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The Reliability of the Improved e-N Method for Transition Prediction as Checked by PSE Method

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Abstract**—**Transition prediction of boundary layers has always been an important problem in fluid mechanics both theoretically and practically, yet notwithstanding the great effort made by many investigators, there is no satisfactory answer to this problem. The most popular method available is so-called e-N method which is heavily dependent on experiments and experience. The author has proposed improvements to the e-N method, so to reduce its dependence on experiments and experience to a certain extent. One of the key assumptions is that transition would occur whenever the velocity amplitude of disturbance reaches 1-2% of the free stream velocity. However, the reliability of this assumption needs to be verified. In this paper, transition prediction on a flat plate is investigated by using both the improved e-N method and the parabolized stability equations (PSE) methods. The results show that the transition locations predicted by both methods agree reasonably well with each other, under the above assumption. For the supersonic case, the critical velocity amplitude in the improved e-N method should be taken as 0.013, whereas in the subsonic case, it should be 0.018, both are within the range 1-2%.

Keywords**—**Boundary layer, e-N method, PSE, Transition

I. Introduction

THE prediction of laminar-turbulent transition in boundary layer has always been an important problem in fluid mechanics both from theoretical and practical points of view. Yet the most popular method for its prediction, the so called e-N method, is largely a semi-empirical method[1].

Su & Zhou[2]-[3] have analyzed the problems existing in the conventional e-N method and proposed certain improvements, so to reduce its dependence on experiments or experience. The improvements include a new transition criterion and some considerations of receptivity. Yet basically, it is still a method relying on linear stability theory under parallel assumption.

Obviously, to make the prediction method reliable and more rational, one has to check, first, how big the error would be in using the parallel assumption; and second, what would be the error in using the new transition criterion, i.e. transition would take place whenever the velocity amplitude of disturbance in the improved e-N method reaches 1-2% of the free stream velocity.

In fact, the first problem mentioned above is not a serious one, because the results from linear stability theory (LST) have already been compared many times with those from direct numerical simulation and results from applying the parabolized stability equation (PSE)[4]-[10]. The conclusion was, when the Reynolds number is large, which is exactly the case for most transition problem, the difference was insignificant. So our main concern would be the second problem, i.e. if the new transition criterion is reasonable.

We would solved this problem by comparing results from applying e-N method and by using PSE, as it has been shown in [11]-[12] that results from applying PSE in predicting the transition location is comparable with those from DNS, provided the initial conditions are the same in both methods. However, there is a profound difference between the e-N method and the PSE method for predicting the transition location. In the e-N method, one seeks to find the T-S wave, whose amplitude reaches the given threshold for transition first among all possible T-S waves, see [2]-[3]. While in PSE, a single T-S wave cannot trigger transition, one has to choose more than one T-S waves, in order to finally trigger transition. So the problem is how to choose the initial set of T-S waves, which is comparable with the single wave in e-N method, and then compare the transition location determined by both methods.

II. Transition Predicted by the Improved e-N Method

The transition of the boundary layer on a flat plate is investigated. The Mach number of the oncoming flow is M=0.3. The Reynolds number is 2000, based on the displacement thickness of the boundary layer at the inlet of the computational domain, the velocity, density and viscosity coefficient of the free-stream. The wall temperature condition is adiabatic. The basic flow is given by the similarity solution.

At first the improved e-N method is used for predicting the transition location. It is done as follows: the computation starts from the location where the amplitude of the disturbance wave can be reasonably estimated, not as in the conventional e-N method that starts from the ZARF or neutral curves. The transition location is so determined that among all the locations that a certain T-S wave's amplitude reaches the threshold amplitude A_{tr} for transition, the most upstream one is the location of transition. We assume that at the inlet of our computational domain, all T-S waves have the same initial velocity amplitude 0.3‰ of the free stream velocity.

The integration starts from the inlet of the computational domain. For subsonic flows, two-dimensional T-S waves dominant the transition. Figure 1 shows the variations of the amplitude of some T-S waves with different frequencies in in the downstream direction. The amplitude A is determined by using (1).

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$$A = A_0 e^{-\int_{x_0}^{x} \alpha_i dx} \quad (1)$$

Where A_0 is the initial velocity amplitude, which is consumed to be 0.3‰, and $-\alpha_i$ represents the amplification rate in the x direction. x_0 is the start location of the integration. The wave that reaches the threshold value A_{tr} first is the dominant one and determines the transition location.

Curves in figure 2 show the location where the amplitude of wave with different frequency reaches the value A_{tr}, and the location of the point on the curve having the smallest value of x is the transition location. For instance, the transition location is x=360 if A_{tr}=0.01, and the corresponding wave frequency is about 0.06.

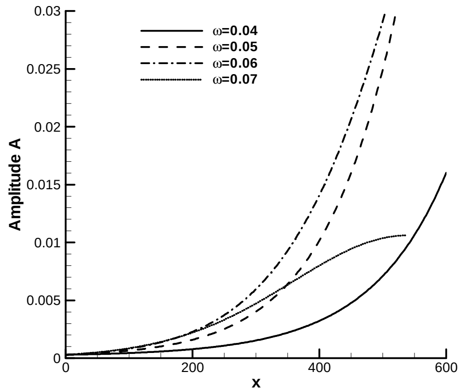

Fig. 1 Amplitude of T-S waves with different frequency

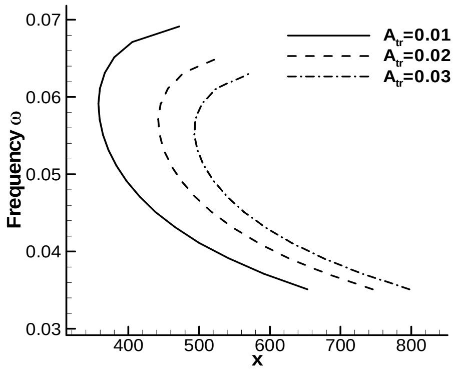

Fig. 2 Results obtained by the e-N method

III. Transition Predicted by Using the PSE

In the PSE method, the disturbance vector φ is expressed as

$$\varphi(x, y, z, t) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \hat{\varphi}_{mn}(x, y) e^{i(-\int_{x_0}^{x} \alpha_{mn}(\bar{x}) d\bar{x} + n\beta z - m\omega t)} \quad (2)$$

Where $\hat{\varphi} = (\hat{\rho}, \hat{u}, \hat{v}, \hat{w}, \hat{T})^T$ is the shape function vector. x, y and z the stream-wise, normal-wise and span-wise coordinates, respectively; t the time, α, β the stream-wise and span-wise wave number, respectively.

The governing equations and the numerical method to solve the equations can be found in many related references, for example in [13].

As mentioned above, in using PSE to predict transition, the initial condition should consist of more than one T-S waves. In the above example, the T-S waves considered in the improved e-N method are 2-D waves. So in the PSE method, we also should have one 2-D wave. Then what should be the other waves？ In nonlinear stability theory, there are famous models for nonlinear instability, i.e. the resonant triad and secondary instability. Both assume a pair of oblique waves having frequency half of the 2-D wave, because the nonlinear interaction between such waves would be the strongest under this assumption. So we also take similar model for our PSE method, i.e. the initial condition consist of a 2-D T-S wave and a pair of oblique T-S waves having frequency the half of the 2-D wave.

At first, it seems there can be infinite set of oblique waves, as it can have arbitrary span-wise wave number. However, in the e-N method, only waves with a real group velocity are considered, here the same principle is adopted. The span-wise wave number of the oblique waves is fixed by the condition $(\partial\alpha/\partial\beta)_i = 0$. The results, i.e. the so found span-wise wave numbers, are shown in Table I. In the improved e-N method as applied above, it is assumed that all waves, no matter what frequencies they have, have the same initial amplitude 0.3‰. In the PSE method for comparison, the same assumption should be used, so the amplitude of either of the two oblique waves should be 0.15‰.

TABLE I
PARAMETERS CHOSEN IN THE COMPUTATION FOR PSE

Cases	Frequency of basic wave	Span-wise wave number of oblique wave
1	0.049	0.21
2	0.051	0.21
3	0.055	0.20
4	0.065	0.17
5	0.069	0.16
6	0.075	0.15

The computation of PSE method ends at the location where the wall shear stress rises abruptly, which is assumed to be the location of transition triggered by that initial wave set, including one 2-D T-S wave and a pair of oblique T-S waves. Figure 3 shows the distribution of the wall friction coefficient C_f in the stream-wise direction. The rise of C_f curve indicates the transition onset.

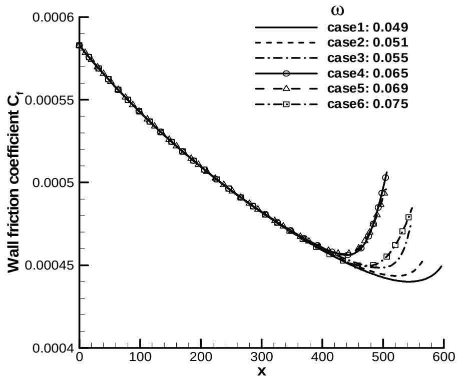

Fig. 3 Distribution of friction coefficient in the stream-wise direction

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Figure 4 shows the results of both the improved e-N method and the PSE methods. The most upstream point on each curve determines the transition location under the respective criteria of transition. For example, for the improved e-N method, the predicted transition location would be *x*=360, 450, 500, corresponding to A_{tr}=0.01, 0.02, 0.03 respectively, while for the PSE method, the transition location predicted is *x*=440. In fact, if in the improved e-N method, A_{tr} is taken to be 0.018, which is within the range 1-2% of free stream velocity as mentioned in the above introduction**,** then the transition location predicted by both methods would be the same. By the way, the frequencies of the 2-D waves triggering the transition in both methods are also roughly close to each other.

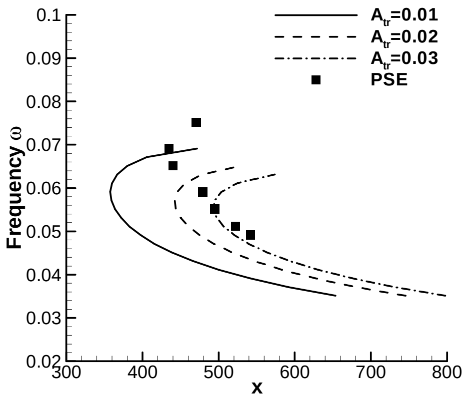

Fig. 4 Transition location predicted by the both methods

Fig.5 shows the velocity amplitude of the 2-D waves with frequencies 0.065 and 0.069, computed by both methods. The frequencies correspond to cases close to the critical case in PSE method. It can be seen that up to the point where transition is predicted by PSE method, the amplitudes of the 2-D wave predicted by both methods are close to each other.

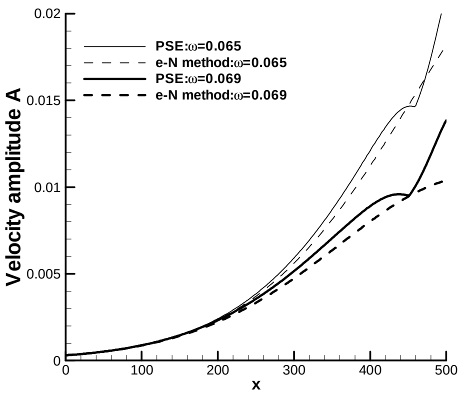

Fig. 5 Development of u_{max}

IV. A Supersonic Case

The transition of a supersonic boundary layer on a flat plate is also investigated in the similar way. The Mach number of the oncoming flow is M=6. The Reynolds number is 36000, defined in the same way as the subsonic case above. The wall temperature condition is adiabatic, too. The initial condition of disturbances is the same as the subsonic case.

The stability character is different from the subsonic case because there are two Mack T-S modes in the boundary layer. For supersonic boundary layers, when the Mach number is larger than 4, then the most unstable wave would be 2-D waves. So the initial disturbance waves in the PSE method are determined by the same way as in the subsonic case above.The parameters of the initial T-S waves are shown in Table II.

TABLE II
PARAMETERS OF T-S WAVES IN THE COMPUTATION FOR SUPERSONIC CASE

Cases	Frequency of 2-D T-S wave	Streamwise wave number of 2-D wave	Span-wise wave number of oblique wave
1	1.6	1.73	1.33
2	1.7	1.83	1.35
3	1.8	1.91	1.38
4	1.9	2.01	1.40
5	2.0	2.14	1.42
6	2.04	2.19	1.43

With the known initial disturbances, we follow their evolution by using PSE. For some supersonic cases the wall friction coefficient has not risen yet when the computation breaks down, inferring that the evolution of the disturbances becomes drastic. Therefore the location where the computation breaks down, or the friction coefficient C_f increases drastically, will be seen as the start of the transition.

The distribution of C_f in the stream-wise direction is shown in fig. 6. The C_f curve does not rise for the case $\omega = 2.0$ when the computation breaks down at *x*=94. A slight rise of C_f curves can be found in the other cases.

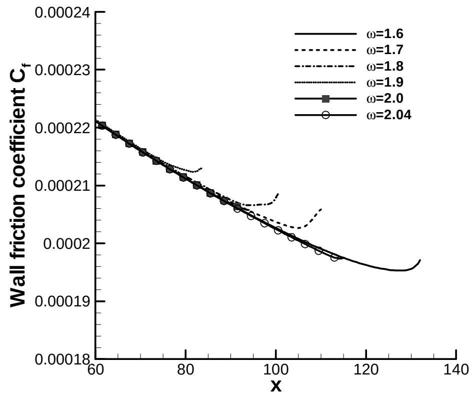

Fig. 6 Distribution of C_f in the stream-wise direction

Figure 7 shows the comparison of the results obtained by both the improved e-N method and the PSE methods. The transition location obtained by the PSE method is *x*=80, which is exactly in the range of transition location predicted by the improved e-N method using the transition criterion A_{tr}=0.01, 0.02, respectively. If in the improved e^N method, A_{tr} is taken to be 0.013, the transition location predicted by both methods would be the same. The transition criterion adopted in the improved e-N method is reasonable. Considering both the subsonic and supersonic cases the transition criteria in the improved e-N could be taken as A_{tr}=0.015. The frequency of the wave triggering the transition is also slightly bigger than that in the improved e-N method, similar to the subsonic case. The reason is thought to be the influence of the nonparallelism and nonlinear effects. More work is underway for further verification.

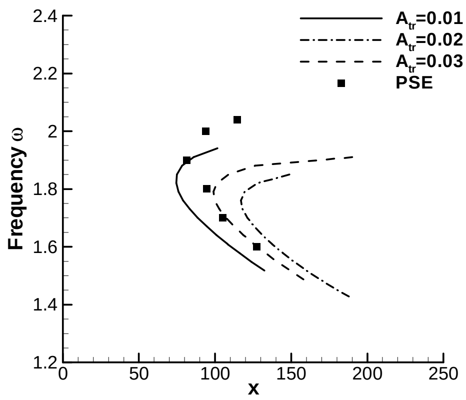

Fig. 7 Transition location predicted by the both methods for the supersonic flow

V. CONCLUSION

The conventional e-N method is a semi-empirical method, with no consideration on receptivity, and no consideration on physical criterion for transition. In the improved e-N method, the initial location of integration and the initial amplitude of the T-S wave are determined by a certain consideration on receptivity mechanism, and the transition criterion is taken to be that the disturbance wave's amplitude reaches 1-2% of the free stream velocity, as indicated from results of DNS for transition. However, it still bears some simplifications, which should be checked by other more sophisticated method. In this paper, PSE method is used to check the results from the improved e-N method. The result of comparison does provide evidence that the improved e-N method bears some rationality. Of course, more work needs to be done, for example, cases of supersonic and hypersonic flows should be studied further, which is underway.

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