Thermoelastic Damping of Inextensional Hemispherical Shell

S. Y. Choi, Y. H. Na, and J. H. Kim

***Abstract*—**In this work, thermoelastic damping effect on the hemi- spherical shells is investigated. The material is selected silicon, and heat conduction equation for thermal flow is solved to obtain the temperature profile in which bending approximation with inextensional assumption of the model. Using the temperature profile, eigen-value analysis is performed to get the natural frequencies of hemispherical shells. Effects of mode numbers, radii and radial thicknesses of the model on the natural frequencies are analyzed in detail. Furthermore, the quality factor (Q-factor) is defined, and discussed for the ring and hemispherical shell.

***Keywords*—**Thermoelastic damping, hemispherical shell, quality factor.

I. Introduction

BEAM, plate and shell structures have been adopted in the wide range of application for the advanced engineering fields. Thus, there are numerous research works up to now, and mechanical systems of a structure should include more accurate substructure with special characteristics.

Hwang[1] reported some experiments on the vibration of a hemi -spherical shell. Both axisymmetrical and asymmetrical modes were excited, and the results were compared with the analytical results. Chung and Lee[2] considered the vibration analysis of a nearly axisym -metric shell structure using a new finite ring element, and developed a FEM program to analyze a Korean bell as an example. Saunders and Paslan[3] studied the inextensional vibration of a sphere-cone shell combination, and compared the theoretical results and experimental data of the natural frequencies. Lee et al. [4] analyzed the free vibration of jointed thin cylindrical-spherical shell structures applying Rayleigh -Ritz method, and then the analytical results were compared with those of the modal test and FEM results.

In order to reduce the vibration amplitude of a structure, the problem of dissipation energy is an important feature. Generally, the damping in metal structures is low relative to the non-metal structures. Berthelot et al.[5] deeply summarized damping analysis of composite materials and structures. Also, Sefrani and Berthelot [6] researched the temperature effect on the damping properties of unidirectional glass fiber composites experimentally. Furthermore, Ganesan and Kadoli[7] studied the linear thermoelastic buckling and free vibration of geometrically perfect hemispherical shells with cutout using semi-analytical method.

For the advanced modeling of structures, there have been a lot of research works on Quality factor(Q-factor) defined as the rate for the kinetic and potential energy to some other form of converted form of irrecoverable energy by various damping mechanisms of micro -structures for thermoelastic damping of the structures. Firstly, Zener [8] presented the analytic and approximate form of Q-factor for the homogeneous, isotropic and uniform beams based on some additional assumptions, obtained a result for the important of the fluctuations of temperature in a vibrating beam. Lifshitz and Roukes [9] studied the refinement of Zener's previous work for thin beams. Using the equations of linear thermoelasticity, the process of fundamental dissipation mechanism in micro- and nanomechanical systems was examined. Using the flexural vibrating beam model, Duwel et al. [10] compared theoretical value of Q-factor to experimental result. Khisaeva and Ostoja-Starzewski [11] examined the damping in micro- / nano-beams considering the finite speed of heat transfer by hyperbolic heat conduction equation. Wong et al. [12] applied the Zener's theory to thin silicon rings, and obtained the theoretical expression of Q-factor, and the theoretical and experimental results show almost equal for practical size of the model. Applying a finite element method, Yi [13] studied the geometric effects on thermoelastic damping in MEMS resonators. To obtain the linear eigenvalue equation, perturbation forms of the temperature and displacements are used. Additionally, using the Fourier reduction method, the order of the problem and computational time can be reduced. Nayfeh and Younis [14] acquired the analytical form of quality factor for the damping with rectangular microplates. Perturbation method was also used to obtain the solution. Meanwhile, Lu et al. [15] presented approximate form of quality factor for the damping in a cylindrical thin shell. The general thermoelastic coupled equations are simplified by using Donnell-Mushtari-Vlasov appr- oaches and approximately solved by Galerkin's method.

In this paper, Q-factor of hemispherical shell is investigated in detail. The thermal expansion only in circumferential direction is considered as an assumption. Then, the temperature

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profile obtained from the heat conduction equation is used. Finally, the Rayleigh energy method is used for obtaining natural frequencies and Q-factors.

II. Formulations

Fig. 1 shows a typical hemispherical thin shell model, where u, v and w denote the displacements in the directions of ϕ, θ and r, respectively.

A. Energy Expressions

The kinetic energy for thin hemispherical shell is expressed as [16]:

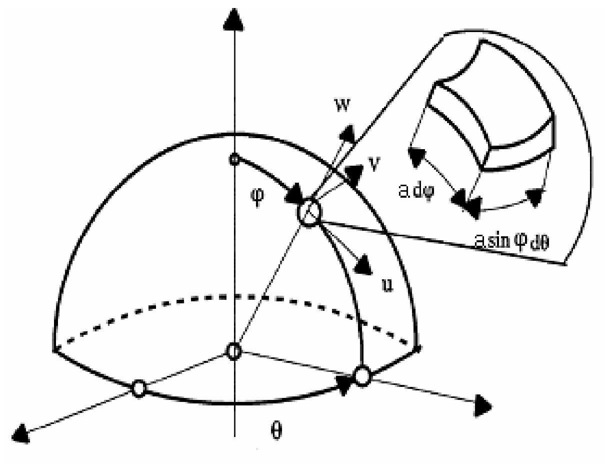

Fig. 1 Hemispherical shell model

$$\mathrm{K} = \frac{1}{2} a\rho h \int_0^{\frac{\pi}{2}} \int_0^{2\pi} (\dot{u}^2 + \dot{v}^2 + \dot{w}^2) \sin\phi \, d\theta \, d\phi \tag{1}$$

The strain energy is expressed as:

$$U = \frac{1}{2} a \int_0^{\frac{\pi}{2}} \int_0^{2\pi} \int_{-\frac{h}{2}}^{\frac{h}{2}} [\sigma_{11}\varepsilon_{11} + \sigma_{22}\varepsilon_{22} + \sigma_{12}\varepsilon_{12}] \sin\phi \, dr \, d\theta \, d\phi \tag{2}$$

where T is the change in the temperature from the ambient temperature T_a. Further σ, ε, h and α are stress, strain, thickness of the shell and the coefficient of thermal expansion, respectively.

The strain of the shell is expressed as:

$$\varepsilon_{\alpha\beta} = \varepsilon^0_{\alpha\beta} + r\kappa_{\alpha\beta} \qquad \text{for} \qquad \alpha, \beta = 1, 2 \tag{3}$$

where ε_0, κ and r stand for membrane strain, curvature of the shell middle surface and local coordinate in radial direction.

Based on inextensional theory of shell, the membrane strains are assumed to be neglected in Eq. (3).

The stress-strain relationship is expressed, and thermal effect is assumed to be considered in the circumferential direction only.

$$\begin{aligned}
\sigma_{11} &= \frac{E}{1-\mu^2}(\varepsilon_{11} + \mu\varepsilon_{22}) \\
\sigma_{22} &= \frac{E}{1-\mu^2}(\varepsilon_{22} + \mu\varepsilon_{11}) - \frac{E\alpha T}{1-\mu} \\
\sigma_{12} &= \frac{E}{2(1+\mu)}\varepsilon_{12}
\end{aligned} \tag{4}$$

Substituting Eq. (3) and (4) to Eq. (2), the strain energy can be expressed as:

$$\begin{aligned}
U &= \frac{1}{2}\frac{Ea}{1-\mu^2} \iiint \{\kappa_{11}^2 + \kappa_{22}^2 + 2\mu\kappa_{11}\kappa_{22} \\
&+ \frac{1}{2}(1-\mu)\kappa_{12}^2\} r^2 \sin\phi \, dr \, d\theta \, d\phi \\
&+ \frac{Ea}{2(1-\mu)} \iiint \varepsilon_{22}\alpha T \sin\phi \, dr \, d\theta \, d\phi
\end{aligned} \tag{5}$$

where κ, μ, E and r are the curvature of the shell middle surface, Poisson's ratio, the Young's modulus and radius of the shell, res- pectively.

For the hemispherical shell, the displacement components u, v and w are assumed as [3]

$$\begin{aligned}
u &= -Da \sin\phi \tan^n \frac{\phi}{2} \sin(n\theta) e^{i\varpi t} \\
v &= Da \sin\phi \tan^n \frac{\phi}{2} \cos(n\theta) e^{i\varpi t} \\
w &= Da(n + \cos\phi) \tan^n \frac{\phi}{2} \cos(n\theta) e^{i\varpi t}
\end{aligned} \tag{6}$$

where n denotes vibration mode number.

The curvature of the shell is expressed as [4]

$$\begin{aligned}
\kappa_{11} &= \frac{1}{a^2}(u_\phi - w_{\phi\phi}) \\
\kappa_{22} &= \frac{1}{a^2}\left[\frac{v_\theta}{\sin\phi} - \frac{w_{\theta\theta}}{\sin^2\phi} + \cot\phi(u - w_\phi)\right] \\
\kappa_{12} &= \frac{1}{a^2}\left[\sin\phi \frac{\partial}{\partial\phi}\left(\frac{v}{\sin\phi} - \frac{w_\theta}{\sin^2\phi}\right) + \frac{u_\theta - w_{\phi\theta}}{\sin\phi}\right]
\end{aligned} \tag{7}$$

where subscripts denote the partial differentiations with respect to the corresponding variables.

Due to the inextensional assumption of the mode, the mode shapes satisfy following relationship.

$$\kappa_{11} + \kappa_{22} = 0 \tag{8}$$

Considering the Eq. (8) into Eq. (5) gives simple expression for the strain energy as,

$$\begin{aligned}
U &= \frac{Ea}{2(1-\mu^2)} \iiint (1-\mu)(\kappa_{12}^2 - \kappa_{11}\kappa_{22}) r^2 \sin\phi \, dr \, d\theta \, d\phi \\
&+ \frac{Ea}{2(1-\mu)} \iiint \varepsilon_{22}\alpha T \sin\phi \, dr \, d\theta \, d\phi
\end{aligned} \tag{9}$$

The maximum kinetic and strain energy can be obtained by substituting Eq. (6) into Eq. (1) and Eq. (9).

In Eq. (9), natural frequency can be obtained directly by using Rayleigh method.

The Lagrange function of the structure is:

$$L = U_{\max} - K_{\max} \tag{10}$$

The Rayleigh's method is applied to Eq. (10) and following relation is obtained.

$$\frac{\partial L}{\partial D} = 0 \tag{11}$$

The natural frequency of the shell can be determined by solving Eq. (11)

B. Thermal Effect

To obtain the temperature profile, the heat conduction equation is used as [12]:

$$\frac{\partial T}{\partial t} - \chi \nabla^2 T = -\frac{E\alpha T_a}{C_v(1-2\zeta)}\frac{\partial \varepsilon}{\partial t} \tag{12}$$

where ∇^2, χ, C_v and T_a denote the Laplacian operator, the thermal diffusivity of the material, the heat capacity per unit volume and the ambient temperature in Kelvin, respectively.

While ε stands for the dilatation as:

$$\varepsilon = \varepsilon_{rr} + \varepsilon_{11} + \varepsilon_{22} \tag{13}$$

Based on the assumption used in Eq. (4), the dilatation can be reduced as:

$$\varepsilon = \varepsilon_{22} \tag{14}$$

Also, Eq. (12) can be lineralized as in Ref. [12]

$$\frac{\partial^2 T}{\partial r^2} - \frac{1}{\chi}\frac{\partial T}{\partial t} = \frac{1}{\chi}\frac{\Delta_E^2}{\alpha^2}\frac{\partial}{\partial t}\varepsilon \tag{15}$$

where Δ_E is the "relaxation strength" of the Young's modulus defined by:

$$\Delta_E = \frac{E\alpha^2 T_a}{C_v} \tag{16}$$

The solution of Eq. (15) can be expressed as:

$$T = T_0 e^{i\omega t}$$

$$T_0 = \frac{\Delta_E}{\alpha}\left\{\frac{D^2 n(n^2-1)\sin(n\theta)\tan^n\left(\frac{\phi}{2}\right)}{a\sin^2\phi}\right\}\left\{r - \frac{\sin(kr)}{k\cos\left(\frac{kh}{2}\right)}\right\} \tag{17}$$

where k is complex value as:

$$k = (1-i)\sqrt{\frac{\omega_{iso}}{2\chi}} \tag{18}$$

In Eq. (18), k is function of isothermal natural frequency (ω_{iso}), and ω_{iso} is assumed to be constant for simplicity as in Ref. [12]

The natural frequencies of the shell are determined by substituting Eq. (17) into Eq. (9) and applying Rayleigh's energy method.

C. Quality Factor of Hemispherical Shell

Substituting Eq. (17) into Eq. (9) yields the thermal strain energy expression. Further applying Rayleigh method gives complex natural frequency of the shell.

Then, the Q-factor can be obtained by definition as:

$$Q^{-1} = 2\left|\frac{\mathrm{Im}(\omega)}{\mathrm{Re}(\omega)}\right| \tag{19}$$

III. Numerical Results and Discussions

In the first section, isothermal natural frequency of the shell is compared with previous results and Q-factor of the circular ring is treated. Second, Q-factor of the shell is considered in detail. Lastly, Q-factors for circular ring and hemispherical shell are compared.

A. Verifications

Isothermal natural frequency can be obtained by applying Eq. (10) and (11) to Eq. (9) neglecting thermal effect as follows:

$$\omega^2 = \frac{n^2(n^2-1)^2 h^2 E}{3(1+\mu)\rho a^4}\frac{\int_0^{\frac{\pi}{2}}\sin^{-3}\phi\tan^{2n}\frac{\phi}{2}d\phi}{\int_0^{\frac{\pi}{2}}\{(n+\cos\phi)^2 + 2\sin^2\phi\}\sin\phi d\phi} \tag{20}$$

This equation can be solved by numerical calculations, then the results are compared with Ref. [17], and the data are almost equal to each other as shown in Fig. 2.

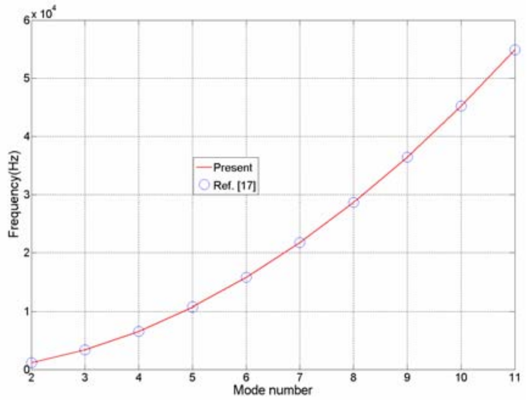

Fig. 2 Isothermal natural frequency of hemispherical shell

Next, Q-factor of circular ring model is considered. Same procedures are applied from Eq. (1) to Eq. (20) with setting $\phi = \dfrac{\pi}{2}$ and neglecting any derivatives with respect to ϕ.

Then results can be expressed as:

$$\omega = \omega_{iso}\sqrt{1+\Delta_E[1+f(\omega)]} \tag{21}$$

where the complex function $f(\omega)$ is given by:

$$f(\omega)=\left[\frac{24}{h^3k^3}\frac{kh}{2}-\tan\left(\frac{hk}{2}\right)\right] \tag{22}$$

Eq. (21) is equal to the result in Ref. [12].

Fig. 3 shows Q-factor of the ring and is exactly same as in Ref. [12].

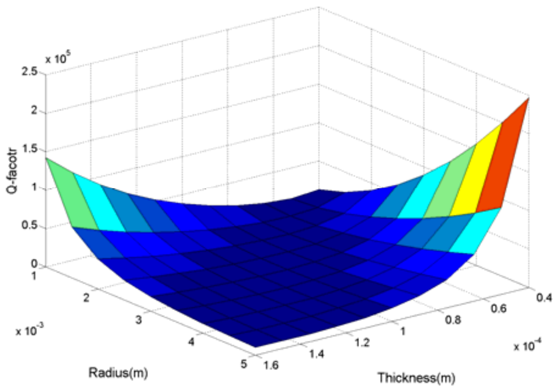

Fig. 3 Q-factor of circular ring for n = 2 mode

B. *Variation of Q-factor with Shell Dimensions*

In this section, the relationships between Q-factor and shell dime –nsions are considered. The Q-factor of the hemispherical shell can be expressed as a function of radius a, radial thickness h and mode number n.

TABLE I
MECHANICAL AND THERMAL PROPERTIES OF SILICON [12]

Symbol	Quantity	
E	Yong's modulus	165Pa
ρ	Density	2330 kg m^{-3}
α	Thermal expansion coefficient	2.6×10^{-6} K^{-1}
C_v	Heat capacity per unit volume	1.64×10^{6} J $m^{-3}K^{-1}$
χ	Thermal diffusivity	$8.6 \times 10^{-5} m^2 s^{-2}$

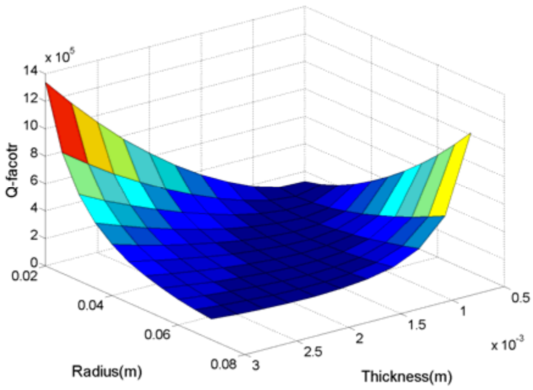

Fig. 4 Q-factor of hemispherical shell for n = 2 mode

In general, the temperature changes associated with vibration are known to be small, it is reasonable to assume the material properties remain constant [12].

Fig. 4 presents Q-factor for n = 2 vibration mode of the shell. The tendency of Q-factor is strongly dependent on the shell dimensions, and separated to two regions of relatively high and low Q-factor.

The regions for larger Q-factor are corresponding to hemispherical shells with (i) larger radius and smaller thickness and (ii) smaller radius and larger thickness.

As stated in Ref. [12], Q-factor value can be determined by proximity of the natural frequency to the maximum damping frequency.

Figs. 5-7 shows the variation of Q-factor for higher modes as n = 3, 4 and 5. With same dimensions, higher modes of vibration show higher Q-factors.

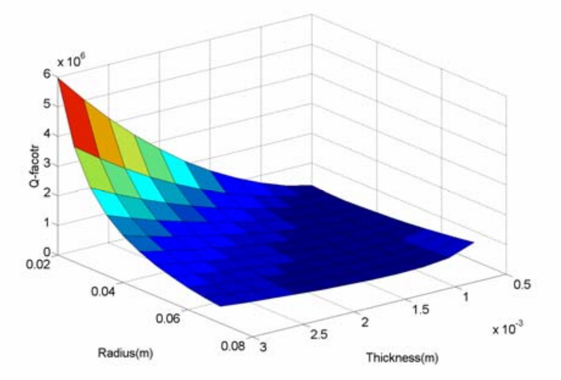

Fig. 5 Variation of Q-factor for n = 3 mode

The patterns of Q-factor dependence on the shell dimensions are similar, however the larger radius and smaller thickness region shifts toward back right-hand corner.

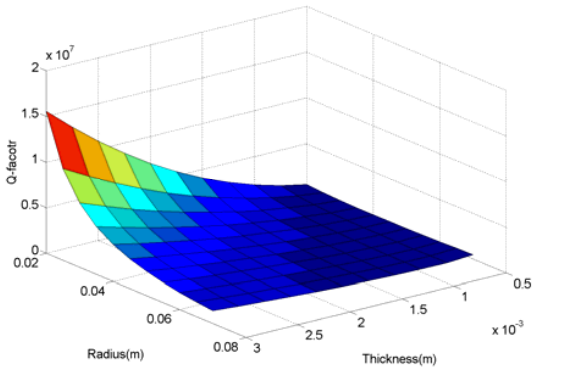

Fig. 6 Variation of Q-factor for n = 4 mode

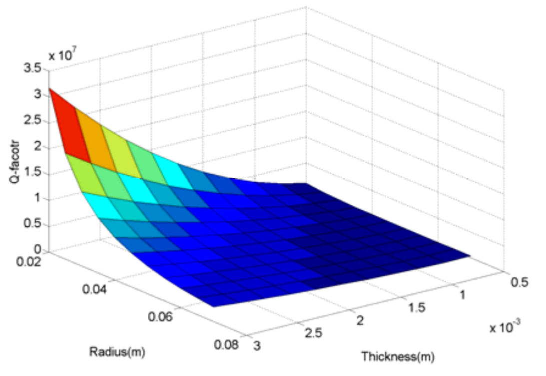

Fig. 7 Variation of Q-factor for n = 5 mode

C. Comparative Study on Q-factors

Figs. 8-9 present percentage differences of Q-factor between circular ring and hemispherical shell.

The percentage difference is defined by:

$$Q_{difference} = \frac{(Q_{Hemisphere} - Q_{Ring})}{Q_{Hemisphere}} \times 100(\%) \tag{23}$$

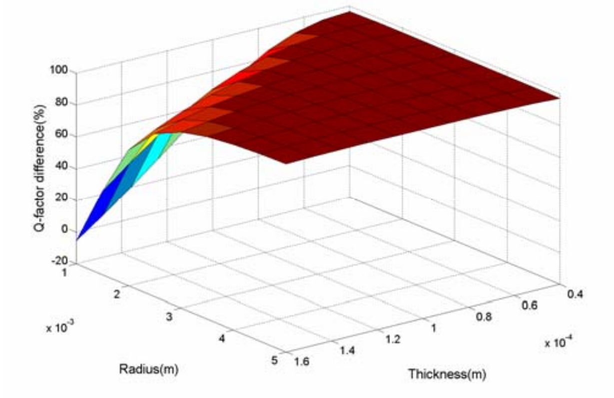

Fig. 8 Difference of Q-factors for n = 2 mode

Fig. 8 shows the difference of Q-factors with dimensions from 1 to 5 mm radius and from 40 to 160 μm. With these dimensions, Q-factor of hemispherical shell is larger within almost all regions.

Fig. 9 is for dimensions from 20mm to 60mm radius and from 0.5 mm to 3mm thickness. In this case the results can be separated to two regions. One of them shows positive percentage value and other shows negative value. Q-factor of hemispherical shell doesn't always present larger value than of circular ring.

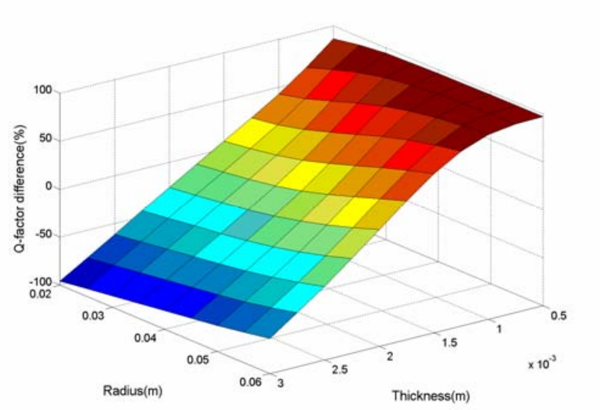

Fig. 9 Difference of Q-factors for n = 2 vibration mode

Thus, geometry of the structure seems to affects the magnitude of Q-factor, also the shape of the Q-factor diagram.

Furthermore, for hemispherical shell and circular ring, the natural frequency is the function of radius, thickness and mode number. The different form of expression for natural frequency corresponding to different geometry results varied shape of Q-factor diagrams.

IV. Conclusion

Analysis of thermoelastic damping and calculating of Q-factor is studied by using Rayleigh energy method. The procedure can be verified by comparing the results with circular ring case.

The dependence of Q-factor variation through the shell dimensions is investigated, and there are regions with higher Q-factor and lower Q-factor. For the natural frequency is the function of radius and thickness of the structure, dimensional property can be a important factor for determining Q-factor Alto the difference of structure geometry can results varied tendency of Q-factor diagram.

Observing the form of expression for the natural frequency of the shell, the further study on effects of material properties is needed.

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