

The Competence of Solving Mathematical Problems in the Formation of Ethical Values

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Abstract—A study and its preliminary results are presented. The research is descriptive and exploratory and it is still in process. Its objective is to develop an assessment method in the field of fostering values using competence mathematics problem solving. This is part of a more extensive research that aims at contributing to educational integration in Latin America, particularly to the development of proposals to link education for citizenship and the mathematics lessons. This is being carried out by research teams of University of Barcelona-España; University Nacional of Costa Rica; University Autónoma of Querétaro-México; Pontificia University Católica of Perú, University Nacional of Villa María- Argentina and University of Los Lagos-Chile, in the context of Andrés Bello Chair for the Association of Latin American Universities. This research was developed and implemented in Chile in 2016, using mixed research methods. It included interviews and a problem-solving math test with ethical values that was administered to students of the secondary education of the regions of Los Ríos and of the Lakes of Chile. The results show the lack of integration between the teaching of values and science discipline.

Keywords—Citizenship, ethical values, mathematics, secondary school, solving problem.

I. INTRODUCTION

THE education for citizenship is part of the curricula of every country. It is given in schools, mainly through one of the following approaches: as an independent subject, as part of another subject or subject field, or as a cross-curricular objective. This multidimensional nature of citizenship is reflected in the curricula of European countries. Schools set objectives related to it, not only in terms of knowledge of its theory, but also in terms of skills to be mastered, and attitudes and values to be fostered [1].

Education for citizenship will be more effective if it is supported by a school environment where students are given the opportunity of experiencing values and principles. One way to do this is linking it to mathematics problem solving. According to [2], mastering mathematics for citizenship does not only involve knowing the mathematics language, facts, concepts, and algorithms, but also knowing more complex processes like the mathematization of situations and problem solving.

Considering the large amount of research in the field of mathematics education, it is assumed that the competencies linked to mathematics contents are the key to specific professional competencies that future teachers should have [3], [4]. On the other hand, there is little research on the

analysis of the training of teachers [5] and the role of mathematics education in the development of cross-curricular competences [6].

Learning values as a cross-curricular competency is a topic that has been discussed lately among families, communities, and companies. It is not hard to find people who state that values have been lost [7]. According to [8], even though knowledge is always useful, it does not guarantee job opportunities or success. It is possible that labor has been given too much importance, and that schools have been oriented towards it and its economic power. This study is important and necessary because it is an answer to a topic that is crucial throughout the educational system. It is also important because it has made the integration of mathematics problem solving and fostering values possible. It is not only about doing mathematics, but also using it in context. In other words, it is not studying statistics or functions in isolation, but in situations where solutions can be given by considering values such as solidarity, participation, education for health, etc.

II. THEORETICAL FRAMEWORK

The development of citizenship in education that includes value education from a cross-curricular and interdisciplinary point of view [9] is conceptualized. To understand values education through mathematics, we start from the affective domain perspective related to achievement in mathematics. Reference [10] recognizes values/moral/ethics as a sub-domain, thus creating a tetrahedral model of interaction between values, emotions, attitudes, and beliefs. Human values, including ethics and morals, according to [11], refer to personal truths or commitments of people.

According to [12], values and beliefs are not the same, even though both constructs are related, and there is plenty of research about beliefs in mathematics education, but little about values. He distinguishes three kinds of basic values in mathematics education: "Values of mathematics", that corresponds to those that have been developed according to the particular culture; "General educational values", linked to the norms of the society and the particular school; "mathematic educational values", present in the curriculum, textbooks, the classroom, etc., as a result of the other groups of values.

Studies that reinforce the conceptual proposal and an empirical verification of the inherent link between the pedagogy of values and the holistic student welfare, including academic improvement, have increased in number and scope [13]-[16]. In some of these studies there is relevant

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information from the research and projects related to the Australian Values Education Programme. Based on evidence, it was concluded that the pedagogy of values has the potential of having a positive effect in the overall learning environment of a school, which causes strengthening the student-teacher relationship, environment, attitudes and values in the classroom, attitudes and behavior of the students, knowing and recognizing students, as well as their accomplishments.

Reference [17] used the California Academic Performance Index as a guide to show a correlation between the fostering of higher order values (character) and the improvement of academic performance. References [18], [19] explain and demonstrate that there is a similar correlation between performance character and moral character, because both are closely linked to the development of personality. Reference [20] presents more evidence of these combined effects by showing that it is the teacher who teaches quality contents in the context of effective pedagogy. He/she builds positive relationships to create strong relationships among students and through the values that arise there, academic performance is improved.

A. Problem Solving and Values

At the beginning, the study of problem solving was centered in the outcomes of the activities of the ones who solved the problems, and in how was it possible to teach more effective problem-solving methods [21]. Afterwards, the focus is shifted to the problem-solving process and the subject that solves the problem. Reference [22] quotes authors [23]-[28] from mathematics and psychology that have been considered pioneers of theories which seek to describe the problem-solving process.

Currently, problem solving from collaborative learning is one of the 21st century skills that from 2015 are measured by the PISA test (Programme for International Student Assessment). According to [29], this is worth developing, not because work in the future will be organized in this way, but because there is a strong belief that, from education, new dynamics of work, learning, and collaboration can be created.

It is evident that problem solving is still an important competence in the process of both teaching and learning, and it is even more important considering the cross-curricular topics of the latest educational reforms [30], [31]. This consideration regarding solving problems has been acknowledged in different educational reforms in Latin America. However, is it possible to state that teachers are used to teaching their discipline considering concepts, procedures, and values? From this research question and considering the Fundamental Cross-curricular Objectives of the Chilean curriculum [32] (OFT, Objetivos Fundamentales Transversales), we formulated the following objective: To design an assessment method in the field of the development of values, considering problem solving in mathematics.

It is clear that problem solving continues to be an important theme of the curriculum and even more so in the perspective of the cross-cutting themes of educational reforms undertaken in recent years. This consideration regarding the resolution

activity has been recognized in several educational reforms in Ibero-America; but, is it possible to affirm that teachers are accustomed to a teaching of the discipline where it manifests the ethic value, in addition to the conceptual and procedural? Based on this research question, we formulate the following objectives:

- to design an evaluation in the field of ethic value development considering the resolution of problems in mathematics,
- to develop a qualitative study of the values aspects, in the second year of Secondary School Education and to integrate the teaching of values and the resolution of problems in mathematics.

The formulated working hypotheses were:

- H1: There is a lack of development of skills in solving problems associated with value situations.
 H2: There is a lack of incorporation of values through the teaching of mathematics.

In this article, for reasons of space and quantity of information, the qualitative development and quantitative results obtained in this project are briefly presented.

III. RESEARCH METHODOLOGY

Mixed-methods of research were used including the administration of a test of mathematic problem solving that included ethics and values. It was considered as the population, students of secondary education pertaining to the regions of Los Ríos and of the Lakes of Chile that comprise seven provinces of the country. The sample was chosen randomly by simple random sampling. It consisted of 285 students with ages of 15 years, who were in the second year of secondary education in Chile. At this level, during the second semester of the academic year 2016, evaluation instruments were applied, which were previously validated in their entirety by content through the judgment of experts in the subject of education and mathematics, accepting a degree of agreement between them or equal to 85%.

Interviews that included situations involving values and questions related to knowledge of OFT's were also carried out. These were literally transcribed in research notes of each research session.

Three interviews were carried out with the all of the students. This was done with the purpose of obtaining through the students' opinion, a direct measure of the appropriation of values and their relation with the resolution of proposed mathematical problems.

The interviews contained 78 questions related to situations of value and eight questions related to the knowledge of the transversal axes, which were transcribed literally in the notes of each research session. These interviews included the presentation of situations related to the following 16 values: sexual education, health education, environmental education, education for peace, autonomy, self-esteem, solidarity and companionship, tolerance and respect for others, sincerity, responsibility, honesty, personal freedom, authenticity and coherence, goodness. The interviews also contained questions related to the transversal axes.

One test was designed, validated and administered to know the student's competence in solving problems, and the extent to which values teaching is encouraged and integrated with the contents of the discipline. This test contains 12 mathematical problem-solving situations involving specific values such as solidarity, fellowship, health education, environmental education, tolerance and respect, justice, teamwork, responsibility, respect for nature, kindness and self-care. These must be recognized by them through the problems that arise.

Levels of achievement were considered in relation to students' progress in solving mathematical problems, and it was estimated according to the Rasch model, adapted by [33] on the same scale with the difficulty of problems. This model posits an interaction between a person and a problem, placing personal performance based on the person's ability and the difficulty of the problem presented.

The degrees of internal discrimination were established through biserial correlation and those of difficulty for each problem and for the test in its entirety, after definition of the universally accepted statistical criteria for such execution. The reliability of the test based on homogeneity was estimated with Cronbach's alpha coefficient and was 0.89, which is considered highly adequate given the nature of the evaluative instrument and its extent.

IV. ANALYSIS OF INFORMATION AND RESULTS

Both the interviews and the application of the test of situations mathematical problems directly associated to values, contributed to the knowledge of the degree of development of the ability to solve problems in mathematics, and to the knowledge of the importance that the students give to the deepening of values considered fundamental in the current educational reform.

A. Problem Solving Test

Below are examples of problem situations and their corresponding responses by students.

Problem: "In one clinic, three physicians treat a total of 75 cancer patients on a weekly basis at no cost. If the first doctor attended five more patients, he would work as much as the second doctor, but if the third doctor gave up five patients, they would attend the same number of people as the second one. How many people attend free of charge each?"

Solve the problem and indicate what values you associate with the problem situation.

Student 1: With the value of solidarity.

Student 2: Equality, because sometimes people do not have enough money to pay the cost of a doctor.

Student 3: With respect and love for people because it is helping them.

Student 4: The question indicates that doctors work hard to do their job.

Problem: "Nutritionists worried about obesity in our country produced a graph that shows the minimum amount of calories per hour and per m² of body surface

area that is needed according to age (or years corresponds to a newborn). Two centuries ago, the mathematician Lagrange developed an interpolating polynomial that allows to "fabricate" a polynomial function which passes through the desired points. For example, with the five points in the graph, you can construct the next function: $C(x) = -0.0046x^4 + 0.2x^3 - 2.95x^2 + 16.36x + 30$, where x is the age in years and c the necessary calories. X axis age in years and Y axis necessary calories.

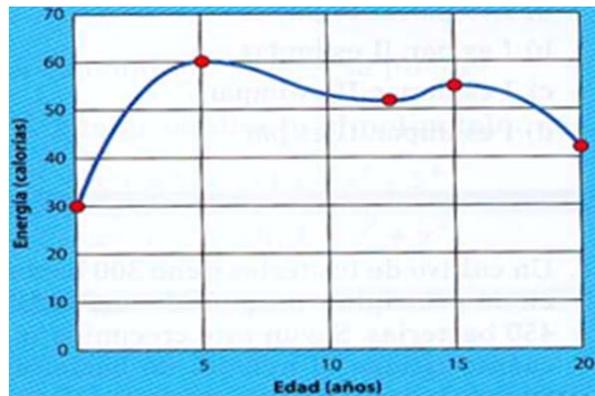


Fig. 1 Estimated calorie need per year by age

How many calories do you need at 14 years old? What happens to the requirements during the first five years of life?"

Solve the problem and indicate what values you associate with the problem situation.

Student 1: With the value of health.

Student 2: With kindness, because there is a concern for obese children.

Student 3: It is associated more than anything with the concern of one person for another.

Student 4: Improve nutrition.

Problem: "In Chile, statistics indicate that for every four Chileans who do not smoke, there are three who do. If it is known that the current population is close to 17,750,000 inhabitants, determine the approximate value of how many people are smokers. Determine how many people are the so-called "passive smokers" who, even if they do not smoke directly, aspire to the smoke that smokers reject, being even more harmful since it contains the most toxic residues of the cigarette".

Solve the problem and indicate what values you associate with the problem situation.

Student 1: To be a good person.

Student 2: Responsibility of everyone.

Student 3: I associate it with freedom value.

Student 4: I would associate it with freedom and the grounds that everyone is responsible for their actions.

Student 5: To teach people that cigarette addiction is very severe, the value I find for this is solidarity.

Problem: If the total area affected by forest fires in the Los

Lagos and Los Ríos regions and by season between 2012 and 2016 was as follows:

TABLE I
SURFACE AFFECTED BY FOREST FIRES

Year	2012	2013	2014	2015	2016
Affected surface throughout the country	40.081	43.595	90.888	101.690	17.179
Surface affected in both regions	16.000	901	40.722	3.703	3.703

What is the percentage of forest fires in both regions with respect to wildfires across the country? Represent it in a pie chart. Determine the average number of fires throughout the country. What measures should be taken to preserve our forests? What care should you take when you are a tourist in a wooded place?

Solve the problem and indicate what values you associate with the problem situation.

- Student 1: Absolute responsibility to take care of forests.
- Student 2: Caring for the environment.
- Student 3: I associate it with honesty.
- Student 4: Respect for the environment.
- Student 5: You cannot make a campfire; we have to take care of our trees, our nature.
- Student 6: Irresponsibility of citizens and lack of maturity and respect for the environment.

Problem: Luis is organizing a dinner for Christmas that will be attended by eight people, so he needs to buy three kilos of meat. He learns that the neighboring family will not have dinner for this occasion, and decides to invite the four members who make up this family. How many more kilos of meat should the host buy?

Solve the problem and indicate what values you associate with the problem situation.

- Student 1: With the value of solidarity.
- Student 2: Solidarity and good feelings, not being selfish.
- Student 3: Humility, good person.
- Student 4: I do not know.
- Student 5: I associate him with solidarity for the gesture he has done.

In general, we can indicate that students only managed to properly recognize the values that are associated with each of the situations mathematical problems at a rate of 35.7%. The remaining percentage, i.e. 64.3%, is distributed among those students who do not adequately recognize the value involved and those who declare not knowing how much to associate the mathematical situation raised. Regardless of mathematical content, 74.2% of students recognize that they do not regularly work in mathematics with problems that include ethical-value situations.

B. Levels of Achievement

The achievement levels obtained by students in the test of problem situations are presented below in Fig. 1.

The average level reached by all students was 13.45%. These results by the nature of the evaluative instrument are considered inadequate, showing that students are not clear

how to use mathematical concepts in a specific situation. It is also evident that the lack of knowledge in terms of properties and use of formulas makes problem solving difficult, and shows that the possibility of doing mathematics in the classroom occupying the resolution of problems with a certain closeness to familiar contexts for students is not an easy task.

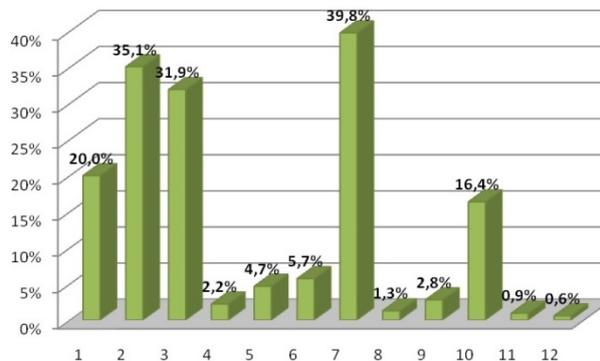


Fig. 2 Percentage of student achievement in each of the problems

C. Interviews

In order to obtain a direct measure through the students' opinion about the degree to which they recognize values-related behaviors, three individual interviews were conducted.

Next, according to the analysis of the interviews, Fig. 3 shows the percentage in which the appropriation of these values in students is manifested.

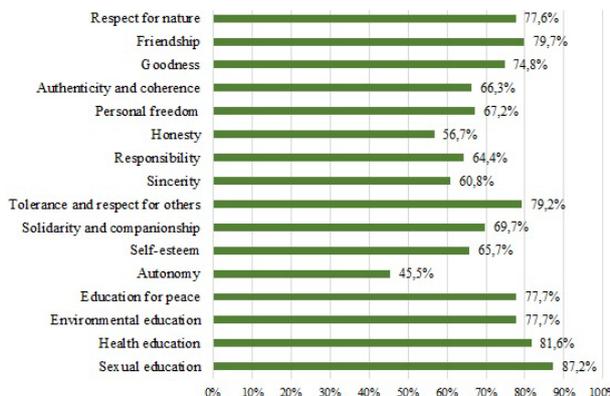


Fig. 3 Percentage of values in students

According to Fig. 3, we can indicate in general that the four highest assignments of the described values correspond to sexual education (87.2%), health education (81.6%), friendship (79.7%) and tolerance and respect for others (79.2%), while the four minors correspond to autonomy (45.5%), honesty (56.7%), sincerity (60.8%) and responsibility (64.4%).

Regarding the mathematics teacher's opportunity to promote positive values and attitudes, in addition to giving tasks in mathematics that involve them, students answer mostly:

"yes, but when there are bad behaviors during the class". "The math teacher advises us more about our future or it makes it clear that if you do not study you will

be a failure in life". As well as the following: "No, they have not given me such tasks; if they have given me exercises, but they do not imply values".

Some of the students state that during math class they have the opportunity to demonstrate their positive values and attitudes, and they argue as follows:

"yes, helping or explaining to my classmates when they do not know how to perform the exercises", "yes, talking more with the teacher and dealing with other types of issues", while others do not have an answer. Finally, most respond that the teacher does not constantly rely on mathematical content to promote positive values and attitudes: "no, he only dedicates himself to passing his subject", "the problems are pure mathematics."

D. Conclusions and Discussion

This study reveals a problem in the treatment of transversal axes, since as a teaching endowed with specific objectives and contents, which must be assumed by the curriculum as a whole, they are not fully integrated into the curricular structure in the different areas of the teaching, which do not contribute significantly to strengthen the ethical-values formation of students, to guide, on the one hand, the process of growth and personal autonomy, and on the other, the way in which students relate with other people and with the society [31], [32].

Their inclusion in the different educational projects in the state secondary schools are not taken into account in the classroom programs, nor are they worked in specific didactic units such as mathematics, they are only worked at particular moments with which they are transformed only in specific and episodic activities not inserted in an important way in the curriculum of the educational establishment.

Specifically in mathematics, we can say that teachers are not accustomed to a teaching of the discipline, where it manifests in addition to the conceptual and procedural value. On the other hand, the problem situations approach revealed the lack of development of skills in problem solving, its dependence and availability with doing, besides the lack of incorporation and appropriation of values through the subject. Therefore, both hypotheses of the study were verified.

Even when the student manifests himself by giving importance to certain values, he is not able to recognize them in the situations presented in mathematical problems. In addition, the students assert that they do not regularly work with mathematics problems that include ethical-value situations, with which it is verified that the teachers miss the opportunity they have to formulate mathematical problems situations that involve the effective appropriation of values in the students, which in the case of autonomy, honesty and sincerity, turn out to be the values of lower appropriation of secondary school students.

To associate the processes of teaching and learning of a discipline with the main ethical values is to close the gap of the contradiction that exists between educating and instructing. It is essential to put aside the current tendency to make a greater emphasis of instruction in relation to

education, which is why the integral training of the student has weakened. It is about finding a balance between the two; it is as important to educate as it is to instruct.

It consists of connecting the contents contributed and developed in academic areas with all those that emanate spontaneously from daily life. It is also achieved when students become individuals capable of transforming reality, but this is only possible with a better understanding of it, acquiring knowledge, and at the same time, thoroughly analyzing the phenomena that happen in everyday life. It is a question, of which, the student assumes a critical and constructive attitude in favor of the development of the fundamental ethical values that comprise the basic values for life and the coexistence that underlies the democratic system.

There is no doubt that educational transversality contributes to students' meaningful learning from the connection of disciplinary knowledge with the social, cultural and ethical themes and contexts present in their environment. Therefore, knowing, doing and being around the environment, self-care and prevention, democratic coexistence, affectivity and sexuality, are integral learning that allow full development as individuals and as society.

In the light of this study, in general we can affirm that, it is essential that the acquisition of academic knowledge, social formation and training in values, make up a whole of the educational process, and in particular, that secondary education can and must, like the family, make an important contribution at the ethical level by promoting in students the capacity to exercise degrees of freedom and autonomy with responsibility, to perform acts of generosity and solidarity, within a framework of recognition and respect for the truth, where personal convictions can coexist with respect for the attitudes and values of others.

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