The Application of Homotopy Method In Solving Electrical Circuit Design Problem

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Abstract-This paper describes simple implementation of homotopy (also called continuation) algorithm for determining the proper resistance of the resistor to dissipate energy at a specified rate of an electric circuit. Homotopy algorithm can be considered as a developing of the classical methods in numerical computing such as Newton-Raphson and fixed point methods. In homotopy methods, an embedding parameter is used to control the convergence. The method purposed in this work utilizes a special homotopy called Newton homotopy. Numerical example solved in MATLAB is given to show the effectiveness of the purposed method

Keywords: electrical circuit homotopy, methods, MATLAB, Newton homotopy.

I. Introduction

Finding the proper resister of an electrical circuit is an essential task in simulating electronics circuits. It involves solving a nonlinear algebraic equation. Numerous electronic circuits employ nonlinear elements. Equations that describe such electronic circuits are nonlinear algebraic equations which we can call them as circuit's equation. Such equations have multiple solutions. There are many methods used to solve such circuit's equations, such as Newton-Raphson method. This method has been proven globally convergent only under unrealistically restrictive conditions [1]. They sometimes fail because it is difficult to provide a starting point sufficiently close to an unknown solution [1-14]. Therefore, many circuit designers experience difficulties in finding the proper resister, especially for bipolar analog integrated circuits [2], [16].

To overcome this convergence problem, globally convergent homotopy methods have been studied by many researchers from various view points [4], [11]. By these studies, the application of the homotopy methods in practical circuit simulation has been remarkably developed electrical circuit design problem with the theoretical guarantee of global convergence. However, the programming of sophisticated homotopy methods is often difficult for non-experts or beginners. [3]

There are several types of homotopy methods, as one of the efficient methods for solving elelctrical circuits design problem, the Newton homotopy (NH) method is well known [6]. For this method, many studies have been formed from various viewpoints [3]. Since the idea of Newton homotopy is introduced, the path following often becomes smooth. However, in this method, the initial point is sometimes far from the solution [3], [16] because it is given as a solution of an electrical circuit design problem.

In this paper, we discuss the use of Newton homotopy method (NHM) to solve the nonlinear algebraic equation of the proper resistance of the resistor in the electrical engineering design problem. Newton homotopy connects a trivial solution of this nonlinear algebraic equation to the desired unknown solution. The proposed method is almost globally convergent algorithm [20]. By numerical examples, it is shown that the proposed method find the proper resistor of an electrical engineering design problem more efficiently than the conventional methods. It is also shown that the proposed method can be easily implemented on MATLAB. [4], [6]

II. Electrical Circuit Design Problem

Electrical engineers often use Kirchhoff's laws to study the steady-state (not time-varying) behavior of electric circuits. Such steady-state behavior appears also in the application of linear algebra. Another important problem involves circuits that are transient in nature where sudden temporal changes take place. Such situation occurs following the closing of the switch in an electrical circuit. In this case, there will be a period of adjustment following the closing of the switch as a new steady-state is reached. The length of this adjustment period is closely related to the storage properties of the capacitor and the inductor. Energy storage may oscillate between these two elements during a transient period. However, resistance in the circuit will dissipate the magnitude of the oscillations.

The flow of current through the resistor causes a voltage drop given by

$$V_R = iR$$

Where i = the current and R = the resistance of the resistor. When R and I have units of ohms and amperes, respectively, V_R has units of volts.

Similarly, an inductor resists changes in current, such that the voltage drop V_L across it is

$$V_L = L\frac{di}{dt}$$

Where L = the inductance. When L and I have units of henrys and amperes, respectively, V_L has units of volts and t has units of seconds.

The voltage drop across the capacitor (V_C) depends on the charge (q) on it:

$$V_C = \frac{q}{C}$$

Where C = the capacitance. When the charge is expressed in units of coulombs, the unit of C is the farad.

Kirchhoff's second law states that the algebraic sum of voltage drops around a close circuit is zero. After the switch is closed we have

$$L\frac{di}{dt} + Ri + \frac{q}{C} = 0$$

However, the current is related to the charge according to

$$i = \frac{dq}{dt}$$

Therefore,

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = 0 \qquad (1)$$

This is a second-order linear ordinary differential equation the can be solved using the methods of calculus. This solution is given by

$$q(t) = q_0 e^{-Rt/2L} \cos\left[\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}\, t\right] \quad (2)$$

Where $t = 0$, $q = 0 = V_0 C$, and V_0 = the voltage from the charging battery. Equation (2) describes the time variation of the charge on the capacitor.

A typical electrical engineering design problem might involve determining the proper resistor to dissipate energy at a specified rate, with known values for L and C.

It is necessary to solve equation (2) for R, with known values of q, q_0, L, and C. However, a numerical approximation technique must be employed because R is an implicit value in equation (2). The Newton-Raphson method can be used for this purpose. Based on the advantages of homotopy methods, this equation will be used to find the value of R. [15].

III. Newton Homotopy Method

First, we describe roughly the theoretical basis of the method. Suppose we need to solve the following equation:

$$f(x) = 0 \qquad (3)$$

where f is a continuously differentiable function from R^n into itself. To find the solution, we can construct a homotopy. The term *homotopy* means a continuous mapping H is defined on the product $R^n \times I$ to R^n, $H: R^n \times I \rightarrow R^n$ where I is the unit interval $[0, 1]$ such that

$$H(x, t) = tf(x) + (1 - t)g(x) \qquad (4)$$

from $g(x) = 0$ to $f(x) = 0$, where the solution of $g(x) = 0$ can be found trivially. For example, choose

$$g(x) = f(x) - f(x^0) \qquad (5)$$

This form of the homotopy is termed the Newton homotopy because some of the ideas behind it come from the work of Sir Isaac Newton himself.

Suppose that x is a function of t, then, from equation (4) and (5), we have

$$H(x,t) = f(x) - (1-t)f(x^0) \qquad (6)$$

Then, the solution trajectory is defined by

$$H(x,t) = f(x) - (1-t)f(x^0) \qquad (7)$$

It is seen from equation (7) that, at $t = 0$, the solution of the equation (7) is already known. For different values of t, the equation will result in different solutions. At $t = 1$, the solution of equation (7) is identical to the desired unknown solution. To follow the above predetermined trajectory, we can differentiate equation (7) with respect to t, yielding

$$H_x(x,t)\dot{x}(t) + H_t(x,t) = 0 \qquad (8)$$

It follows, from equation (8), that we can find the desired solution by using any existing ordinary differential equation solver to integrate

$$\dot{x}(t) = -H_x^{-1}H_t = 0 \; ; x = x^0$$

up to $t = 1$, provided that is nonsingular for all t in the interval $[0, 1]$ [6].

IV. Proposed Method

Based on the above theory, propose a new application of Newton homotopy for solving the equation of electrical circuit design problem as follows:

$$H(R, t) = f(R) + (1-t)f(R^0)$$

where R is the resistance of the resister. It is observed that the first term of the above equation is equal to the equation of electrical circuit design problem and R^0 is the initial value for the resistance R. We solve the homotopy equation

$$H(R, t) = f(R) + (1-t)f(R^0) = 0$$

The value of R can be obtained using the following mathematical formula (Euler's homotopy formula):

$$H_R(R,t)\dot{R}(t) + H_R(R,t) = 0$$
$$\dot{R}(t) = -H_R^{-1}H_t = 0 \,; R = R^0 \qquad (9)$$

Thus, the proposed method is summarized as follows:
1. Solve the homotopy equation to obtain the initial value R^0.
2. Choose Δt.
3. Follow the trajectory curve from $t = 0$ to $t = 1$ to find the desired solution using Euler's homotopy formula.

V. Numerical Example

Solve the equation of electrical circuit design problem for R if the charge is dissipated to 1 percent of its original value $q_0 = 0.01q$ in t = 0.05 s, with $L = 5\,H$ and $C = 10^{-4}\,F$.

Solution

We apply the algorithm in IV to find the value of R.
1. Solving the homotopy equation gives the initial value as any real number. We suggest that $R^0 = 0$.
2. Choose $\Delta t = 0.25$.
3. Implement the formula (9) in MATLAB for 4 iterations. The solution at $t = 0$ is the value $R = 328.1515\,\Omega$.

If we solve this equation of electrical circuit design problem using, for example, bisection method, twenty-one iterations give $R = 328.1515\,\Omega$.

VI. Conclusion

We have described how Newton homotopy can be exploited to find the value of the resistance R in an electrical circuit design problem. This homotopy method is conceptually very simple has almost globally convergent characteristics. Another advantage is with certain number of iterations, the desired solution will be obtained. Based this advantage, in this method, we didn't need to control the value of the error as in

the classical methods as Bisection, fixed-point and Newton-Raphson methods. In particular, the proposed method is especially attractive compared with the existing methods. Naturally, the proposed method take more computing time, this is the common disadvantage of homotopy methods. At the expense of computing speed we can achieve a much wider convergent region.

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