

# The Application of the Queuing Theory in the Traffic Flow of Intersection

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**Abstract**—It is practically significant to research the traffic flow of intersection because the capacity of intersection affects the efficiency of highway network directly. This paper analyzes the traffic conditions of an intersection in certain urban by the methods of queuing theory and statistical experiment, sets up a corresponding mathematical model and compares it with the actual values. The result shows that queuing theory is applied in the study of intersection traffic flow and it can provide references for the other similar designs.

**Keywords**—Intersection, Queuing theory, Statistical experiment, System metrics.

## I. INTRODUCTION

WITH the development of economy, vehicles maintain a substantial increase in volume of China, queuing phenomenon is so common in road traffic. Intersection is the main concentrated area of stream of people and vehicles; also, it is one infrastructure construction that connecting the roads to make it play network functions. In daily life, traffic congestion responses to the intersection directly. It is so clear that road intersection will be the bayonet of traffic capacity and safety. Therefore, it is significant to study the intersection flow to improve the congested traffic and maintain social order.

In the early 20th century, queuing theory originated from the Danish engineer Erlang's study of telephone exchange efficiency of communication system. After the section world war, especially with the rapid development of computer and communication technology, queuing theory got attention and developed fast, also, it became an important branch of operations research and its corresponding disciplines theory and reliability theory were developed.

In the mid-1930-s, queuing theory was recognized one important subject when W.Feller recommended birth and death process. In the early 1950s, D.G.Kendall researched queuing theory systematically by the method of Markov chain and made it develop further. In the 1960s, the projects studied complicatedly in queuing theory, it is so difficult to get the exact solution that people began to study the approximate method [1], [6], [8].

In the traffic engineering, 1936, Adams considered the

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This paper is supported by Shandong province university scientific research project (No. J13LN34) and Qingdao science and technology development project, China (No. KJZD-13-27-JCH).

pedestrian delay problem by queuing theory that the intersection which not set the traffic signals, then, queuing theory had been widely used in traffic control. Such as the study of vehicle delay, traffic capacity, configuration light time, the design and management of traffic facilities for the park and station and so on.

At present, queuing system model has been widely used in all kinds of management system. Such as production management, inventory management, business management, transportation, banking, medical services, computer design and performance evaluation, and so on.

## II. BASIC KNOWLEDGE

Queuing theory is the mathematical theory and method of queuing system (stochastic system). In daily life, people will encounter all sorts of queuing problems, such as, standing at bus stops, going to hospital, and going to the ticket office to buy the tickets and so on. In these problems, bus and passengers, doctor and patients, conductor and the buyers forms a queuing system or service system respectively; the former can be regarded as service agencies and the latter can be regarded as customers.

The queue can be tangible queue may also be intangible queue. For example, several passengers make telephone call to order train tickets at the same time, if a passenger is on the phone, can only wait for the other passengers, this form of queue is invisible. The people or some objects can be the queue, such as semi-finished products for processing in the production line, machine waiting for maintenance, and the information waiting for computing center to process, etc.

Queuing theory consists of three parts: input process, queuing rules and service agencies. The schematic diagram as follows:

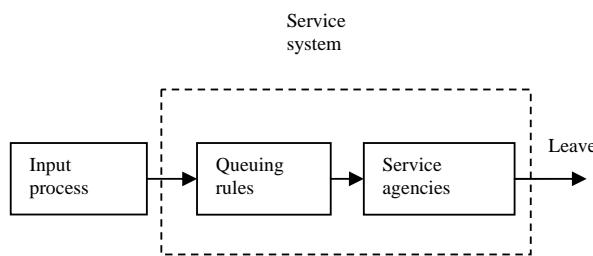


Fig. 1 The composition of queuing system

Queuing theory mainly studies three aspects:

- 1) Statistical inference: in this part, it mainly sets up the

mathematical model based on data, solves the problem by appropriate method of Queuing theory, and achieves the rationalization of queuing system.

- 2) The inertia of system: namely the probability of regularity of quantity index about queuing, mainly concludes: the distribution of the waiting time of a customer, busying period distribution, the distribution of the queuing length that the customer waiting and so on. It mainly includes two states: the steady state under statistical equilibrium; instantaneous state.
- 3) The system optimization problems: its purpose is to make all systems produce best results, design correctly, and move effectively. In general, the system optimization problem is divided into two categories: the system design optimization and the system control optimization. The former is called a static optimization problem, which goal is to make the system achieve maximum benefit, or under a certain index, the system is the most economical. The latter is called dynamic optimization problem. It is to say, for a given system, how to run to make a objective function value to the optimal.

### III. MODELING

Queuing theory can be divided into single channel queuing system and multi-channel queuing system. This paper mainly researches the performance index under the steady state.

#### A. Single Channel Queuing System

The single channel queuing system is called  $M/M/1$  system. Assume that customers arrive randomly, follows Poisson distribution,  $\lambda$  is the average arrival rate,  $\mu$  is the average output rate,  $\rho = \frac{\lambda}{\mu}$  is traffic intensity or utilization coefficient [6].

When  $\rho < 1$ , the arrival rate is less than the rate of output, then the intersection traffic will be smooth. If  $\rho \geq 1$ , the arrival rate is greater than the rate of output and then the queuing length will be infinity, the system is not steady. Therefore,  $\rho < 1$  is the necessary and sufficient condition for the system to be steady.

Combined with the Little formula, the quantity indexes of single channel queuing system can be obtained, as follows:

- 1) The probability of no vehicle in the system:

$$P_0 = 1 - \rho$$

- 2) The probability of  $n$  vehicles in the system:

$$P_n = \rho^n (1 - \rho)$$

- 3) The average number of vehicles in the system:

$$L = \frac{\rho}{1 - \rho}$$

- 4) The average queuing length of vehicles in the system:

$$L_q = L \cdot \rho$$

- 5) The average staying time of vehicles in the system:

$$W = \frac{L_q}{\lambda} + \frac{1}{\mu}$$

- 6) The average waiting time of vehicles in the system:

$$W_q = W - \frac{1}{\mu}$$

#### B. Multi-Channel Queuing System

The multi-channel queuing system is called  $M/M/N$  system. Its traffic intensity is  $\frac{\rho}{N}$  which is different from the single channel queuing system. The system is stable when  $\frac{\rho}{N} < 1$ , otherwise, it is not. At the same time,  $\frac{\rho}{N} < 1$  is the necessary and sufficient condition for the system to be steady[8].

- 1) The probability of no vehicles in the system:

$$P_0 = \frac{1}{\sum_{k=0}^{N-1} \frac{\rho^k}{k!} + \frac{\rho^N}{N!(1-\rho/N)}}$$

- 2) The probability of  $k$  vehicles in the system:

$$\begin{cases} P_k = \frac{\rho^k}{k!} P_0 & (\text{when, } k < N) \\ P_k = \frac{\rho^k}{N!N^{k-N}} P_0 & (\text{when, } k \geq N) \end{cases}$$

- 3) The average queuing length of vehicles in the system:

$$L_q = \frac{\rho^{N+1}}{N!N} \cdot \frac{P_0}{(1-\rho/N)^2}$$

- 4) The average number of vehicles in the system:

$$L = L_q + \rho$$

- 5) The average staying time of vehicles in the system:

$$W = \frac{L_q}{\lambda} + \frac{1}{\mu}$$

- 6) The average waiting time of vehicles in the system:

$$W_q = \frac{L_q}{\lambda}$$

### C. Establish the Statistical Law of Intersection

Takes the traffic of several intersections of Laoshan District of Qingdao for example, especially, the Shenzhen intersection. We note the numbers of vehicles in every direction when the traffic lights change each cycle [2]. The data can be divided into 5 groups,  $X_t$  is the number of vehicles and  $f_t$  is the time that belongs to the every part of vehicles. Now we list the statistical result in the west as shown in the Table I:

TABLE I						
TIME : 2014/4/15	10:00-11:00	DIRECTION: WEST				
The number of vehicles $X_t$	12-14	15-17	18-20	21-23	24-26	above 27
Times $f_t$	7	12	10	8	7	1

We validate the number of the arrived vehicles in the input process weather obey the Poisson distribution by the  $\chi^2$  hypothesis testing method [7].

First, it needs to estimate the parameter  $\lambda$  in Poisson distribution by the maximum likelihood method.

Assume the whole

$$X \sim \pi(\lambda)$$

$$P(X = k) = \frac{\lambda^k}{k!} e^{-\lambda}, k = 0, 1, 2, \dots \quad (1)$$

Then the likelihood function of parameter  $\lambda$ :

$$L(\lambda) = \prod_{i=1}^n P(X = x_i) = \prod_{i=1}^n \frac{\lambda^{x_i}}{x_i!} e^{-\lambda} = \frac{\lambda^{\sum x_i}}{x_1! \dots x_n!} e^{-n\lambda} \quad (2)$$

Take the logarithm on both sides and the likelihood equation is obtained:

$$\frac{d \ln L(\lambda)}{d \lambda} = -n + \frac{1}{\lambda} \sum_{i=1}^n x_i = 0 \quad (3)$$

Solve it:

$$\hat{\lambda} = \frac{1}{n} \sum_{i=1}^n x_i = \bar{x} \quad (4)$$

Also

$$\frac{d^2 \ln L(\lambda)}{d \lambda^2} \Big|_{\lambda=\bar{x}} = \frac{-n \bar{x}}{\lambda^2} \Big|_{\lambda=\bar{x}} = -\frac{n}{\bar{x}} < 0 \quad (5)$$

So the maximum likelihood estimator of parameter

$\lambda$  is  $\hat{\lambda} = \bar{x}$ .

The average arrival rate is 18.8 per cycle based the Table I.

Apart, the probability is  $P(X_t) = \frac{\lambda^{X_t}}{X_t!} e^{-\lambda}$  when the number

of vehicles is  $X_t$ , the probability is  $P_n = \sum_{k=a_{n-1}}^{a_n} \frac{\lambda^k}{k!} e^{-\lambda}$  of each group,  $a_{n-1}$  is the lower limit of the  $n-1$  group,  $a_n$  is the upper limit of the  $n-1$  group,  $\bar{f}_n = 45P_n$  is the theoretical frequency,  $\lambda$  is the average number of arrival vehicles.

From the above data and formulas, we can calculate the  $\chi^2$  value:

$$\chi^2 = \sum_{n=0}^5 \frac{(f_n - \bar{f}_n)^2}{\bar{f}_n} = 3.092$$

Because of estimating a

parameter  $\lambda$  when calculates the probability,  $r = 1$ . The degree of freedom is  $k - r - 1 = 4$ ,  $\alpha = 0.05$  is selected, referring the Chi-square distribution table,  $\chi^2_{0.95}(6-1-1) = \chi^2_{0.95}(4) = 9.488$ ,  $\chi^2 < \chi^2_{0.95}(4)$ . So the number of the arrived vehicles per unit time obeys the Poisson distribution. And the other 3 directions can be verified through the same method, but their parameters are different.

### IV. APPLICATION EXAMPLE

It takes money and people to cut or add the fixed lanes that the number of the lanes should be confirmed in the beginning of construction design [3]. Weather the existing establishment of the lanes is reasonable, the model can validate it.

We select the 3 lanes of Shenzhen intersection in one direction, take the vehicle flow of April 13, 2014 to April 15, 2014 as research objects. Suppose the time of every vehicle through the intersection is 5s in view of pedestrian and traffic singles. Measure the vehicle flow of April 13, 2014 from 10:00 to 11:00 am is 734, the vehicle flow of April 13, 2014 from 10:00 to 11:00am is 795, the vehicle flow of April 15, 2014 from 10:00 to 11:00am is 847. Because of undergoing 45 cycles in one hour, now note the data of the vehicles flow of 3 cycles as shown in the Table II:

TABLE II THE MEASURED DATA								
time	1	2	3	4	5	6	7	8
13 vehicles	51	50	43	46	47	51	55	45
14 vehicles	45	52	54	48	53	67	58	60
15 vehicles	75	54	67	52	54	63	47	60
time	9	10	11	12	13	14	15	sum
13 vehicles	57	45	55	56	44	55	44	734
14 vehicles	49	55	60	45	60	50	39	795
15 vehicles	62	59	46	56	58	52	52	847

Takes the average traffic volume as standard, we use the model to validate the existing establishment of driveways is reasonable or not.

1) If the driveway is single, then

$$\lambda = \frac{792}{3600} = \frac{11}{50}, \mu = \frac{1}{5}, \rho = \frac{\lambda}{\mu} = \frac{11}{10} > 1$$

- 2) Conversely, the system metrics [4] of intersection as shown in Table III, the probability of k vehicles in the system is shown in Table IV:

TABLE III  
THE SYSTEM METRICS OF INTERSECTION

system	metrics			
	$L_q$	$L$	$W$	$W_q$
$M / M / 2$	0.405	1.505	6.841	1.841
$M / M / 3$	0.240	1.340	6.089	1.089
$M / M / 4$	0.009	1.109	5.042	0.042

TABLE IV  
THE PROBABILITY OF K VEHICLES IN THE SYSTEM

system	$P_k$			
	$P_0$	$P_1$	$P_2$	$P_3$
$M / M / 2$	0.290	0.319	0.176	0.097
$M / M / 3$	0.327	0.360	0.198	0.073
$M / M / 4$	0.332	0.365	0.201	0.074

system	$P_k$			
	$P_4$	$P_5$	$P_6$	$P(k > 6)$
$M / M / 2$	0.053	0.029	0.016	0.009
$M / M / 3$	0.027	0.010	0.004	0.001
$M / M / 4$	0.020	0.006	0.002	0

By analyzing the data in the Table III, the service indicators of system are in decline with the increasing of the driveways. Therefore the increasing of the lanes has a positive impact on the vehicle flow. By analyzing the data in the Table IV, the probability of six or more vehicles is 0 in the intersection in the system of  $M / M / 4$ . So the 4 driveways is the first selection of the design. At the same time, the establishment of the driveway needs to consider various factors. The more lanes, the shorter the vehicles queue length. But it will be unnecessary waste in some degree if the scale of the construction is large, the large investment, and the high operating costs. So the system of  $M / M / 3$  is the ideal selection to ensure the smooth and fast traffic and save resources. This application example only proves it is feasible that the model can be used to confirm the number of the lanes by analyzing the vehicle flow of the intersection.

## V. CONCLUSION

The paper sets up the queuing model, analyses the traffic flow [5] of Shenzhen intersection through analyzing the queuing theory deeply, and uses the model to analyze the settings of the lane that based on the certain degree of accuracy. From the paper, the theoretical data is consistent with the reality. Therefore, it is economic that the method of the system metrics in confirming the number of the lanes of the intersection and it can provide references for similar design.

## REFERENCES

- [1] Yingshi Niu. Operations research (M). Shan xi: the press of the Xian Jiaotong University.2006.
- [2] Woye Liu. Hongchen Wu. Zhenyu Wu. The application of queuing theory in the traffic engineering. Mathematical statistics and management. 1999.15(1):19-22.
- [3] Xiaohua Bian. Jinsong Zhuang. Dayi Qu. Jian Yang. The analysis of the statistical method of vehicle queue of the signalized intersection. Journal of Qingdao Technological. 2011.32(6):86-90.
- [4] Zheng Huaping. He Xia. The analysis of the traffic flow of the intersection based on the queuing theory. Scientific and technological information. 2010.35:377-378.
- [5] Tao Chen. Zefei Tan. Yao Liu. The improving method of the intersection traffic problems of the urban road. Traffic engineering. 2013.7:56-58.
- [6] Baoqun Duan. Hongsheng Xi. Yaping Zhou. Queuing system performance analysis and Markov control process(M). Hefei: the press of University of science and technology of China, 2004.
- [7] Fasheng Liu. Queuing theory and the customer service system of bank (J). The journal of Shandong Jiaotong University, 2003(1):83-86.
- [8] Chuanlai Lu. The queuing theory (The second edition) (M). Bei jing. The press of Beijing institute of posts and telecommunications, 2009.