams because structural exam needs longer time for marking and grading than objective exams)
- 5) No students are scheduled into two consecutive exams whether in timeslots or days. (Consecutiveness)

Based on the above constraints, we then formulated a mathematical model that represents each requirement. The problem of UETP in UMT consists of a set of:

1. Notation

a. Sets:

- E, Set of exams, e;
- T, Set of available timeslots, t;
- S, Set of students, s;
- E_s, Exams taken by the same students;
- E_{Large}, Exams with large number of students;
- T_{Early}, Timeslots at the earliest time of exams;

TABLE I
THE PREFERENCE OF THE COMMUNITIES

No.	Constraint	Disagree	Neutral	Agree
1.	All courses must be scheduled into examination timetable. (Completeness)	92 24.9%	70 18.9%	208 56.2%
2.	Student cannot be scheduled into two exams at the same time. (Conflict)	14 3.8%	15 4.1%	341 92.2%
3.	Same exams must be scheduled at the same timeslot.	124 33.5%	66 17.8%	180 48.6%
4.	Exam with large number of students must be scheduled early in the timetable.	33 8.9%	59 15.9%	278 75.1%
5.	Exam must be spread evenly throughout the timetable.	6 1.6%	19 5.1%	345 93.2%
6.	Special treatment should be provided for handicapped students.	13 3.5%	22 5.9%	335 90.5%
7.	The number of invigilator must suit the number of student.	34 9.2%	47 12.7%	289 78.1%
8.	Lecturers in charge of the subject are scheduled to be in charged in invigilating the exams.	18 4.9%	27 7.3%	325 87.8%
9.	Students to have only one exam in a day.	6 1.6%	16 4.3%	348 94.1%
10.	No two exams consecutively in a day.	45 12.2%	46 12.4%	279 75.4%
11.	No two exams in the same day.	90 24.3%	64 17.3%	216 58.4%
12.	No core subject exam is scheduled twice in a day.	98 26.5%	63 17%	209 56.5%
13.	No students are scheduled to exams in two days consecutively.	56 15.1%	77 20.8%	237 64.1%
14.	Student can be schedule to finish all exams late	74 20%	87 23.5%	209 56.5%
15.	Students have one-day gap (to rest) before the next exams.	26 7%	35 9.5%	309 83.5%
16.	Maximum two-day gap for students before the next exams.	27 7.3%	53 14.3%	290 78.4%
17.	Three-day gap between exams is provided.	76 20.5%	72 19.5%	222 60%
18.	Exam cannot be scheduled at 8.00 A.M.	111 30%	79 21.4%	180 48.6%
19.	Exam can be scheduled at 9.00 A.M.	50 13.5%	65 17.6%	255 68.9%
20.	No exams are scheduled during lunch hour (1-2 pm).	49 13.2%	45 12.2%	276 74.6%
21.	No student can be scheduled for a night exam from 8 pm – 10 pm.	46 12.4%	35 9.5%	289 78.1%
22.	All students must be scheduled in rotation between morning, afternoon and evening exams.	138 37.3%	94 25.4%	138 37.3%
23.	No exams can be scheduled during weekend.	133 35.9%	56 15.1%	181 48.9%
24.	During weekend, exam cannot be scheduled in Saturday only.	142 38.4%	66 17.8%	162 43.8%
25.	Exams during religious activity such as Friday prayer time should be avoided.	34 9.2%	22 5.9%	314 84.9%
26.	Students cannot be assigned to exams for any subject during holiday.	39 10.5%	28 7.9%	303 81.9%
27.	Students cannot be assigned to exams for any subject during holiday.	76 20.5%	57 15.4%	237 64.1%
28.	Exams are scheduled in a room that can only fit one course.	78 21.1%	95 25.7%	197 53.2%
29.	Same exams taken by different program	100	83	187

No.	Constraint	Disagree	Neutral	Agree
	must be scheduled in the same room.	27%	22.4%	50.5%
30.	Student can be schedule in the same room with other courses for the same exams if the room is large (DSM).	35 9.5%	49 13.2%	286 77.3%
31.	Same exams taken by different program can be scheduled in different room only if there are not enough seats.	55 14.9%	76 20.5%	239 64.6%
32.	Student should not be scheduled to a room with not enough seats.	41 11.1%	86 23.2%	243 65.7%
33.	Only exams with same length period can be schedule in the same room.	36 9.7%	54 14.6%	280 75.7%
34.	Only exam of same period is scheduled in the same room.	36 9.7%	57 15.4%	277 74.9%
35.	Some exams may require specific room to be scheduled.	39 10.5%	75 20.3%	256 69.2%
36.	Larger hall (DSM) must be assign for exams first before considering a smaller room (BK).	32 8.6%	50 13.5%	288 77.8%

- E_{Pre} , Set of exams with different features (e_a , e_b) that need to be scheduled first before another.

b. Parameters:

- $P_{e,t}$, The preference of the communities.

c. Decision Variable:

$$\sum_{t \in T} \chi_{e,t} = 1, \quad \forall e$$

$$\chi_{e,t} = \begin{cases} 1, & \text{if an exams, } e \text{ is assigned to timeslots, } t \\ 0, & \text{otherwise} \end{cases}$$

2. Objective Function

The objective function is to maximize the preference, $P_{e,t}$ assigned to the examination such that $P_{e,t}$ is the preference of communities as to have exams as they desired.

Maximize

$$Z = \sum_e \sum_t P_{e,t} X_{e,t} \quad (1)$$

Subject to:

$$\sum_t X_{e,t} = 1, \quad \forall e \quad (2)$$

$$\sum_e X_{e,t} \leq 1, \quad \forall t \quad \forall s \in S \quad (3)$$

$$\sum_e^{E_{Large}} \sum_t^{T_{Early}} X_{e,t} \leq 1, \quad \forall t \quad (4)$$

$$X_{e_a,t} - \sum_{t=t+1}^T X_{e_b,t} = 0 \quad (5)$$

$$\forall (e_a, e_b) \in E_{Pre} \quad \text{and} \quad \forall t \in \{1, 2, \dots, t-1\}$$

$$\sum_t (X_{e_i,t} + X_{e_j,t+1}) \leq 1 \quad \forall (e_i, e_j) \in E_s \quad (6)$$

$$X_{e,t} \in \{0, 1\} \quad (7)$$

Objective function (1) is to maximize the preference of communities for examination. Constraint (2) is to make sure that all exams are completely assigned to a timeslot. Next, constraint (3) is the conflict constraint that ensures that no students were to take more than one exam at the same time. Constraint (4) is assigning the exam that has large number of enrolment at the earliest timeslot of exams to allow longer marking time. For constraint (5), the precedent constraint, allows an exam to be scheduled before one another. Last but not least, constraint (6) is consecutiveness constraint which ensures that no students are assigned in two or more consecutive exams per day. Constraint (7) is the constraint that allows the decision variable $\chi_{e,t}$ to be binary (either 0 or 1).

C. Discussion

Based on our analysis on the survey, UMT communities have so many preferences that the UMT scheduler could consider during the scheduling process. This can be seen as almost every respondent agreed to the constraint asked in the survey. Especially, the constraint about students having only one exam per day and fair spreading of exams, it receives more than 90% agree rate by the respondents. This constraint may only be a soft constraint, but it can also bring a major difference in examination timetable arrangement as having only one exam in a day and equally distributed exams may help student to become more focused and to have better concentration. Although the scheduler is allowed to only include the necessary constraint during the process, the consideration of including more preferences can be appreciated by the communities.

From the modeling result, we managed to identify the five basic constraints that must be included in the timetable from the discussion with the scheduler in UMT. As mentioned, the five constraints are only the basic constraints that are used during the scheduling process before considering any other restriction. In the survey, we have identified other constraints or preferences of the communities that need to be included during the modeling process if we aim to produce a high quality timetable.

V. CONCLUSION

In this paper, we have investigated and discussed on the problem regarding to the examination timetabling as a whole and also focusing on the mathematical modeling development in UMT. Based on our observation from the results, the timetabling communities in UMT, mostly the students have their own opinion on the preferences and what is best for their examination schedule. The university itself has some conditions that need to be satisfied before considering the other options. Thus, we have managed to develop the new basic model for the university as to fulfill these conditions. The model in this paper will be the basic guidelines for our future work.

In the future, we will investigate and discuss intensely all outcomes from our survey and develop a new model that will include most communities' preference in our research. The constraint that is not yet modeled will be considered in the

future.

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