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The Determination of Rating Points of Objects with Qualitative Characteristics and their Usage in Decision Making Problems

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Abstract**—**The paper presents the method developed to assess rating points of objects with qualitative indexes. The novelty of the method lies in the fact that the authors use linguistic scales that allow to formalize the values of the indexes with the help of fuzzy sets. As a result it is possible to operate correctly with dissimilar indexes on the unified basis and to get stable final results. The obtained rating points are used in decision making based on fuzzy expert opinions.

Keywords**—**complete orthogonal semantic space, qualitative characteristic, rating points.

I. Introduction

RATING points systems are widely used in various human activities (educational process, economics, techniques, etc.) and are of great importance in decision making problems. These systems make it possible to get available and timely information in the form of an aggregative index and to use it in decision making problems. The complexity of obtaining rating points for objects with qualitative characteristics results from the general complexity of the quantitative assessment of qualitative characteristics. This complexity is also associated with the necessity of taking into account characteristics and judgements of the surveyors who take decisions based on their personal assessment. As a rule, the qualitative characteristics are scored in different scales and are often incomparable in principle. The elements of these scales (as a rule, order-type scales) are transformed into scores. Such transformation needs some substantiation because stability of the final findings depends on it. The following example will make the point clear. Let us suppose that two objects got 4 and 3 points for one characteristic and 4 and 5 points for the other characteristic correspondingly. As a result of two assessments each object gets the same total score that equals 8. The conclusion is made that they have similar rating points and similar rating correspondingly. Since the order-type scale is used while assessing objects' qualitative characteristics, strictly increasing transformation Φ of this scale will be applied, that is acceptable: $\Phi(3)=3, \Phi(4)=4, \Phi(5)=7$. It is known [1] that an acceptable transformation of the values of the assessed quality characteristic is such a transformation that retains subject matter of the type of assessment involved. In accordance with the transformation applied the total score remained the same for object 1 while it changed for object 2 and has become equal 10 points. Thus the rating point of the second object has increased. The stability of the results after the acceptable transformation is violated that testifies to the fact that transformation of verbal scales' elements into scores needs some substantiation. Fuzzy set theory makes it possible to avoid the above-mentioned problems in rating systems creation. Presenting separate indexes in the form of the fuzzy sets defined in a uniform universal set and correct manipulation with their membership functions provides for adequate and stable rating points. The obtained rating points are suggested to be used in decision making problems, that guarantee successful functioning of objects with qualitative characteristics.

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II. Problem formulation

The original task while making rating points with qualitative characteristics is the task of formalization the data obtained during their assessment. The solution of this problem lies in creating expert assessment models in a uniform universal set. In the context of the instruments of the fuzzy sets theory semantic space with a wide sphere of practical applications (expert systems, decision-making support intellect systems, data analysis and complex process management) may serve as these models [2, 3].

A semantic space is a linguistic variable with a fixed term-set [4].

A linguistic variable is a set of five

$$\{X, T(X), U, V, S\},$$

where X - is a name of a variable; $T(X)=\{X_i, i=\overline{1,m}\}$- a term-set of variable X, i.e. a set of terms or names of linguistic values of variable X (each of these values is a fuzzy variable with a value from a universal set U); V - is a syntactical rule that gives names of the values of a linguistic variable X; S - is a semantic rule that gives to every fuzzy variable with a name from $T(X)$ a corresponding fuzzy subset of a universal set U [4].

A fuzzy variable is a triplet

$$\{X, U, \widetilde{A}\},$$

where X - is a name of a variable; U - its domain (a universal set) ; $\widetilde{A}$ - a fuzzy set of a universal set that describes possible values of a fuzzy variable [4].

The theoretic research of semantic spaces' properties aimed at adequacy improvement of the expert assessment models and their utility for practical tasks solution has made it possible to formulate the valid requirements to the membership functions $\mu_l(x), l = \overline{1,m}$ of their term-sets $T(X) = \{X_l, l = \overline{1,m}\}$ [2]:

1. For every $X_l, l = \overline{1,m}$ there is $\widehat{U}_l \neq \varnothing$, where $\widehat{U}_l = \{x \in U : \mu_l(x) = 1\}$ is a point or an interval.
2. Let $\widehat{U}_l = \{x \in U : \mu_l(x) = 1\}$, then $\mu_l(x), l = \overline{1,m}$ does not decrease to the left of $\widehat{U}_l$ and does not increase to the right of $\widehat{U}_l$.
3. $\mu_l(x), l = \overline{1,m}$ have maximum two points of discontinuity of the first type.
4. For every $x \in U$ $\sum_{l=1}^{m} \mu_l(x) = 1$.

The semantic spaces, whose membership functions meet the mentioned requirements were named complete orthogonal semantic spaces (COSS) [2].

Thus, the first task of this paper is to make COSS based on the data of the expert assessment of the objects' characteristics.

The second task is to determine fuzzy, crisp and interval (with a designated confidence level) rating points of the objects with qualitative characteristics. Fuzzy rating points are suggested to be used for clusterization of the objects with the expert information as a basis as well as for working out operation instructions.

III. Problem Solution

A group of N objects will be considered. These objects are being assessed for the characteristic X in the verbal scale with the levels $X_l, l = \overline{1,m}$, $m \geq 2$, that are ordered according to the intensity of manifestation of the characteristic X. The levels of the applied verbal scale uniquely specify term-set - $T(X) = \{X_1, X_2, ... X_m\}$. For a universal set COSS $U = [0,1]$ is selected. Point $x = 0$ corresponds to the total absence of characteristic X manifestation and that is why it is considered a typical point of term X_1, point $x = 1$ corresponds to total presence of characteristic X and that is why it is considered a typical point of term X_m.

Membership functions of term $X_l, l = \overline{1,m}$ shall be designated correspondingly by $\mu_l(x), l = \overline{1,m}$. The number of objects with the characteristic X intensity evaluated by the level of $X_l, l = \overline{1,m}$ shall be designated by $n_l, l = \overline{1,m}$ and $\frac{n_l}{N}, l = \overline{1,m}$ by $a_l, l = \overline{1,m}$, $\sum_{l=1}^{m} a_l = 1$, $\min(a_1, a_2)$ shall be designated by b_1, $\min(a_{l-1}, a_l, a_{l+1}), l = \overline{2, m-2}$ by $b_l, l = \overline{2, m-2}$, and $\min(a_{m-1}, a_m)$ by b_{m-1}.

Trapezoid membership functions (i.e. membership function graph is a trapezium) were chosen for modelling. Then

$$\mu_1(x) \equiv \left(0, a_1 - \frac{b_1}{2}, 0, b_1\right)$$

$$\mu_l(x) \equiv \left(\sum_{i=1}^{l-1} a_i + \frac{b_{l-1}}{2}, \sum_{i=1}^{l} a_i + \frac{b_l}{2}, b_{l-1}, b_l\right),$$

$l = \overline{2, m-2}$,

$$\mu_{m-1}(x) \equiv \left(\sum_{i=1}^{m-2} a_i + \frac{b_{m-2}}{2}, 1 - a_m - \frac{b_{m-1}}{2}, b_{m-2}, b_{m-1}\right)$$

$$\mu_m(x) \equiv \left(1 - a_m - \frac{b_{m-1}}{2}, 1 - a_m + \frac{b_{m-1}}{2} b_{m-1}, 0\right).$$

The first two parameters in brackets are abscissas of the apexes of the trapezium upper bases that are graphs of the corresponding membership functions; while the last two parameters are the lengths of the left and right trapezium wings correspondingly.

Thus, the authors offer the method of formalization of the verbal scales' elements, which allows to transform them not into scores, but into fuzzy numbers. All fuzzy numbers are defined on uniform universal set. Obtained linguistic scale (with fuzzy numbers elements) is adjusted for specific group of objects, that allows to make a comparative analysis of intensity of manifestation of the qualitative characteristics for different group of objects.

A group of N objects is being considered which are being evaluated by characteristics X_j, $j = \overline{1,k}$. Let X_{lj}, $l = \overline{1, m_j}$ be levels of the verbal scales that are used correspondingly to evaluate these characteristics. COSS with the names X_j, $j = \overline{1,k}$ and term-sets X_{lj}, $l = \overline{1, m_j}, j = \overline{1,k}$ were created. Membership function of fuzzy number $\widetilde{X}_{lj}$ that corresponds to the lth term of the j th COSS will be designated by $\mu_{lj}(x)$, $l=\overline{1,m_j}$, $j=\overline{1,k}$.

A fuzzy number [5] $\widetilde{A}$ is a fuzzy set with the membership function

$$\mu_{\widetilde{A}}(x): R \to [0,1].$$

Fuzzy numbers $\widetilde{X}_{lj}$, $l=\overline{1,m_j}$, $j=\overline{1,k}$ or their membership functions $\mu_{lj}(x)$, $l=\overline{1,m_j}$, $j=\overline{1,k}$ are referred as objects' points. Characteristic X_j point of the n th object will be designated by $\widetilde{X}_j^n$ or $\mu_j^n(x) \equiv \left(a_{j1}^n, a_{j2}^n, a_{jL}^n, a_{jR}^n\right)$, $n=\overline{1,N}$, $j=\overline{1,k}$. Fuzzy number $\widetilde{X}_j^n$ with membership function $\mu_j^n(x)$ equals one of fuzzy numbers $\widetilde{X}_{lj}$, $l=\overline{1,m_j}$, $j=\overline{1,k}$.

Weight coefficients of the evaluated characteristics will be designated by $\omega_j, j=\overline{1,k}$, $\sum_{j=1}^{k}\omega_j = 1$.

Fuzzy rating point of the n th object, $n=\overline{1,N}$ in the frame of characteristics X_j, $j=\overline{1,k}$ is determined as a fuzzy number

$$\widetilde{A}_n = \omega_1 \otimes \widetilde{X}_1^n \oplus \ldots \oplus \omega_k \otimes \widetilde{X}_k^n \qquad (1)$$

with membership function

$$\mu_n(x) \equiv \left(\sum_{j=1}^{k}\omega_j a_{j1}^n, \sum_{j=1}^{k}\omega_j a_{j2}^n, \sum_{j=1}^{k}\omega_j a_{jL}^n, \sum_{j=1}^{k}\omega_j a_{jR}^n \right), \qquad (2)$$

$n=\overline{1,N}$.

A confidence interval for precise rating point x_n, that classifies index X_j, $j=\overline{1,k}$ manifestation for the n th object, $n=\overline{1,N}$ will be determined.

At the confidence level $\mu_n(x_n) \ge \alpha, 0<\alpha<1$ we get:

$$\begin{array}{c}\sum_{j=1}^{k}\omega_j a_{j1}^n - (1-\alpha)\sum_{j=1}^{k}\omega_j a_{jL}^n \le x_n \le \\ \sum_{j=1}^{k}\omega_j a_{j2}^n + (1-\alpha)\sum_{j=1}^{k}\omega_j a_{jR}^n \end{array}. \qquad (3)$$

Fuzzy numbers $\widetilde{A}_n, n=\overline{1,N}$, $\widetilde{B}_1 = \omega_1 \otimes \widetilde{X}_{11} \oplus \ldots \oplus \omega_k \otimes \widetilde{X}_{1k}$, $\widetilde{B}_m = \omega_1 \otimes \widetilde{X}_{m_1 1} \oplus \ldots \oplus \omega_k \otimes \widetilde{X}_{m_k k}$ will be defuzzied by the method of gravity center [1].

The obtained precise numbers will be designated by $A_n, n=\overline{1,N}, B_1, B_m$.

Number $A_n, n=\overline{1,N}$ is called a rating point of the characteristic X_j, $j=\overline{1,k}$ manifestation for the n th object, $n=\overline{1,N}$.

The normed rating point of the n th object will be found with the following formula

$$E_n = \frac{A_n - B_1}{B_m - B_1}, n=\overline{1,N}. \qquad (4)$$

The rating point $E_n, n=\overline{1,N}$ will be called a medium degree of intensity of the characteristics X_j, $j=\overline{1,k}$ manifestation for the n th object, $n=\overline{1,N}$.

As a result of assessment of all the characteristics every object should be assigned one of the existing qualification levels $Y_l, l=\overline{1,p}$ in order of their rating increase.

A COSS with the term-set $Y_l, l=\overline{1,p}$ will be made according to the method described above.

Membership functions of the fuzzy numbers $\widetilde{Y}_l, l=\overline{1,p}$, that correspond to terms $Y_l, l=\overline{1,p}$ will be designated respectively by $\nu_l(x) \equiv \left(a_1^l, a_2^l, a_L^l, a_R^l\right), l=\overline{1,p}$.

To assign one of the qualification levels $Y_l, l=\overline{1,m}$ to the n th object it is necessary to identify a fuzzy number $\widetilde{A}_n, n=\overline{1,N}$ with the membership function $\mu_n(x)$, $n=\overline{1,N}$ with one of the fuzzy numbers $\widetilde{Y}_l, l=\overline{1,m}$ with membership functions $\nu_l(x), l=\overline{1,p}$.

With this aim identification indexes will be calculated

$$\beta_n^l = \frac{\int_0^1 \min(\nu_l(x), \mu_n(x))dx}{\int_0^1 \max(\nu_l(x), \mu_n(x))dx}, \ l=\overline{1,p}, \ n=\overline{1,N}. \qquad (5)$$

If $\beta_n^j = \max_l \beta_n^l$, then $Pos\left(\widetilde{A}_n = \widetilde{Y}_j\right)$ is calculated.

If $Pos\left(\widetilde{A}_n = \widetilde{Y}_j\right) = \gamma$, then the n th object is assigned qualification level Y_j with possibility γ.

The obtained fuzzy rating points are suggested to be used for object's clusterization based on the expert opinions regarding the importance of certain characteristics for the corresponding cluster. The following statement can serve as an example of such statements: "For the object's belonging to i th cluster characteristics from the first group are not very important , characteristics from the second group are rather important,…and characteristics from r th group are very important", $i=\overline{1,r}$.To formalize the linguistic terms "not important at all", "rather unimportant", "not very important", "rather important", "important", "very important" fuzzy numbers $\tilde{C}_1,...,\tilde{C}_6$ with the following corresponding membership functions may be used without limiting the continuity:

$$\mu_1(x)\equiv(0,\ 0,\ 0.2),\ \ \mu_2(x)\equiv(0.2,\ 0.2,\ 0.2),$$
$$\mu_3(x)\equiv(0.4,\ 0.2,\ 0.2),\ \mu_4(x)\equiv(0.6, 0.2,\ 0.2),$$
$$\mu_5(x)\equiv(0.8, 0.2, 0.2), \mu_6(x)\equiv(1, 0.2, 0).$$

The corresponding fuzzy rating points of the n th object for the first, second and so on r th group of characteristics will be designated by $\tilde{A}_n^1,...\tilde{A}_n^v$. Then according to the expert opinion a fuzzy number $\tilde{R}_n^i$ will be a fuzzy rating point of the n th object in the frame of the i th cluster:

$$\tilde{R}_n^i\equiv\tilde{C}_3\otimes\tilde{A}_n^1\oplus\tilde{C}_4\otimes\tilde{A}_n^2\oplus...\oplus\tilde{C}_6\otimes\tilde{A}_n^v,$$

$n=\overline{1,N}, i=\overline{1,r}$ with the membership function $\mu_n^i(x)$.

The rating points of other clusters for all the objects are obtained in a similar way in accordance with the expert opinions. The comparison of the obtained results is made on the basis $\tilde{R}_n^i, n=\overline{1,N}, i=\overline{1,r}$. For this fuzzy sets $I^i, i=\overline{1,r}$ are determined at the index set $\{1,2,...,N\}$. Membership functions' values of these sets $\mu_i(n), n=\overline{1,N}, i=\overline{1,r}$ are interpreted as belonging degree of the n th object to i th cluster. If $\sup_n x:\mu_n^i(x)=1,\ \ n=\overline{1,N}$ belongs to $\tilde{R}_k^i(x)$, then k th object is considered to be a typical representative of i th cluster and $\mu_i(k)=1$. The values $\mu_i(n), n=\overline{1,N}$ with $n\neq k$ are calculated in the following way:

$$\mu_i(n)=\max_x\min\left(\mu_n^i(x),\mu_k^i(x)\right).$$

If there are several typical representatives of i th cluster (for example, they are $k_1,k_2,...,k_p$ objects), then the values $\mu_i^l(n), l=\overline{1,p}, i=\overline{1,r}, n=\overline{1,N}, n\neq k_l$ are calculated in the following way:

$$\mu_i^l(n)=\max_x\min\left(\mu_n^i(x),\mu_{k_l}^i(x)\right).$$

At last

$$\mu_i(n)=\max_l\mu_i^l(n), n=\overline{1,N}, n\neq k_l, l=\overline{1,p}.$$

Numerical example.

The developed methods were used to determine rating points of the students [6] using the indexes of their progress in math. logic, physics, probability theory and history. Their knowledge was evaluated with traditional poor (2), satisfactory (3), good (4) and excellent (5) grades. We considered all the characteristics to have the same weight coefficients.

The evaluation results of ten students were entered into Table 1.

TABLE I
POINTS OF FIVE STUDENTS

	1	2	3	4	5
Math. logic	4	3	5	5	5
Physics	3	4	3	4	4
Prob. theory	4	4	4	3	5
History	3	3	4	4	4

The fuzzy rating points and normed rating points were calculated by the developed methods. The traditional rating points were calculated as sum of the points multiplied by their weight coefficients. The results are presented in Table 2. The fuzzy rating points are presented by their parameters there.

TABLE II
RATING POINTS OF FIVE STUDENTS (KNOWLEDGE)

	1	2	3	4	5
Fuzzy rating points	0.426 0.532 0.113 0.097	0.487 0.568 0.109 0.058	0.519 0.686 0.106 0.113	0.674 0.747 0.088 0.128	0.709 0.839 0.132 0.145
Normed rating points E_n	0.497	0.441	0.527	0.694	0.722
Traditional rating points	3.5	3.5	4	4	4.5

From Table 2 it is clear that the results obtained with fuzzy rating points considerably expand the results obtained with the average points. For example, students 1 and 2, 3 and 4 correspondingly have the same traditional points, but different normed rating points.

Thus the developed methods can be successfully applied for determination of the rating points of students.

Fuzzy rating points and normed rating points of psychophysiological and personal characteristics of five students, that have been found according to the developed methods will be considered. The results are presented in Tables 3 and Table 4.

TABLE III
RATING POINTS OF FIVE STUDENTS (PSYCHOPHYSIOLOGICAL CHARACTERISTICS)

	1	2	3	4	5
Fuzzy rating points	0.312 0.376 0.114 0.204	0.271 0.342 0.094 0.126	0.341 0.396 0.029 0.095	0.498 0.539 0.126 0.103	0.613 0.698 0.115 0.118
Normed rating points E_n	0.343	0.290	0.367	0.517	0.648

TABLE IV
RATING POINTS OF FIVE STUDENTS (PERSONAL CHARACTERISTICS))

	1	2	3	4	5
Fuzzy rating points	0.574 0.598 0.104 0.136	0.468 0.488 0.112 0.142	0.602 0.625 0.109 0.134	0.732 0.768 0.096 0.094	0.635 0.687 0.116 0.211
Normed rating points E_n	0.585	0.484	0.610	0.732	0.635

The obtained fuzzy rating points were used for student's clusterization. Four clusters were considered. First cluster – "knowledge is very important, psychophysiological characteristics are rather important and personal characteristics are very important". Second cluster – "knowledge is not very important, psychophysiological characteristics are rather important and personal characteristics are very important". Third cluster – "knowledge is rather unimportant, psychophysiological characteristics are important and personal characteristics are rather unimportant". Fourth cluster – "knowledge is not important at all, psychophysiological characteristics are rather important and personal characteristics are important".

Membership functions' values $\mu_i(n), n=\overline{1,5}, i=\overline{1,4}$ were calculated and presented in Table 5.

TABLE V
MEMBERSHIP FUNCTIONS' VALUES $\mu_i(n), n=\overline{1,5}, i=\overline{1,4}$

	1	2	3	4	5
1	0.53	0.45	0.73	0.96	1
2	0.61	0.76	0.85	1	1
3	0.77	0.68	0.82	1	0.98
4	0.79	0.66	0.83	1	1

IV CONCLUSION

The method was developed to determine rating points of objects in the frame of several characteristics assessed in verbal scales.

The novelty of the method lies in the fact that the initial information is formalized with the methods of fuzzy sets theory.

Such formalization allows to present dissimilar data in common abstract form and to operate correctly with them by their membership functions.

The transfer to the linguistic values of the characteristics makes it possible to determine fuzzy, interval (with the given confidence level), point and normed rating points. The practical application of the developed method has demonstrated its viability and validity.

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