

# Time-Delay Estimation Using Cross- $\Psi_B$ -Energy Operator

Z. Saidi, A.O. Boudraa, J.C. Cexus, and S. Bourennane

**Abstract**—In this paper, a new time-delay estimation technique based on the cross  $\Psi_B$ -energy operator [5] is introduced. This quadratic energy detector measures how much a signal is present in another one. The location of the peak of the energy operator, corresponding to the maximum of interaction between the two signals, is the estimate of the delay. The method is a fully data-driven approach. The discrete version of the continuous-time form of the cross  $\Psi_B$ -energy operator, for its implementation, is presented. The effectiveness of the proposed method is demonstrated on real underwater acoustic signals arriving from targets and the results compared to the cross-correlation method.

**Keywords**—Teager-Kaiser energy operator, Cross-energy operator, Time-Delay, Underwater acoustic signals.

## I. INTRODUCTION

Time-delay (TD) estimation between signals in the presence of noise and interference is a problem of importance in areas such as communications (radio transmitter,...), sonar, radar, biomedicine and geophysics [1]-[4]. For example, in sonar signal processing the TD between the observed signals is used to estimate the sources (targets) ranges and bearings. TD is often referred to by TD estimation (TDE), time of arrival (TDOA), and time-of-arrival difference (TOAD). A common method for measuring the TD involves cross correlating the receiver outputs; an estimate of the TD is given by the argument that maximizes the cross-correlation (CC) function. The CC is linear similarity measure. However, often it is not possible to avoid nonlinearities of the sensors used. Furthermore, in practice the interaction between the signals, generated by sensors, may not be linear, and thus the maximum of CC does not, necessarily, corresponds to maximum of interaction. Consequently the resulting TD value is problematic. To tackle this problem we present a similarity method that takes into account the nonlinearity of the signals and their interaction. This method is based on cross  $\Psi_B$ -energy operator which is a nonlinear measure, recently proposed in [5]. Furthermore this operator is well suited for nonstationary signals [6].  $\Psi_B$ -energy operator is derived from a second energy-like function, called cross Teager-Kaiser energy

operator (CTKEO) [7]-[8], which measures the interaction between two real time functions.  $\Psi_B$  measures how much a signal is present in another one. Based on a nonlinear operator,  $\Psi_B$ , the method may be viewed as a nonlinear matched filtering.

In this paper we propose to use the  $\Psi_B$  method in the acoustic array processing to estimate the TD. The proposed method is compared to the experimental, considered as the standard, and cross-correlation TD estimation methods on real underwater acoustic data.

## II. $\Psi_B$ -BASED TIME DELAY ESTIMATION

### A. Cross- $\Psi_B$ -Energy Operator

Recently the CTKEO has been extended to complex-valued signals and an operator called  $\Psi_B$  introduced [5]. Let  $x$  and  $y$  be two complex signals,  $\Psi_B$  is defined as follows [5]:

$$\begin{aligned}\Psi_B(x,y) &= 0.5[\Psi_C(x,y) + \Psi_C(y,x)] \\ &= 0.5[\dot{x}^* \dot{y} + \dot{x} \dot{y}^*] - 0.25[x\ddot{y}^* + x^* \ddot{y} + y\ddot{x}^* + y^* \ddot{x}]\end{aligned}$$

where  $\Psi_C(x,y) = 0.5[\dot{x}^* \dot{y} + \dot{x} \dot{y}^*] - 0.5[x\ddot{y}^* + x^* \ddot{y}]$ . It has been shown that the cross  $\Psi_B$ -energy operator of  $x$  and  $y$  is equal to the cross-Teager energies of their real and imaginary parts [5]:

$$\Psi_B(x,y) = \Psi_B(x_r, y_r) + \Psi_B(x_i, y_i)$$

$$\Psi_B(x_k, y_k) = \dot{x}_k \dot{y}_k - 0.5[x_k \ddot{y}_k + \ddot{x}_k y_k], \quad k \in \{r, i\}$$

where  $x(t) = x_r(t) + jx_i(t)$  and  $y(t) = y_r(t) + jy_i(t)$ .

Where  $j$  denotes the imaginary unit. To discretize  $\Psi_B$ , two-sample backward difference is used. The aim is to obtain a discrete operator, noted  $\Psi_{Bd}$ , closely related to continuous version of  $\Psi_B$  and operating on discrete-time signals  $x(n)$  and  $y(n)$ . Thus, we replace  $t$  by  $nT_s$  ( $T_s$  is the sampling period),  $x(n)$  with  $x(nT_s)$  or simply  $x(n)$ ,  $\ddot{x}(t)$  with  $[x(n) - 2x(n-1) + x(n-2)]/T_s^2$  and  $\dot{x}(t)$  with  $[x(n) - x(n-1)]/T_s$ . Then, for  $k \in \{r, i\}$  we have:

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$$\Psi_B(x_k(t), y_k(t)) \mapsto x_k(n-1)y_k(n-1)/T_s^2 - 0.5[x_k(n)y_k(n-2) + y_k(n)x_k(n-2)]/T_s^2$$

$$\Psi_B(x_k(t), y_k(t)) \mapsto \Psi_{Bd}(x_k(n-1), y_k(n-1))/T_s^2$$

where  $\mapsto$  denotes the mapping from continuous to discrete. Thus from  $\Psi_B$  we obtained  $\Psi_{Bd}$  shifted by one sample to the right and scaled by  $T_s^{-2}$ . If we ignore the one-sample shift and the scaling by  $T_s^{-2}$ , one can transform  $\Psi_B(x(t), y(t))$  into  $\Psi_{Bd}(x(n), y(n))$  as follows:

$$\Psi_B(x(t), y(t)) \mapsto \Psi_{Bd}(x_r(n), y_r(n)) + \Psi_{Bd}(x_i(n), y_i(n))$$

$$\Psi_{Bd}(x_k(n), y_k(n)) = x_k(n)y_k(n) - 0.5[x_k(n+1)y_k(n-1) + x_k(n-1)y_k(n+1)]$$

It is easy to see that the two-sample forward difference gives the same result. However, three-sample symmetric difference leads to more complicated expression. Note that if  $x=y$ , the operator  $\Psi_{Bd}$  is reduced to the discrete Teager-Kaiser operator  $\Psi_{Bd}(x(n), x(n)) = x^2(n) - x(n-1)x(n+1)$ .

**B. Time-Delay Estimation Problem**

Consider the signal from a remote source being received in the presence of noise at two spatially separated receivers. The time histories of the receiver outputs, denoted by  $r_m(t)$  and  $r_k(t)$ , are given by

$$r_m(t) = s(t) + n_m(t)$$

$$r_k(t) = A.s(t - (k-m)D) + n_k(t)$$

where  $s(t)$  is the signal waveform,  $n_m(t)$  and  $n_k(t)$  are the noise waveforms at the respective receivers,  $A$  is an attenuation factor and  $D$  is the difference in the wavefront arrival times at two consecutive receivers ( $k=2, m=1$ ), TD.

**Proposition:**

Suppose that the noises  $n_m(t)$  and  $n_k(t)$ , and the signals  $r_m(t)$  and  $r_k(t)$  are mean square differentiable and mutually uncorrelated. Then

$$E[\Psi_B(r_m(t), r_k(t))] = A.E[\Psi_B(s(t), s(t-\tau))] \tag{1}$$

where  $\tau = (k-m).D$  and  $E[.]$  is the expectation operator.

**Proof:**

$n_m(t)$  and  $r_m(t)$  have zero-mean and are uncorrelated. Then,

$$E[n_m(t).n_k(t)] = 0 \Rightarrow E\left[n_m(t).\frac{dn_k(t)}{dt}\right] = -E\left[n_k(t).\frac{dn_m(t)}{dt}\right]$$

$s(t)$  and  $n_m(t)$  are independent. Then

$$E[s(t).n_k(t)] = E[s(t)].E[n_k(t)] = 0 \Rightarrow E\left[\frac{ds(t)}{dt}.\frac{dn_k(t)}{dt}\right] = -\frac{1}{2}E\left[s(t).\frac{d^2n_k(t)}{dt^2}\right] - \frac{1}{2}E\left[n_k(t).\frac{d^2s(t)}{dt^2}\right]$$

Similarly, for  $s(t-\tau)$  and  $n_m(t)$  we can write

$$E\left[\frac{ds(t)}{dt}.\frac{dn_m(t)}{dt}\right] = -\frac{1}{2}E\left[s(t-\tau).\frac{d^2n_m(t)}{dt^2}\right] - \frac{1}{2}E\left[n_m(t).\frac{d^2s(t)}{dt^2}\right]$$

where  $\frac{ds(t-\tau)}{dt} = \frac{ds(t)}{dt}$

$$\Psi_B(r_m(t), r_k(t)) = \frac{d[s(t)+n_m(t)]}{dt} \cdot \frac{d[a.s(t-\tau)+n_k(t)]}{dt} - \frac{1}{2}\left[(s(t)+n_m(t))\left(a.\frac{d^2s(t)}{dt^2} + \frac{d^2n_k(t)}{dt^2}\right) + (a.s(t-\tau)+n_k(t))\left(a.\frac{d^2s(t)}{dt^2} + \frac{d^2n_m(t)}{dt^2}\right)\right]$$

$$E[\Psi_B(r_m(t), r_k(t))] = A.E[\Psi_B(s(t), s(t-\tau))] + 2.E\left[\frac{dn_k(t)}{dt}.\frac{ds(t)}{dt}\right] + 2A.E\left[\frac{dn_m(t)}{dt}.\frac{ds(t)}{dt}\right] + 2E\left[\frac{dn_m(t)}{dt}.\frac{dn_k(t)}{dt}\right]$$

$$E\left[\frac{dn_m(t)}{dt}.\frac{dn_k(t)}{dt}\right] = E\left[\lim_{h \rightarrow 0} \frac{n_m(t+h)-n_m(t)}{h} \cdot \lim_{h' \rightarrow 0} \frac{n_k(t+h')-n_k(t)}{h'}\right]$$

$$= E\left[\lim_{h \rightarrow 0} (n_k(t+h).n_m(t+\alpha.h))\right] - E\left[\lim_{h \rightarrow 0} (n_k(t+h).n_m(t))\right]$$

$$- E\left[\lim_{h \rightarrow 0} (n_k(t).n_m(t+\alpha.h))\right] = -E[n_k(t).n_m(t)] = 0$$

where  $h' = \alpha.h$  and  $\alpha$  is a constant.

Similarly,

$$E\left[\frac{dn_m(t)}{dt}.\frac{ds(t)}{dt}\right] = E\left[\frac{dn_k(t)}{dt}.\frac{ds(t)}{dt}\right] = 0$$

and (1) results.

Let  $t_0 = (k-m).D$  be the unknown time representing the delay of the received signal. Let  $T = [T_{min}, T_{max}]$  be the possible range of values for  $t_0$ . For any given  $t_0 \in T$  the maximum of interaction between  $r_m(t)$  and  $r_k(t)$  measured with the cross  $\Psi_B$ -energy operator is given by

$$T_B = \arg \max_{t \in T} [\int_T \Psi_B(r_m(t), r_k(t))] \tag{2}$$

Thus the quadratic detector,  $\Psi_B$ , calculates the interaction between the received signal and all possible time-shifted versions of the transmitted signal and picks the largest energy interaction as the decision. The location of the peak is the estimate of the unknown parameter  $T_B = \Delta d/c$  where and  $\Delta d = d_k - d_m$  is the path length difference (Figure 1).

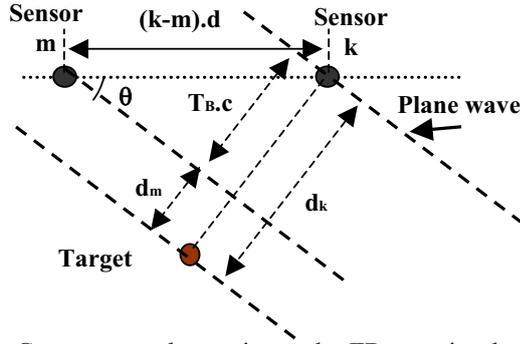


Figure 1: Geometry used to estimate the TD associated with plane waves.

### C. Pseudo-code

Both the reference signal,  $s(m)$ , and the received one,  $r(n)$ , are smoothed using Savitzky-Golay filter [9]. The complex form of the signals are obtained using Hilbert transform. The quadratic operator involves the following steps:

#### Inputs:

Reference signal:  $s(m) = s_a(m) + js_b(m)$ ,  $m \in \{1, 2, \dots, w\}$

Received signal:  $r(n) = r_a(n) + jr_b(n)$ ,  $n \in \{1, 2, \dots, N\}$

#### Output: $T_B$ (time-delay)

For  $k=1$  to  $(N-w+1)$  //  $k$  is the current window

$g^k(n-k+1) \leftarrow r(n)$ ,  $n \in F = \{k, w+k-1\}$

$\Psi_r^1 \leftarrow \Psi_{B_d}(s_a(l), g_a^k(l))$ ,  $l \in \{1, \dots, w\}$

$\Psi_i^1 \leftarrow \Psi_{B_d}(s_b(l), g_b^k(l))$

$\Psi_{B_d}(s(l), g^k(l)) \leftarrow \Psi_r^1 + \Psi_i^1$

Compute the sum  $I(k)$  of  $\Psi_{B_d}$  values over  $F$ :

$$I(k) \leftarrow \sum_{l=k}^{w+k-1} \Psi_{B_d}(s(l), g^k(l))$$

EndFor

$$T_B \leftarrow \underset{1 \leq k \leq M}{\operatorname{argmax}} [I(k)]$$

where  $\leftarrow$  denotes the affectation operation.

## III. RESULTS

TD  $\Psi_B$  method is tested on underwater acoustic signals. A linear array composed of  $n$  uniformly spaced sensors (Figure 1) is used. Each sensor received signals arriving from the sources (targets) supposed sufficiently distant from the receivers. In this case, the wavefronts are assumed to be planar. The TD,  $T_B$ , between the wave fronts impinging upon

the  $m^{\text{th}}$  and the  $k^{\text{th}}$  sensors are given by the experimental method as follows:

$$T_B = (k-m)d \cdot \sin\theta / c \quad (3)$$

where  $d$  is the distance between two consecutive receivers ( $k=2, m=1$ ),  $c$  is the celerity in the medium and  $\theta$  is the direction of arrival of a plane wave (Figure 1). TD is estimated on real acoustical data measured in a tank with a linear array of twenty sensors ( $n=20$ ) where air-full cylindrical objects are buried under the sand. Two cylinders for Data 1 and one cylinder for Data 2:  $c=1485\text{ms}^{-1}$ ,  $d=2\text{mm}$ ,  $\theta_{\text{Data1}}=22^\circ$ ,  $\theta_{\text{Data2}}=64.15^\circ$ , sample number=2000 points, sampling rate=10MHz. 2D plots, samples-sensor, of Figure 2(b) and 3(b) represent sensors output of signals (echoes) arriving from two (Data 1) and one (Data 2) cylinders respectively. Signals are smoothed using a third-order Savitzky-Golay filter over a moving window of width 51. Both the  $\Psi_B$  and the CC methods, applied to filtered signals, are implemented in the time-domain. 1D plots, amplitude-samples, of Figure 2(a) and 3(a) represent the output of the first sensor, selected as reference signal  $s(n)$ , for Data 1 and Data 2 respectively. The Experimental, CC and  $\Psi_B$  estimated time delays, for all array sensors, are plotted, as a function of the position indexes of the sensors along the array, in Figures 4(a)-(b) for Data 1 and Data 2 respectively. In each case delay estimation is performed between the first sensor of the array and the remaining ones. Root mean square error (RMSE) between pair of sensors, calculated using equation (4), for Data 1 and Data 2 is reported in Table I.

$$\text{RMSE}_{(\Psi_B - \text{Exp})} = \sqrt{\frac{\sum_{i=1}^n (TD_{\Psi_B}(i) - TD_{\text{Exp}}(i))^2}{n-1}} \quad (4)$$

As shown in Figure 4(b), for Data 2 there is a parfait agreement (except for sensor 2 where the error is one sample) between the  $\Psi_B$  and the experimental method (Eq. (3)). This is confirmed by the RMSE value (0.0526). The RMSE of the CC method is 3.42 times higher than that of the  $\Psi_B$  method. Figure 4(a) shows that, for Data 1, the estimated TD by the  $\Psi_B$  and the CC deviate moderately from TD values given by Eq. (3). Note that the  $\Psi_B$  performs slightly better than the CC with RMSE value of 0.217. For both Data 1 and Data 2, globally, the  $\Psi_B$  performs better than the CC method. This may suggest, even partially, the nonlinear relationship between the signals that linear method such as the CC cannot account for. The mismatch between expected TD values and the  $\Psi_B$  TD ones may be due to the error in the estimation of the bearing angles  $\theta_{\text{Data1}}$  and  $\theta_{\text{Data2}}$ , and the  $c$  value which depends on the temperature of the propagation media. It is important to keep in mind that there is no a golden method for evaluating the TD estimation. The method based on Eq. (3) can be used, in the far field case, as reference method if we have good measures of  $\theta$  and  $d$ .

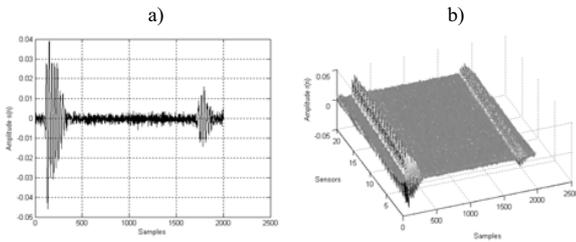


Fig. 2: Two buried targets example. (a) The selected reference signal  $s(n)$ . (b) Time-sensors representation of Data 1.

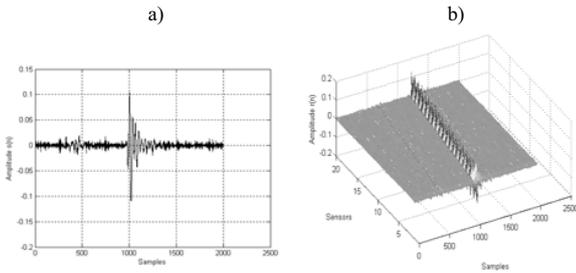


Fig. 3: Two buried targets example. (a) The selected reference signal  $s(n)$ . (b) Time-sensors representation of Data 2.

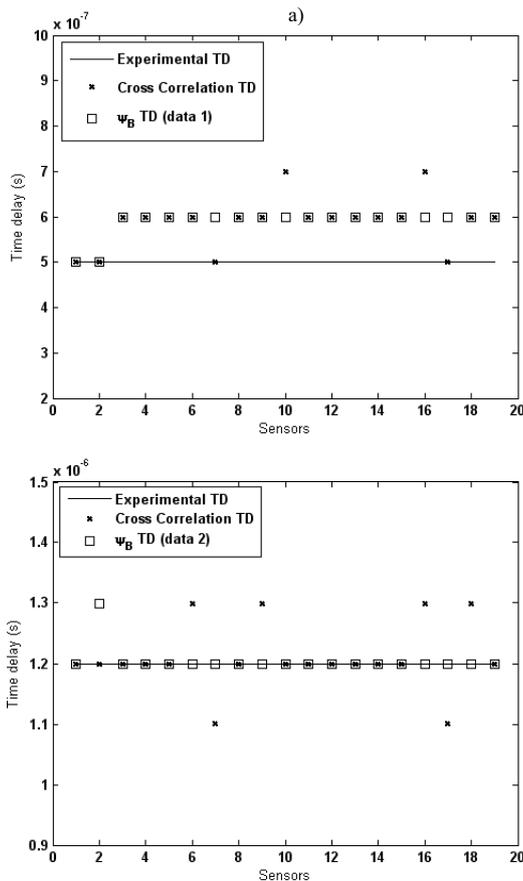


Fig. 4: TD estimations by experimental, CC and  $\Psi_B$  methods. Estimated TD versus the sensors position indexes along the

array in Data 1 (a) and Data 2 (b) respectively.

**Table I**

RMSE between  $\Psi_B$  and experimental TD, CC and experimental TD for Data 1 and Data 2.

Data 1		Data 2	
$RMSE_{(\Psi_B-Exp)}$	$RMSE_{(CC-Exp)}$	$RMSE_{(\Psi_B-Exp)}$	$RMSE_{(CC-Exp)}$
0.2170	0.250	0.0526	0.180

IV. CONCLUSION

In this paper, a new method for TD estimation, based on  $\Psi_B$  operator is introduced.  $\Psi_B$  measures how much a signal is present in another one. As the CC method, the  $\Psi_B$  is simple and very easy to implement efficiently. A discrete version of the continuous-time of  $\Psi_B$  for its implementation is presented. Presented preliminaries results on real underwater acoustic signals are very close to that of the experimental method. The obtained results show that the  $\Psi_B$  method globally performs better than the CC method. To confirm the effectiveness of the  $\Psi_B$  method must be evaluated with a large class of signals and in different experimental conditions such as high noise levels, sampling rates and sample sizes. As future work, we plan to estimate TD in the case of non uniform array. We also plan to modify the approach to tackle the problem of mutually correlated noises. It is also interesting to use TD estimated by  $\Psi_B$  in order to correct sensor gain and phase uncertainties.

ACKNOWLEDGMENT

Authors would like to thank Dr. Jean Pierre Sessarego from CNRS-LMA, Marseille, France, for making data available. We also thank Pr. Thierry Chonavel from ENST-Bretagne, France for his helpful suggestions and valuable comments.

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