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# The Effects of Peristalsis on Dispersion of a Micropolar Fluid in the Presence of Magnetic Field

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*Abstract*—The paper presents an analytical solution for dispersion of a solute in the peristaltic motion of a micropolar fluid in the presence of magnetic field and both homogeneous and heterogeneous chemical reactions. The average effective dispersion coefficient has been found using Taylor's limiting condition under long wavelength approximation. The effects of various relevant parameters on the average coefficient of dispersion have been studied. The average effective dispersion coefficient increases with amplitude ratio, cross viscosity coefficient and heterogeneous chemical reaction rate parameter. But it decreases with magnetic field parameter and homogeneous chemical reaction rate parameter. It can be noted that the presence of peristalsis enhances dispersion of a solute.

*Keywords*—Peristalsis, Dispersion, Chemical reaction, Magnetic field, Micropolar fluid

## I. Introduction

PERISTALSIS is a natural mechanism of transport for many physiological fluids. This is achieved by the passage of progressive waves of area contraction or expansion along the boundary of a fluid-filled distensible tube. Different physiological phenomena, such as the flow of urine from kidney to the bladder through ureters, transport of food material through the digestive tract, movement of spermatozoa in the ductus efferentes of the male reproductive tract and cervical canal and the transport of ovum in the fallopian tube, take place by the mechanism of peristalsis. Some biomedical instruments such as blood pumps in dialysis and the heart lung machine use this principle. Peristaltic transport of a toxic liquid is used in nuclear industry to avoid contamination of the outside environment. The industrial use of this pumping mechanism in roller/finger pumps to pump slurries and corrosive fluids is well known. Several studies have been made on peristalsis with reference to mechanical and physiological situations. (Shapiro et al. [1], Fung and Yih [2], Misra and Pandey [3], [4], Mishra and Rao [5], Radhakrishnamacharya [6]).

Most of bio-fluids such as blood exhibit the behavior of non-Newtonian fluids. Hence, the study of peristaltic transport of non-Newtonian fluids may help to have better understanding of the biological systems. Radhakrishnamacharya [6] studied long wavelength approximation to peristaltic motion of a power law fluid. Another non-Newtonian fluid that received considerable attention of researchers is micropolar fluid. The main advantage of using this fluid model compared to other non-Newtonian fluids is that it takes care of the rotation of fluid particles by means of an independent kinematic vector called the microrotation vector. To mention some studies, Srinivasacharya et al. [7], Muthu et al. [8] investigated the influence of wall properties on the peristaltic motion of micropolar fluid. Sankad et al. [9] studied long wavelength approximation to peristaltic motion of micropolar fluid with wall effects.

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Magnetohydrodynamics (MHD) is the science which deals with the motion of highly conducting fluids in the presence of a magnetic field. The motion of the conducting fluid across the magnetic field generates electric currents which change the magnetic field and the action of the magnetic field on these currents gives rise to mechanical forces which modify the flow of the fluid (Mekheimer [10]). MHD flow of a fluid in a channel with elastic, rhythmically contracting walls (peristaltic flow) is of interest in connection with certain problems of the movement of conductive physiological fluids (example: the blood and blood pump machines) (Hayat et al. [11] and Srinivasacharya and Mekonnen [12]). Currently, studies on peristaltic motion in magnetohydrodynamic (MHD) flows of electrically conducting physiological fluids have become a subject of growing interest for researchers. This is due to the fact that such studies are useful particularly for getting a proper understanding of the functioning of different machines used by clinicians for pumping blood (Misra et al. [13]). Misra et al. [13] pointed out that theoretical researches with an aim to explore the effect of a magnetic field on the flow of blood in atherosclerotic vessels also find application in a blood pump used by cardiac surgeons during the surgical procedure.

The process of dispersion of a solute in fluids flowing through channels or pipes has been extensively investigated because of its important applications in various chemical and biological systems. The study of such problem was initiated by Taylor [14]–[16], who presented an analysis to discuss dispersion of a soluble salt when ejected to a stream of solvent flowing slowly through a tube. This analysis was later generalized and extended by many researchers to study dispersion of solute in Newtonian or non-Newtonian fluid flows under various situations (Aris [17], Dutta et al. [18], Shukla et al. [19] and Peeyush and Agarwal [20] and Philip and Peeyush [21]). The effects of homogeneous and/or heterogeneous chemical reactions on the dispersion of a solute have also been studied by numerous authors under different conditions (Gupta and Gupta [22], Ramana Rao and Padma [23], [24], Padma and Ramana Rao [25], Shukla et al. [19], and Philip and Peeyush [21]).

The effect of peristalsis on dispersion in the presence of magnetic field has not received any attention. It is realized that magnetic field and peristalsis may have significant effect on the dispersion of a solute in the flow of conducting fluid and this may lead to better understanding of the flow situation in physiological systems. The objective of this paper is to study the effect of peristalsis on the dispersion of a solute in micropolar fluid in the presence of magnetic field. Using long wavelength approximation and Taylor's approach, closed form solution has been obtained for the dispersion coefficient for both the cases of homogeneous first-order irreversible chemical reaction and combined first-order homogeneous and heterogeneous chemical reactions. The effects of various relevant parameters on the average effective dispersion coefficient are studied.

## II. Mathematical Formulation

Consider the dispersion of a solute in peristaltic flow of an electrically conducting micropolar fluid in a channel of width $2d$ and with flexible walls on which traveling sinusoidal waves of long wavelength are imposed. A uniform magnetic field $B_0$ is applied to the fluid normal to the walls of the channel. Cartesian coordinate system $(x, y)$ is chosen with the $x$-axis aligned with the center line of the channel. The traveling waves are represented by (Fig.1)

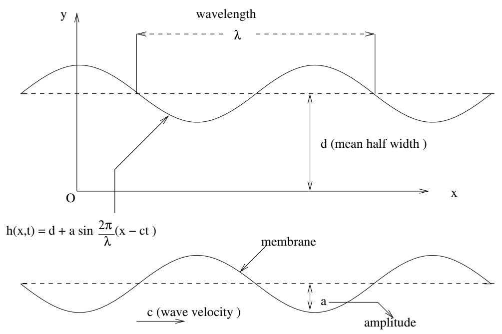


Fig. 1. Geometry of the problem.

$$y = \pm h = \pm\left[d + a\sin\frac{2\pi}{\lambda}(x - ct)\right] \tag{1}$$

where $a$ is the amplitude, $c$ is the wave speed and $\lambda$ is the wavelength of the peristaltic wave.

Under long wavelength approximation, the equations governing the peristaltic motion of incompressible micropolar fluid for the present problem are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2}$$

$$-\frac{\partial p}{\partial x} + \left(\frac{2\mu+\kappa}{2}\right)\frac{\partial^2 u}{\partial y^2} + \kappa\frac{\partial g}{\partial y} - \sigma B_0^2 u = 0 \tag{3}$$

$$-\frac{\partial p}{\partial y} = 0 \tag{4}$$

$$-2\kappa g + \gamma\frac{\partial^2 g}{\partial y^2} - \kappa\frac{\partial u}{\partial y} = 0 \tag{5}$$

where $u(x, y, t)$ and $v(x, y, t)$ are the velocity components in the $x$ and $y$ directions respectively, $g(x, y, t)$ is the microrotation component in the direction normal to both $x$ and $y$ axes, $\mu$ is the viscosity coefficient of classical fluid dynamics, $\kappa$ and $\gamma$ are the new viscosity coefficients for micropolar fluids, $\sigma$ is the electrical conductivity of the fluid and $B_0$ is a uniform magnetic field.

We assume that the walls are inextensible so that only lateral motion takes place and the horizontal displacement of the wall is zero.

Thus, the no-slip boundary conditions for the velocity and microrotation are given by

$$u = 0, \qquad g = 0 \qquad at \qquad y = \pm h. \tag{6}$$

Solving (3)-(5) under the boundary conditions (6), the velocity is given by

$$u(y) = -\frac{1}{\sigma B_0^2}\frac{\partial p}{\partial x}\frac{1}{S_3^*}\left[m_1^* S_2^* \sinh(m_2^* h)\cosh(m_1^* y) - m_2^* S_1^* \sinh(m_1^* h)\cosh(m_2^* y) + S_3^*\right] \tag{7}$$

where

$$\begin{aligned}
m_1^* &= \sqrt{(\delta_1^* + \sqrt{(\delta_1^*)^2 - 4\delta_2^*}\,)/2},\\
m_2^* &= \sqrt{(\delta_1^* - \sqrt{(\delta_1^*)^2 - 4\delta_2^*}\,)/2},\\
\delta_1^* &= \frac{2(2\mu\kappa + \gamma\sigma B_0^2)}{\gamma(2\mu+\kappa)}, \qquad \delta_2^* = \frac{4\kappa\sigma B_0^2}{\gamma(2\mu+\kappa)},\\
S_1^* &= \sigma B_0^2/\kappa - (m_1^*)^2(2\mu+\kappa)/2\kappa,\\
S_2^* &= \sigma B_0^2/\kappa - (m_2^*)^2(2\mu+\kappa)/2\kappa,\\
S_3^* &= m_2^* S_1^* \sinh(m_1^* h)\cosh(m_2^* h) - m_1^* S_2^* \cosh(m_1^* h)\sinh(m_2^* h)
\end{aligned}$$

Further, the mean velocity is defined by

$$\bar{u} = \frac{1}{2h}\int_{-h}^{+h} u(y)dy. \tag{8}$$

Substituting (7) in (8) we get,

$$\bar{u} = -\frac{1}{\sigma B_0^2}\frac{\partial p}{\partial x}\frac{1}{S_3^*}\left[(S_1^* + S_2^*)\sinh(m_1^* h)\sinh(m_2^* h)/h + S_3^*\right]. \tag{9}$$

If we now consider convection across a plane moving with the mean speed of the flow, then relative to this plane, the fluid velocity is given by

$$\begin{aligned}
u_x &= u - \bar{u}\\
&= -\frac{1}{\sigma B_0^2 S_3^*}\frac{\partial p}{\partial x}\{m_1^* S_2^* \sinh(m_2^* h)\cosh(m_1^* y)\\
&\quad - m_2^* S_1^* \sinh(m_1^* h)\cosh(m_2^* y) - (S_2^* - S_1^*)\sinh(m_1^* h)\sinh(m_2^* h)/h\}.
\end{aligned} \tag{10}$$

### A. Diffusion with a Homogeneous First-order Chemical Reaction

It is assumed that a solute diffuses and simultaneously undergoes a first order irreversible chemical reaction in peristaltic transport of a micropolar fluid with isothermal conditions. Assuming $\frac{\partial^2 C}{\partial x^2} << \frac{\partial^2 C}{\partial y^2}$, the equation for the concentration

$C$ of the solute for the present problem satisfies the diffusion equation

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}=D\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right)-k_1 C \tag{11}$$

where $D$ is the molecular diffusion coefficient and $k_1$ is the first order reaction rate constant. For typical values of physiologically relevant parameters of this problem, it is realized that $\bar{u}\approx c$. Using this condition and following Taylor [14]–[16], we assume partial equilibrium is maintained , and then making use of the following dimensionless quantities

$$\theta=t/\bar{t},\quad \bar{t}=\lambda/\bar{u},\quad \eta=y/d,\quad \xi=(x-\bar{u}t)/\lambda,\quad H=h/d \tag{12}$$

(10) reduces to

$$\begin{aligned}u_x=-\frac{d^2}{\mu S_3}\frac{1}{(H^*)^2}\frac{\partial p}{\partial x}\{&m_1S_2\sinh(m_2H)\cosh(m_1\eta)\\&-m_2S_1\sinh(m_1H)\cosh(m_2\eta)\\&-(S_2-S_1)\sinh(m_1H)\sinh(m_2H)/H\}.\end{aligned} \tag{13}$$

where

$$\begin{aligned}S_3&=S_3^*d=m_2S_1\sinh(m_1H)\cosh(m_2H)\\&-\ m_1S_2\cosh(m_1H)\sinh(m_2H),\\m_1&=m_1^*d=\sqrt{(\delta_1+\sqrt{\delta_1^2-4\delta_2})/2},\\m_2&=m_2^*d=\sqrt{(\delta_1-\sqrt{\delta_1^2-4\delta_2})/2},\\S_1&=S_1^*d^2=(1/\mu_1)\left((H^*)^2-m_1^2(2+\mu_1)/2\right),\\S_2&=S_2^*d^2=(1/\mu_1)\left((H^*)^2-m_2^2(2+\mu_1)/2\right),\\\delta_1&=\delta_1^*d^2=M^2N^2+(2/(2+\mu_1))(H^*)^2,\\\delta_2&=\delta_2^*d^4=M^2N^2(H^*)^2,\\M&=2d\,(\mu/\gamma)^{1/2},\quad N=(\mu_1/(2+\mu_1))^{1/2},\\\mu_1&=\kappa/\mu,\ (H^*)^2=\sigma B_0^2d^2/\mu.\end{aligned}$$

and $H^*$ is magnetic field parameter (Hartmann number). Further, (11) becomes

$$\frac{\partial^2 C}{\partial\eta^2}-\frac{k_1d^2}{D}C=\frac{d^2}{\lambda D}u_x\frac{\partial C}{\partial\xi}. \tag{14}$$

Assuming that there is no absorption at the walls, the boundary conditions for the concentration $C$ are

$$\frac{\partial C}{\partial\eta}=0 \qquad for \qquad \eta=\pm H=\pm[1+\epsilon\sin(2\pi\xi)] \tag{15}$$

where $\epsilon=a/d$ is the amplitude ratio.

Assuming that $\partial C/\partial\xi$ is independent of $\eta$ at any cross-section and solving (14) under the boundary conditions (15), the solution for the concentration of the solute $C$ is given as

$$\begin{aligned}C(\eta)=A\cosh(\alpha\eta)-\frac{d^4}{\mu\lambda DS_3}\frac{1}{(H^*)^2}\frac{\partial C}{\partial\xi}\frac{\partial p}{\partial x}\\\times\left\{\frac{m_1S_2\sinh(m_2H)\cosh(m_1\eta)}{m_1^2-\alpha^2}-\frac{m_2S_1\sinh(m_1H)\cosh(m_2\eta)}{m_2^2-\alpha^2}\right.\\\left.+\frac{(S_2-S_1)}{\alpha^2H}\sinh(m_1H)\sinh(m_2H)\right\}\end{aligned} \tag{16}$$

where

$$\begin{aligned}A=\frac{d^4}{\mu\lambda DS_3}\frac{1}{(H^*)^2}\frac{\partial C}{\partial\xi}\frac{1}{L}\sinh(m_1H)\sinh(m_2H)\\\times\left\{\frac{m_1^2S_2}{m_1^2-\alpha^2}-\frac{m_2^2S_1}{m_2^2-\alpha^2}\right\},\end{aligned} \tag{17}$$

$\alpha=d(k_1/D)^{1/2}$ and $L=\alpha\sinh(\alpha H)$.

The volumetric rate $Q$ at which the solute is transported across a section of the channel of unit breadth is defined by

$$Q=\int_{-H}^{+H}Cu_x d\eta. \tag{18}$$

Substituting (13) and (16) in (18), we get the volumetric rate $Q$ as

$$Q=-\frac{2d^6}{\lambda\mu^2D}\frac{\partial C}{\partial\xi}\left(\frac{\partial p}{\partial x}\right)^2F(\xi,\epsilon,\alpha,\mu_1,M,H^*) \tag{19}$$

where

$$\begin{aligned}F(\xi,\epsilon,\alpha,\mu_1,M,H^*)=\frac{1}{S_3^2}\frac{1}{(H^*)^4}\\\times\left\{\frac{m_1^3S_2^2\sinh(m_1H)\sinh^2(m_2H)}{\alpha\sinh(\alpha H)(m_1^2-\alpha^2)^2}B_1\right.\\-\frac{m_1^2m_2S_1S_2\sinh^2(m_1H)\sinh(m_2H)}{\alpha\sinh(\alpha H)(m_1^2-\alpha^2)(m_2^2-\alpha^2)}B_2\\-\frac{m_1^2S_2(S_2-S_1)}{\alpha^2H(m_1^2-\alpha^2)}\sinh^2(m_1H)\sinh^2(m_2H)\\+\frac{m_2^2S_1(S_2-S_1)}{\alpha^2H(m_2^2-\alpha^2)}\sinh^2(m_1H)\sinh^2(m_2H)\\-\frac{m_1m_2^2S_1S_2\sinh(m_1H)\sinh^2(m_2H)}{\alpha\sinh(\alpha H)(m_1^2-\alpha^2)(m_2^2-\alpha^2)}B_1\\+\frac{m_2^3S_1^2\sinh^2(m_1H)\sinh(m_2H)}{\alpha\sinh(\alpha H)(m_2^2-\alpha^2)^2}B_2\\-\frac{m_1S_2^2\sinh^2(m_2H)}{4(m_1^2-\alpha^2)}(2m_1H+\sinh(2m_1H))\\-\frac{m_2S_1^2\sinh^2(m_1H)}{4(m_2^2-\alpha^2)}(2m_2H+\sinh(2m_2H))\\+\frac{m_1m_2S_1S_2}{(m_1^2-m_2^2)}\sinh(m_1H)\sinh(m_2H)\left(\frac{1}{m_1^2-\alpha^2}+\frac{1}{m_2^2-\alpha^2}\right)B_3\\+\frac{S_2(S_2-S_1)}{H(m_1^2-\alpha^2)}\sinh^2(m_1H)\sinh^2(m_2H)\\\left.-\frac{S_1(S_2-S_1)}{H(m_2^2-\alpha^2)}\sinh^2(m_1H)\sinh^2(m_2H)\right\}\end{aligned} \tag{20}$$

$$\begin{aligned}B_1&=m_1\cosh(\alpha H)\sinh(m_1H)-\alpha\cosh(m_1H)\sinh(\alpha H),\\B_2&=m_2\cosh(\alpha H)\sinh(m_2H)-\alpha\cosh(m_2H)\sinh(\alpha H),\\B_3&=m_1\cosh(m_2H)\sinh(m_1H)-m_2\cosh(m_1H)\sinh(m_2H).\end{aligned}$$

Comparing (19) with Fick's law of diffusion, we find that the solute is dispersed relative to a plane moving with the mean speed of the flow with an effective dispersion coefficient $D^*$ given by

$$D^* = 2\frac{d^6}{\mu^2 D}(\frac{\partial p}{\partial x})^2 F(\xi,\epsilon,\alpha,\mu_1,M,H^*) \quad (21)$$

Let the average of $F$ be $\overline{F}$ and is defined by

$$\overline{F} = \int_0^1 F(\xi,\epsilon,\alpha,\mu_1,M,H^*)d\xi. \quad (22)$$

*B. Diffusion with Combined Homogeneous and Heterogeneous Chemical Reaction*

We now discuss the problem of diffusion with a first-order irreversible chemical reaction taking place both in the bulk of the medium (homogeneous) as well as at the walls (heterogeneous) of the channel which are assumed to be catalytic to chemical reaction. The diffusion equation is same as given by (11), i.e.,

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} = D\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right) - k_1 C.$$

The differential material balance at the walls (Philip and Peeyush [21]) gives the boundary conditions

$$\frac{\partial C}{\partial y} + fC = 0 \quad at \quad y = h = \left[d + a\sin\frac{2\pi}{\lambda}(x - \bar{u}t)\right], \quad (23)$$

$$\frac{\partial C}{\partial y} - fC = 0 \quad at \quad y = -h = -\left[d + a\sin\frac{2\pi}{\lambda}(x - \bar{u}t)\right]. \quad (24)$$

If we introduce the dimensionless variables (12) and assume the limiting condition of Taylor [14]–[16], the diffusion equation remains as (14) but subject to the boundary conditions

$$\frac{\partial C}{\partial \eta} + \beta C = 0 \quad for \quad \eta = H = [1 + \epsilon\sin(2\pi\xi)] \quad (25)$$

$$\frac{\partial C}{\partial \eta} - \beta C = 0 \quad for \quad \eta = -H = -[1 + \epsilon\sin(2\pi\xi)] \quad (26)$$

where $\beta = fd$ is the heterogeneous reaction rate parameter corresponding to catalytic reaction at the walls.
The solution of (14) satisfying the boundary conditions (25) and (26) is

$$C(\eta) = A'\cosh(\alpha\eta) - \frac{d^4}{\mu\lambda D S_3}\frac{1}{(H^*)^2}\frac{\partial C}{\partial \xi}\frac{\partial p}{\partial x} \times\left\{\frac{m_1 S_2\sinh(m_2 H)\cosh(m_1\eta)}{m_1^2-\alpha^2} - \frac{m_2 S_1\sinh(m_1 H)\cosh(m_2\eta)}{m_2^2-\alpha^2} + \frac{(S_2-S_1)}{\alpha^2 H}\sinh(m_1 H)\sinh(m_2 H)\right\} \quad (27)$$

where

$$A' = \frac{d^4}{\mu\lambda D S_3}\frac{1}{(H^*)^2}\frac{\partial C}{\partial \xi}\frac{\partial p}{\partial x}\frac{1}{L'}\left\{\frac{m_1^2 S_2\sinh(m_2 H)\sinh(m_1 H)}{m_1^2-\alpha^2} - \frac{m_2^2 S_1\sinh(m_1 H)\sinh(m_2 H)}{m_2^2-\alpha^2} + \frac{\beta m_1 S_2\sinh(m_2 H)\cosh(m_1 H)}{m_1^2-\alpha^2} - \frac{\beta m_2 S_1\sinh(m_1 H)\cosh(m_2 H)}{m_2^2-\alpha^2} + \frac{\beta(S_2-S_1)\sinh(m_1 H)\sinh(m_2 H)}{\alpha^2 H}\right. \quad (28)$$

and $L = \alpha\sinh(\alpha H) + \beta\cosh(\alpha H)$

Substituting (27) and (13) in (18), we get

$$Q = -2\frac{d^6}{\lambda\mu^2 D}\frac{\partial C}{\partial \xi}(\frac{\partial p}{\partial x})^2 G(\xi,\epsilon,\alpha,\beta,\mu_1,M,H^*) \quad (29)$$

where

$$\begin{aligned}
G(\xi,\epsilon,\alpha,\beta,\mu_1,M,H^*) = {} & \frac{1}{S_3^2}\frac{1}{(H^*)^4} \\
\times\Bigg\{\frac{1}{L'}\Bigg[ & \frac{m_1^3 S_2^2\sinh(m_1 H)\sinh^2(m_2 H)}{(m_1^2-\alpha^2)^2}B_1 \\
& - \frac{m_1^2 m_2 S_1 S_2\sinh^2(m_1 H)\sinh(m_2 H)}{(m_1^2-\alpha^2)(m_2^2-\alpha^2)}B_2 \\
& - \frac{(S_2-S_1)}{\alpha H\,\mathrm{csch}(\alpha H)}\left(\frac{m_1^2 S_2}{m_1^2-\alpha^2} - \frac{m_2^2 S_1}{m_2^2-\alpha^2}\right)\sinh^2(m_1 H)\sinh^2(m_2 H) \\
& - \frac{m_1 m_2^2 S_1 S_2\sinh(m_1 H)\sinh^2(m_2 H)}{(m_1^2-\alpha^2)(m_2^2-\alpha^2)}B_1 \\
& + \frac{m_2^3 S_1^2\sinh^2(m_1 H)\sinh(m_2 H)}{(m_2^2-\alpha^2)^2}B_2 \\
& + \beta\frac{m_1^2 S_2^2\cosh(m_1 H)\sinh^2(m_2 H)}{(m_1^2-\alpha^2)^2}B_1 \\
& - \beta\frac{m_1 m_2 S_1 S_2\cosh(m_1 H)\sinh(m_1 H)\sinh(m_2 H)}{(m_1^2-\alpha^2)(m_2^2-\alpha^2)}B_2 \\
& - \beta\frac{m_1 S_2(S_2-S_1)}{\alpha H(m_1^2-\alpha^2)}\cosh(m_1 H)\sinh(m_1 H)\sinh^2(m_2 H)\sinh(\alpha H) \\
& - \beta\frac{m_1 m_2 S_1 S_2\cosh(m_2 H)\sinh(m_1 H)\sinh(m_2 H)}{(m_1^2-\alpha^2)(m_2^2-\alpha^2)}B_1 \\
& + \beta\frac{m_2^2 S_1^2\cosh(m_2 H)\sinh^2(m_1 H)}{(m_2^2-\alpha^2)^2}B_2 \\
& + \beta\frac{m_2 S_1(S_2-S_1)}{\alpha H(m_2^2-\alpha^2)}\cosh(m_2 H)\sinh^2(m_1 H)\sinh(m_2 H)\sinh(\alpha H) \\
& + \beta\frac{m_1 S_2(S_2-S_1)}{\alpha^2 H(m_1^2-\alpha^2)}\sinh(m_1 H)\sinh^2(m_2 H)B_1 \\
& - \beta\frac{m_2 S_1(S_2-S_1)}{\alpha^2 H(m_1^2-\alpha^2)}\sinh^2(m_1 H)\sinh(m_2 H)B_2 \\
& - \beta\frac{(S_2-S1)^2}{\alpha^3 H^2}\sinh^2(m_1 H)\sinh^2(m_2 H)\sinh(\alpha H)\Bigg]
\end{aligned}$$

$$
\begin{aligned}
&- \frac{m_1 S_2^2 \sinh^2(m_2 H)}{4(m_1^2-\alpha^2)} \left(2m_1 H + \sinh(2m_1 H)\right) \\
&\quad - \frac{m_2 S_1^2 \sinh^2(m_1 H)}{4(m_2^2-\alpha^2)} \left(2m_2 H + \sinh(2m_2 H)\right) \\
&\quad\quad + \frac{S_2(S_2-S_1)}{H(m_1^2-\alpha^2)} \sinh^2(m_1 H)\sinh^2(m_2 H) \\
&+ \frac{m_1 m_2 S_1 S_2}{(m_1^2-m_2^2)} \sinh(m_1 H)\sinh(m_2 H) \left(\frac{1}{m_1^2-\alpha^2} + \frac{1}{m_2^2-\alpha^2}\right) B_3 \\
&\quad\quad - \frac{S_1(S_2-S_1)}{H(m_2^2-\alpha^2)} \sinh^2(m_1 H)\sinh^2(m_2 H) \Bigg\} \qquad (30)
\end{aligned}
$$

Comparing (29) with Fick's Law of Diffusion, we find that the solute is dispersed relative to a plane moving with the mean speed of the flow with an effective dispersion coefficient $D^*$ given by

$$D^* = 2\frac{d^6}{\mu^2 D}\left(\frac{\partial p}{\partial x}\right)^2 G(\xi, \epsilon, \alpha, \beta, \mu_1, M, H^*) \qquad (31)$$

The average of $G$ denoted by $\overline{G}$ is defined as

$$\overline{G} = \int_0^1 G(\xi, \epsilon, \alpha, \beta, \mu_1, M, H^*) d\xi. \qquad (32)$$

## III. Results and Discussion

The effects of various parameters on the average effective dispersion coefficient can be observed through the functions $\overline{F}(\xi, \epsilon, \alpha, \mu_1, M, H^*)$ (for homogeneous case) and $\overline{G}(\xi, \epsilon, \alpha, \beta, \mu_1, M, H^*)$ (for combined homogeneous and heterogeneous case) given by equations (22) and (32), respectively. The expressions for $\overline{F}$ and $\overline{G}$ have been obtained by numerical integration using MATHEMATICA software for different values of relevant parameters and presented graphically. The important parameters involved in the expressions are: the amplitude ratio $\epsilon$, the homogeneous reaction rate parameter $\alpha$, the heterogeneous reaction rate parameter $\beta$, the cross viscosity coefficient $\mu_1$, the other micropolar parameter $M$ and the Hartmann number (or magnetic field parameter) $H^*$.

### *A. Homogeneous Chemical Reaction*

Figs. 2-5 show the effects of various parameters on dispersion in the presence of homogeneous chemical reaction in the bulk of the medium. It can be observed that the average effective coefficient of dispersion decreases with homogeneous chemical reaction rate parameter $\alpha$ (Figs. 2-5). This result is expected since increase in $\alpha$ leads to increasing number of moles of solute undergoing chemical reaction, which results in the decrease of dispersion. The result that dispersion decreases with $\alpha$ agrees with previous results obtained by Gupta and Gupta [22], Dutta et al. [18], Ramana Rao and Padma [23], [24], Padma and Ramana Rao [25], Shukla et al. [19]. Further, average dispersion decreases with magnetic field parameter (or Hartmann number) $H^*$ (Fig. 2). The result that dispersion decreases with Hartmann number $H^*$ agrees with the results obtained by Ramana Rao and Padma [23], [24]. However, $\overline{F}$ increases with micropolar parameters $M$ and $\mu_1$ (Figs. 3 and 4, respectively) and amplitude ratio $\epsilon$ (Fig. 5). The result that dispersion increases with $\epsilon$ shows that dispersion is more in the presence of peristalsis.

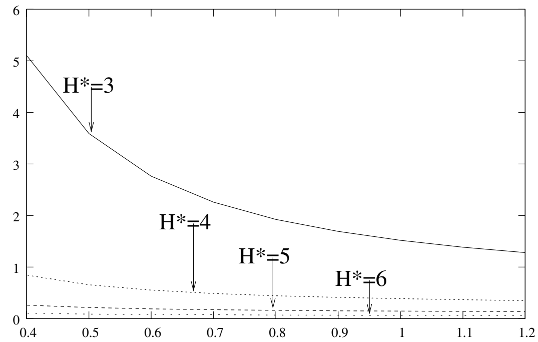


Fig. 2. Effect of $H^*$ on $\overline{F}$ for $M = 5.0$, $\mu_1 = 10.0$ and $\epsilon = 0.2$.

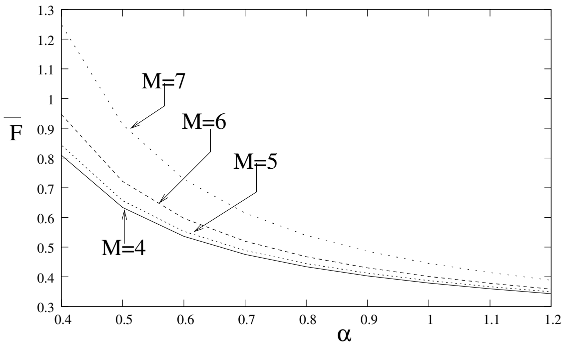


Fig. 3. Effect of $M$ on $\overline{F}$ for $H^* = 4.0$, $\mu_1 = 10.0$ and $\epsilon = 0.2$.

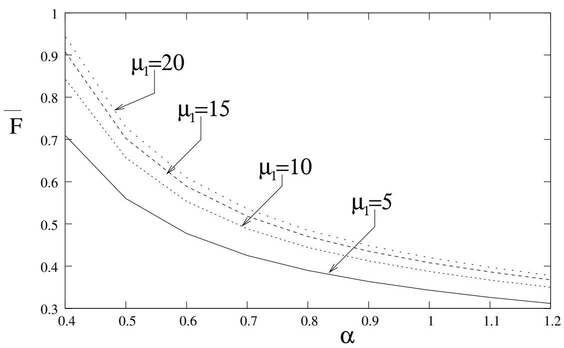


Fig. 4. Effect of $\mu_1$ on $\overline{F}$ for $H^* = 4.0$, $M = 5.0$ and $\epsilon = 0.2$.

### *B. Combined Homogeneous and Heterogeneous Chemical Reactions*

Figs. 6-10 show the effects of various parameters on dispersion for the case of combined first-order homogeneous and heterogeneous chemical reactions both in the bulk and at the walls. Average effective dispersion coefficient $\overline{G}$ decreases with magnetic field parameter $H^*$ (Fig.6), micropolar parameter $M$ (Fig.7) and homogeneous chemical reaction rate parameter $\alpha$ (Fig. 10). But it increases with cross viscosity coefficient $\mu_1$ (Fig. 8) and amplitude ratio $\epsilon$ (Fig.9). Further, Figs. 6-10 show that dispersion increases with heterogeneous chemical reaction rate parameter $\beta$. The increase with $\beta$ is less significant for higher values of heterogeneous chemical reaction rate parameter ($\beta \geq 6$). Similarly, the result that

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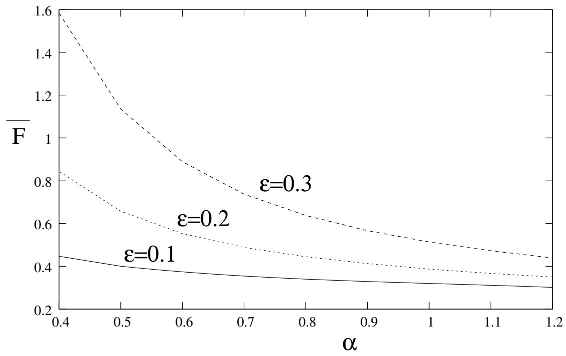


Fig. 5. Effect of $\epsilon$ on $\overline{F}$ for $H^* = 4.0$, $M = 5.0$ and $\mu_1 = 10.0$.

dispersion increases with amplitude ratio shows that dispersion increases in situations where peristalsis takes place.

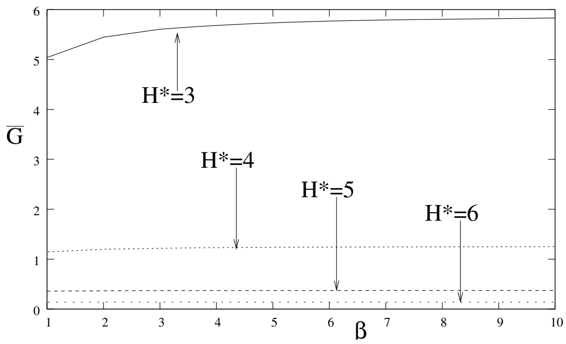


Fig. 6. Effect of $H^*$ on $\overline{G}$ for $M = 5.0$, $\mu_1 = 10.0$, $\alpha = 1.0$ and $\epsilon = 0.2$.

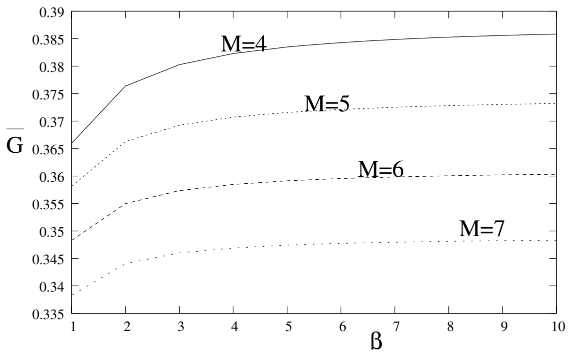


Fig. 7. Effect of $M$ on $\overline{G}$ for $H^* = 4.0$ , $\mu_1 = 10.0$, $\alpha = 1.0$ and $\epsilon = 0.2$.

## IV. Conclusion

The dispersion of a solute in peristaltic motion of a micropolar fluid in the presence of magnetic field with both homogeneous and heterogeneous chemical reactions has been studied under long wavelength approximation and Taylor's limiting condition. It is observed that average effective coefficient of dispersion decreases with magnetic field parameter (or Hartmann number) $H^*$ and homogeneous chemical reaction rate parameter $\alpha$. But dispersion increases with amplitude ratio

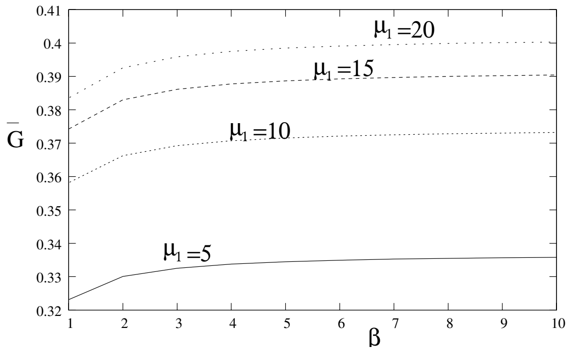


Fig. 8. Effect of $\mu_1$ on $\overline{G}$ for $H^* = 4.0$ , $M = 5.0$, $\alpha = 1.0$ and $\epsilon = 0.2$.

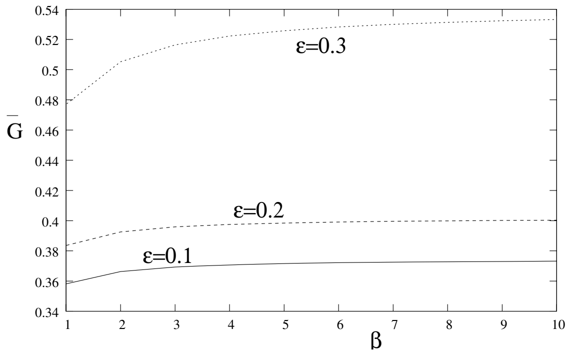


Fig. 9. Effect of $\epsilon$ on $\overline{G}$ for $H^* = 4.0$ , $M = 5.0$, $\mu_1 = 10.0$, and $\alpha = 1.0$.

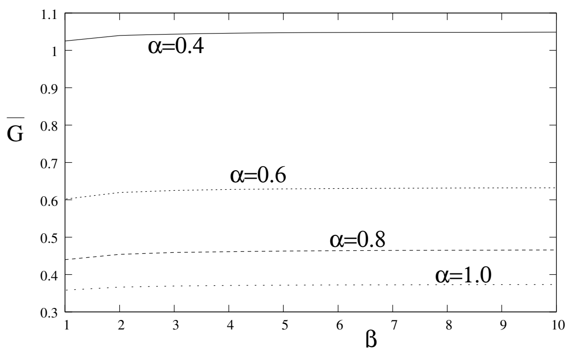


Fig. 10. Effect of $\alpha$ on $\overline{G}$ for $H^* = 4.0$ , $M = 5.0$, $\mu_1 = 10.0$ and $\epsilon = 0.2$.

$\epsilon$, cross viscosity coefficient $\mu_1$ and heterogeneous chemical reaction rate parameter $\beta$. In both the cases, it can be noted that dispersion is more in the presence of peristalsis.

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