

Thermal Elastic Stress Analysis of Steel Fiber Reinforced Aluminum Composites

M. R. Haboğlu, A. Kurşun, Ş. Aksoy, H. Aykul, N. B. Bektaş

Abstract—Athermal elastic stress analysis of steel fiber reinforced aluminum laminated composite plate is investigated. Four sides of the composite plate are clamped and subjected to a uniform temperature load. The analysis is performed both analytically and numerically. Laminated composite is manufactured via hot pressing method. The investigation of the effects of the orientation angle is provided. Different orientation angles are used such as $[0^\circ/90^\circ]_s$, $[30^\circ/-30^\circ]_s$, $[45^\circ/-45^\circ]_s$, and $[60^\circ/-60^\circ]_s$. The analytical solution is obtained via classical laminated composite theory and the numerical solution is obtained by applying finite element method via ANSYS.

Keywords—Laminated Composites, Thermo Elastic Stress, Finite Element Method.

I. INTRODUCTION

TWO or more materials combine to form a composite material which has an enhanced mechanical properties. Composite materials are widely used in many applications such as marine and aerospace industry. Especially for the need of high strength at low weights, composite materials are used. A composite material consists of two main material types: (i) A reinforcement material which is generally in fiber form (e.g. e-glass fibers, carbon-fibers and metal fibers) and (ii) a matrix material (e.g. polyester, epoxy and metal matrix) which provides bounds between the reinforcement fibers. Analyzing the mechanical properties of composite materials is crucial because, the mechanics of composite materials cannot be standardized due to the variations in production processes. Therefore, Kanoute et al. claimed that the mechanical properties of composite materials cannot be analyzed by determining the properties of the compounds separately [1]. They suggested to assume the composite as a homogenous material which can be analyzed as a whole.

Metal matrix composite (MMC) is a composite material which contains metal as the matrix compound. The matrix (e.g. aluminum) is a ductile material and has higher elasticity when compared to the reinforcement material which has also ductile material properties [2]. The reinforcement (e.g. steel fiber) material has higher strength and toughness when compared to the matrix material. An MMC such as steel fiber and aluminum matrix composite higher mechanical properties than aluminum and lighter than steel. Thus, its usage in many

applications endures.

There are several studies regarding the mechanical properties of metal matrix composites. Sayman investigated the elastic-plastic stress analysis of stainless steel fiber reinforced aluminum metal composites by finite element technique [2]. His study resulted that, metal fiber reinforced metal matrix composites show higher mechanical properties when compared to the other reinforcement materials. Meijer et al. also applied finite element analysis on metal matrix composites [3]. They concluded that mechanical response of the composite material strongly depends on the inclusion geometry. When materials that have sharp corners are used, the stress/strain areas are localized at specific positions meaning that soft corners lead to more homogenous stress distributions. In addition to that, Arnold et al. studied the effects of fiber architecture (shape and distribution) to the elastic and inelastic response of the MMCs [4]. They used different fiber arrangements such as square, hexagonal, rectangular or random array while using different fiber shapes such as circular, square or cross-shaped. They concluded that MMCs with square arranged fibers give higher mechanical properties when compared to the hexagonal arranged fibers. They also investigated the effect of fiber volume fraction (Vf) on the stress/strain behavior of the material and resulted in that the higher the fiber volume fraction, the better the stress/strain behavior.

Steel fiber reinforcements with aluminum metal matrix applications are highly used in the literature [2]-[7]. Sayman and Aksoy applied a hole to a stainless steel fiber reinforced aluminum metal matrix composite [5]. They claimed that the maximum residual stresses are obtained on the upper and lower surfaces. The circular hole causes the expansion of the plastic region in the direction fibers around it.

The temperature and orientation angles are important on the mechanical properties of MMCs. Sayman and Özer studied the mechanical properties of steel fiber reinforced aluminum matrix composites by applying a parabolic temperature distribution along the specimen in order to monitor the effect of the temperature [6]. They performed their experiments with 0 and 90 degrees of orientation angles and claimed that plastic strain is maximum when the orientation angle is 0 degree. The specimen was a cantilever beam which is fixed at both ends. In conclusion, the residual stresses increased as the stiffness of the material was increased. Additionally, Sayman obtained an analytical solution to see the effect of temperature [7]. He pointed out the importance of thermal stresses due to their potential of causing plastic yielding or material failure. Another conclusion was that the residual stresses at the upper

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and lower surfaces are maximum along the z direction. Finally he claimed that increasing the stiffness causes more stress production. Liu et al. also concluded that the elasto-plastic behavior of MMCs and affected by the thermal residual stresses [8]. Another study showed that when matrix materials with temperature dependent elastic modulus and proportional limit are used, there is a slight change in the stress/strain curve [3].

There are other studies in the literature that used different types of matrix materials instead of metal. For example Sayman and Çallıoğlu used thermoplastic composite beams which have steel fiber reinforcements with thermoplastic matrix material [9]. However, Sayman claimed in his study that metal fiber reinforced metal matrix composites show higher mechanical properties than thermoplastic composites [2].

In this study, mechanical properties of steel fiber reinforced aluminum metal matrix composites with varying orientation angles are analyzed both analytically and numerically.

II. STRESS ANALYSIS

The thermal behavior of a unidirectional composite layer can be represented by two thermal expansion coefficients which are “ α_1 ” and “ α_2 ” as shown in Fig. 1. Those coefficients are related to the geometrical and physical properties of the parts that form the composite layer.

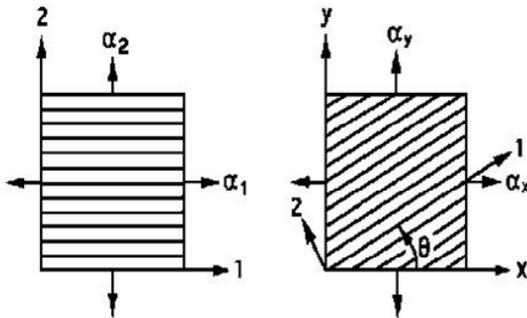


Fig. 1 Schematic of the thermal expansion coefficients

Equation (1) represents the thermal expansion coefficient of a continuous fiber reinforced composite layer.

$$\alpha_1 = \frac{E_f \alpha_f V_f + E_m \alpha_m V_m}{E_f V_f + E_m V_m} \quad (1)$$

which can also be written in the form;

$$\alpha_1 = \frac{(E\alpha)_1}{E_1} \quad (2)$$

where;

E_f, E_m : Elastic modulus of the fiber and the matrix,

α_f, α_m : Thermal expansion coefficients of the fiber and

the matrix,

V_f, V_m : Volumes of the fiber and the matrix

and $(E\alpha)_1$ represents the numerator of the (1).

The equation below gives the Poisson's ratio of the composite layer which can be found by adding the Poisson's ratios of the fiber and the matrix based on their volumetric ratios.

$$\nu_{12} = \nu_f V_f + \nu_m V_m \quad (3)$$

where ν_f, ν_m and ν_{12} represent the Poisson's ratios of the fiber, matrix and the composite layer respectively.

If a reference temperature T_0 is taken as the initial temperature, the temperature difference ΔT is $T - T_0$. This temperature difference causes thermal deformation on the composite layer. If we assume that thermal expansion coefficients are constant and take the thermal deformations separately, the thermal deformation of the composite layer is as shown in Fig. 2 and (4)-(6).

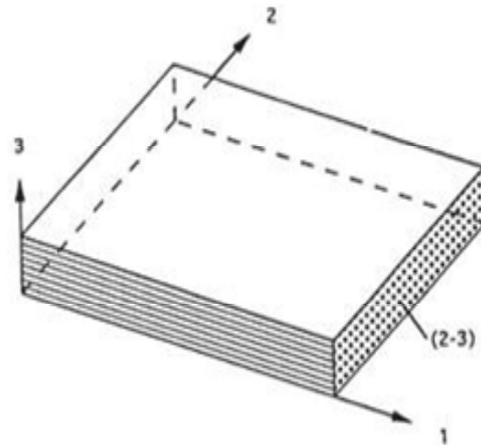


Fig. 2 Thermal deformation on the composite layer

$$\epsilon_1^T = \alpha_1 \Delta T \quad (4)$$

$$\epsilon_2^T = \alpha_2 \Delta T \quad (5)$$

$$\epsilon_{12}^T = 0 \quad (6)$$

The composite plate consisting of reinforcement materials having the orientation angles of $[90 / -90 / -90 / 90]$, $[30 / -30 / -30 / 30]$, $[45 / -45 / -45 / 45]$ and $[60 / -60 / -60 / 60]$ is fixed at all the edges and subjected to a constant ΔT temperature load along the thickness profile as shown in Fig. 3.

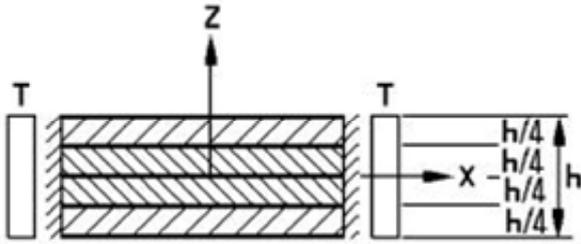


Fig. 3 Temperature load on the composite layer which is fixed at all the edges

Taking N_x , N_y , N_{xy} as resultant forces, M_x , M_y , M_{xy} as resultant moments for unit length and $[A]$, $[B]$ and $[D]$ as the layer rigidity matrices, according to the classical shell theorem, the stress/strain equations can be written as follows:

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = [A] \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + [B] \begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix} - [T^*] \Delta T \quad (7)$$

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = [B] \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + [D] \begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix} - [T^{**}] \Delta T \quad (8)$$

where T^* and T^{**} are thermal load and moment components. Because of the boundary conditions and symmetric structure of the layers, moment components are equal to "0". Therefore, the elements forming the $[T^*]$ matrix are equal to "0" as well. For that $[T^*]$ matrix can be written as follows:

$$[T^*]T = \int \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{21} & Q_{22} & Q_{26} \\ Q_{16} & Q_{26} & Q_{66} \end{bmatrix}_j \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix}_j T dz \quad (9)$$

Taking α_1 and α_2 as the axial thermal expansion coefficients of the components and θ as the orientation angle, the directional coefficients of the layer are indicated in (10)-(14).

$$\alpha_x = \alpha_1 m^2 + \alpha_2 n^2 \quad (10)$$

$$\alpha_y = \alpha_1 n^2 + \alpha_2 m^2 \quad (11)$$

$$\alpha_{xy} = 2mn(\alpha_1 - \alpha_2) \quad (12)$$

where,

$$m = \cos \theta \quad (13)$$

$$n = \sin \theta \quad (14)$$

Because of the symmetry, there is no trajectory which also

means that k_x , k_y and k_{xy} are equal to "0" in (6), (7). Thus, those equations can be written in the form of (15).

$$\begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} - \begin{Bmatrix} T_1^* \\ T_2^* \\ T_3^* \end{Bmatrix} \Delta T = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \quad (15)$$

For the derivation of the strain vector from (15), following equations can be used:

$$\varepsilon_x^0 = \frac{R_2 U_2 - R_3 U_1}{R_2^2 - R_1 R_3} \Delta T \quad (16)$$

$$\varepsilon_y^0 = \frac{R_2 U_1 - R_1 U_2}{R_2^2 - R_1 R_3} \Delta T \quad (17)$$

$$\gamma_{xy}^0 = -\frac{-T_3^* \Delta T + A_{16} \varepsilon_x^0 + A_{26} \varepsilon_y^0}{A_{66}} \quad (18)$$

where,

$$R_1 = A_{11} A_{66} - A_{16}^2 \quad (19)$$

$$R_2 = A_{12} A_{66} - A_{16} A_{26} \quad (20)$$

$$R_3 = A_{22} A_{66} - A_{26}^2 \quad (21)$$

$$U_1 = A_{16} T_3^* + A_{66} T_1^* \quad (22)$$

$$U_2 = -A_{26} T_3^* + A_{66} T_2^* \quad (23)$$

The stress components of each layer can be expressed by using the strain equations.

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix}_j = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}_j \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}_j \quad (24)$$

Here, term "j" represents the number of layers that the composite material has and the components of the matrix are indicated below.

$$\bar{Q}_{11} = m^4 Q_{11} + n^4 Q_{22} + 2m^2 n^2 Q_{12} + 4m^2 n^2 Q_{66} \quad (25)$$

$$\bar{Q}_{22} = n^4 Q_{11} + m^4 Q_{22} + 2m^2 n^2 Q_{12} + 4m^2 n^2 Q_{66} \quad (26)$$

$$\bar{Q}_{12} = m^2 n^2 Q_{11} + m^2 n^2 Q_{22} + (m^4 + n^4) Q_{12} - 4m^2 n^2 Q_{66} \quad (27)$$

$$\bar{Q}_{16} = m^3 n Q_{11} - m n^3 Q_{22} + (m n^3 - m^3 n) Q_{12} + 2(m n^3 - m^3 n) Q_{66} \quad (28)$$

$$\bar{Q}_{26} = mn^3Q_{11} - m^3nQ_{22} + (m^3n - mn^3)Q_{12} + 2(m^3n - mn^3)Q_{66} \quad (29)$$

$$\bar{Q}_{66} = m^2n^2Q_{11} + m^2n^2Q_{22} - 2m^2n^2Q_{12} + (m^2 - n^2)^2Q_{66} \quad (30)$$

Rigidity and Poisson's ratio are expressed in (31)-(33).

$$\nu_{21} = \nu_{12} \frac{E_2}{E_1} \quad (31)$$

$$Q_{11}^j = \frac{E_1^j}{1 - \nu_{12}^j \nu_{21}^j} \quad (32)$$

$$Q_{12}^j = \frac{\nu_{21}^j E_1^j}{1 - \nu_{12}^j \nu_{21}^j} = \frac{\nu_{12}^j E_2^j}{1 - \nu_{12}^j \nu_{21}^j} \quad (33)$$

$$Q_{22}^j = \frac{E_2^j}{1 - \nu_{12}^j \nu_{21}^j} \quad (34)$$

$$Q_{66}^j = G_{12}^j \text{ and } Q_{16}^j = Q_{26}^j = 0 \quad (35)$$

The yield strength of the material to the thermal stresses can be expressed by finding the stress components via (36).

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2 - n^2 \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} \quad (36)$$

III. RESULTS AND DISCUSSION

A thermo-elastic analysis is carried out both analytically and numerically. Analytical solution is expressed in the previous section. The composite materials are subjected to a thermal load while they are simply supported at all the edges.

The numerical solution is performed by using ANSYS. The material properties, temperatures and fixed edges are given as inputs to ANSYS. Thermal stress values resulting from both numerical and analytical solutions for different orientation angles are given in Tables I-III.

TABLE I
ANALYTICAL AND NUMERICAL STRESS VALUES IN THE X DIRECTION FOR DIFFERENT ORIENTATION ANGLES AT T=11.26°C

Orientation Angle	T [°C]	σ_x [MPa] Analytical	σ_x [MPa] Numerical [MPa]
(0°/90°)s	11.26	-24.9	-24.8
(30°/-30°)s	11.26	-24.67	-24.361
(45°/-45°)s	11.26	-24.45	-24.142
(60°/-60°)s	11.26	-24.22	-23.923

TABLE II
ANALYTICAL AND NUMERICAL STRESS VALUES IN THE Y DIRECTION FOR DIFFERENT ORIENTATION ANGLES AT T=11.26°C

Orientation Angle	T [°C]	σ_y [MPa] Analytical	σ_y [MPa] Numerical
(0°/90°)s	11.26	-23.99	-23.704
(30°/-30°)s	11.26	-24.22	-23.923
(45°/-45°)s	11.26	-24.45	-24.142
(60°/-60°)s	11.26	-24.67	-24.361

TABLE III
ANALYTICAL AND NUMERICAL SHEAR STRESS VALUES FOR DIFFERENT ORIENTATION ANGLES AT T=11.26°C

Orientation Angle	T [°C]	τ_{xy} [MPa] Analytical	τ_{xy} [MPa] Numerical
(0°/90°)s	11.26	0	0
(30°/-30°)s	11.26	-0.39	-0.37957
(45°/-45°)s	11.26	-0.45	-0.43824
(60°/-60°)s	11.26	-0.39	-0.37957

As it can be seen from Figs. 1-3, analytical and numerical solutions are convenient. This result assures that a numerical approach to the stress analysis of the steel fiber reinforced aluminum matrix composite materials is applicable.

Additionally, it can be concluded from the tables that maximum stress distribution at x direction is seen for the materials that have (0°/90°)s orientation angle while the maximum stress distribution at y direction is seen for the materials that have (60°/-60°)s orientation angle. Those materials also have no shear stresses as it can be seen from Table III.

Finally, it is seen that while the stress values along the x direction is higher for orientation angles (0°/90°) s and (30°/-30°) s, the stress values along the x direction is lower than y direction for the orientation angle (60°/-60°) s. When the orientation angle is (45°/-45°) s, the stress values along x and y directions are the same.

IV. CONCLUSIONS

In this study, a comparison of the numerical and analytical analysis of the thermal stresses of steel fiber reinforced aluminum matrix composites is performed. Numerical solutions are carried out via ANSYS.

The results conclude that analytical and numerical solutions are convenient which provides the permission to use numerical stress analysis for steel fiber aluminum matrix composites. Another result can be expressed by investigating the effect of the orientation angle. Materials that have the orientation angle of (0°/90°)s have higher stress values when compared to the other orientation angles. Again for the same orientation angle, the shear stress values are equal to "0" which means there is no thermal shear for that angle.

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