

# Temporal Signal Processing by Inference Bayesian Approach for Detection of Abrupt Variation of Statistical Characteristics of Noisy Signals

Farhad Asadi, Hossein Sadati

**Abstract**—In fields such as neuroscience and especially in cognition modeling of mental processes, uncertainty processing in temporal zone of signal is vital. In this paper, Bayesian online inferences in estimation of change-points location in signal are constructed. This method separated the observed signal into independent series and studies the change and variation of the regime of data locally with related statistical characteristics. We give conditions on simulations of the method when the data characteristics of signals vary, and provide empirical evidence to show the performance of method. It is verified that correlation between series around the change point location and its characteristics such as Signal to Noise Ratios and mean value of signal has important factor on fluctuating in finding proper location of change point. And one of the main contributions of this study is related to representing of these influences of signal statistical characteristics for finding abrupt variation in signal. There are two different structures for simulations which in first case one abrupt change in temporal section of signal is considered with variable position and secondly multiple variations are considered. Finally, influence of statistical characteristic for changing the location of change point is explained in details in simulation results with different artificial signals.

**Keywords**—Time series, fluctuation in statistical characteristics, optimal learning.

## I. INTRODUCTION

**S**IGNAL processing is applied in broad range of science from neuroscience to dynamical system applications. Moreover nowadays an advanced sensory device is able to gathering reliably data from systems. These sensors generate high dimensional data streams which can be modeled as signals of system.

Signal processing is essential for understanding of the phenomena in systems that produce the data. There are local positions in the complex noisy signal where the behavior in the data values has changed sharply. They may show the situation in temporal zone when external inputs or events have caused the dynamical system to behave differently.

The problem of detecting local changes in signals has been mostly researched in the class of segmentation and scaling where segmentation of the signal is modeled by a known function. Subsequently, abrupt changes are detected as the points in signal where two near segments of the signal are

connected. Generally complex systems have critical variation that is called change point at which the system changes from one state to another abruptly. In these situations, signal processing analysis for making inference about state evolution of system is useful. This detection of variation in noisy signal is more difficult due to the noisy components of interferences [1]-[4]. However, for modeling and understanding of parameters on signal, uncertainty detection and removing are important. For instance in learning theory we should adjust different learning rates to signal for explaining of variation [5]. In some references like [6] is used a neural network model for explaining the behavior of signal and fitting of learning rates parameters. Moreover, in other fields, like process control [6], [7] cardiograph analysis [8], [9] and EEG signal processing [10], [11], different signal processing algorithms are made for removing and detection of uncertainty. In above references, the structure of signal processing is offline which needs the entire data for prediction [12], [13]. In other hand, online approaches have some important constraints such as needing to high computational time for removing and segmentation of signal [14], [15]. Generally, for detecting of variation in statistical data, decomposition of signal is a vital step. Moreover, in dynamical system when a statistical characteristic of system is changed hidden states of system is going to bifurcation.

In some algorithms, a distance threshold is programmed for detection of these variations when variation is bigger of this threshold [16], [17]. In this paper different artificial noisy signal with predefined statistical characteristics in different temporal zone of signal is programmed and then a Bayesian inference approach for detection of these variations is used and performance of algorithm for detection in different noisy signals is investigated.

Finally, the structure of article is as follows. Firstly, overall structure of algorithm is shown schematically in Section II and main formulation of algorithm is reported. Then, setup of simulations and whole characteristics of artificial data series and approaches are discussed. Furthermore, data characteristics for each segment between change points are introduced. Different simulations for detecting change point location are done and effect of data characteristics is explained and investigated numerically.

## II. STRUCTURE ALGORITHM OF STEP DETECTION

This paper is related with the problems of finding abrupt changes in signal. And overall summary of steps in algorithm

Farhad Asadi has Master degree in Mechanical engineering in dynamic and control, K. N. Toosi, Tehran, Iran (phone: +989357430949, e-mail: farhadasadi1990@gmail.com).

Hossein Sadati is Assistant professor in Mechanical Engineering at K. N. Toosi, University, Tehran, Iran (e-mail: sadati@kntu.ac.ir)

is shown in Fig. 1. In this approach, a signal is segmented to time series and density of signal is depending to a value that varies at unknown time instants. Then signal is sampled with parameters that depend to probability of changing abrupt variation in signal. So a posterior run length is made so inference of the unknown change point is needed. The posterior run length can be formulated as:

$$p(x_{t+1}|x_{1:t}) = \sum_{r_t} p(x_{t+1}|x_{1:t}, r_t) p(r_t|x_{1:t}) = \sum_{r_t} p(x_{t+1}|x_t^{(r)}) p(r_t|x_{1:t}) \quad (1)$$

where  $x_t^{(r)}$  is to the last  $r_t$  observations of  $x$  in signal, and the run length posterior can be calculated by normalizing the joint likelihood:  $p(r_t|x_{1:t}) = \frac{p(r_t, x_{1:t})}{\sum_{r_t} p(r_t, x_{1:t})}$ .

The joint likelihood can be updated using a scheme by (2):

$$\begin{aligned} \gamma_t &= p(r_t|x_{1:t}) = \\ \sum_{r_{t-1}} p(r_t|r_{t-1}, x_{1:t}) &= \sum_{r_{t-1}} p(r_t, x_t|r_{t-1}, x_{1:t-1}) p(r_{t-1}|x_{1:t-1}) = \\ &= \sum_{r_{t-1}} p(r_t|r_{t-1}) p(x_t|r_{t-1}, x_t^{(r)}) p(r_{t-1}, x_{1:t-1}) \quad (2) \end{aligned}$$

It is common to estimate the (log) marginal likelihood of at time  $T$ , as it can be disjointed into the one-step-ahead predictive likelihoods.

$$\log p(x_{1:T}|\theta) = \sum_{t=1}^T \log p(x_t|x_{1:t-1}, \theta) \quad (3)$$

In our simulation, signal is produced noisy by adding Gaussian random noise to it. Thus, sample-by-sample variation of signal characteristics can show both noise and abrupt change. However, the positions of change-points in signal are not determined. Thus, for computing next abrupt change, all possible run lengths in signal should be considered and weight them by the probability of the run length parameter.

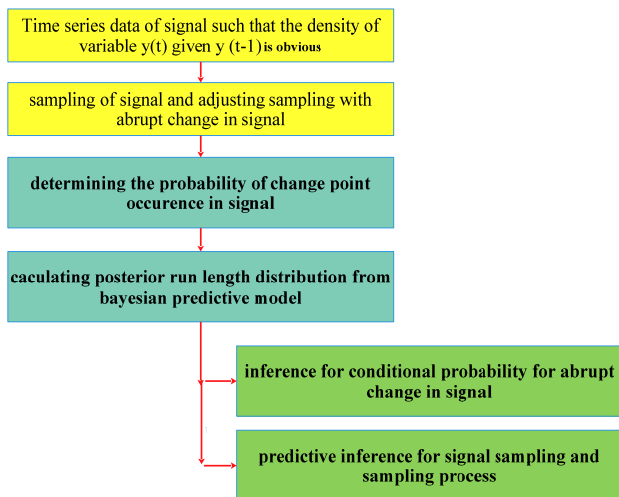


Fig. 1 Overall structure of algorithm

### III. SETUP OF SIMULATIONS AND WHOLE CHARACTERISTICS OF ARTIFICIAL SIGNALS AND APPROACH

In this section the algorithm is simulated on some specifically simulated signal. In order to simulate the algorithm, artificial signal with known change point positions, various signal to noise ratios (SNR) and segmentations of data are implemented and algorithm is applied on the data to see how effectively the algorithm finds the correct segments and locations of change points under different circumstances. This signal is produced by modeling of Gaussian distribution with changing variance of data. Such modeling arises in many biological signals like detecting pattern of electromyography signal. In signal processing application most researchers use the Bayesian method for finding change-point detection in signals [18], [22]. In this paper, Bayesian online inference in models of data series is constructed by change-points, and it is tested on simulated data.

The purpose of this study is to represent the influence of signal characteristics for finding the change-point detection in artificial datasets. The simulations of this paper are implemented in two sections. Firstly, the estimation of a single change-point with fluctuating position is discussed. Secondly, the case of multiple change-points in signal is simulated. Such artificial signals represent test scenario for detecting abrupt changes in localization of change point.

In first section, one built-in change point is applied to signal. Signal is divided in two segments which include SNR and mean value of data. An important problem, in change-point detection, is detection of the similar locations in many different statistical features of the data. Another important aspect of these simulations is to find critical and correlated behavior of signal by allowing fluctuations in the precise location of the change-points.

### IV. SIGNAL CHARACTERISTICS FOR EACH SEGMENT BETWEEN CHANGE POINTS

Number and position of change points, and value of the variance and mean value parameters are assumed to be known in simulations. Details of variance and mean value and location of change point in artificial signals are reported in Tables I and II.

### V. INVESTIGATION OF A SINGLE FLUCTUATING POSITION OF CHANGE-POINT IN SIGNAL

First case study of our simulation is plotted in Fig. 1. This signal is segmented in two parts and change point location is applied to signal in position of step 100 and mean value of data in across of this point is changed to mean value of 1 to -2 and variance of signal in two segmentations of signal is same. In Fig. 2 distribution of change point location is shown and it is evident that the prediction of location for change point is recognized properly in step 100. This simulation demonstrates that when there is a correlation for event in across of the critical transition or change point then variations of mean value of signal have more influence than variance of data. For this simulation the numbers of nodes in run-length distribution

as a function of time are plotted in Fig. 3.

TABLE I  
DATA CHARACTERISTICS IN SINGLE CHANGE-POINT CONDITION

Simulation number		Variance value in each segmentation		Mean value in each segmentation		Location of proper change point
#1	Step(1-100)	4	Step(1-100)	1	100	
	Step(100-200)	4	Step(100-200)	-2		
#2	Step(1-100)	4	Step(1-100)	1	100	
	Step(100-200)	3	Step(100-200)	-2		
#3	Step(1-100)	4	Step(1-100)	1	100	
	Step(100-200)	2	Step(100-200)	-2		
#4	Step(1-100)	4	Step(1-100)	1	100	
	Step(100-200)	3	Step(100-200)	-4		
#5	Step(1-100)	4	Step(1-100)	1	100	
	Step(100-200)	2	Step(100-200)	-4		

TABLE II  
DATA CHARACTERISTICS IN MULTIPLE CHANGE-POINT CONDITION

	Simulation #1	Simulation #2
Variance value in each segmentation	Step(1-100)=4	Step(1-100)=4
	Step(100-200)=2	Step(100-200)=3
	Step(200-300)=4	Step(200-300)=4
	Step(300-400)=2	Step(300-400)=3
	Step(400-500)=4	Step(400-500)=4
Mean value in each segmentation	Step(1-100)=1	Step(1-100)=1
	Step(100-200)=-2	Step(100-200)=-2
	Step(200-300)=1	Step(200-300)=1
	Step(300-400)=-5	Step(300-400)=-5
	Step(400-500)=3	Step(400-500)=3
Location of proper change point	Step(1-100)=100	Step(1-100)=100
	Step(100-200)=200	Step(100-200)=200
	Step(200-300)=300	Step(200-300)=300
	Step(300-400)=400	Step(300-400)=400
	Step(400-500)=500	Step(400-500)=500

Characteristics of different simulations in next sections are described in Tables I and II. One problem of this approach is that on different steps, the algorithm may find change point in different locations which have a small posterior probability of being a proper change point, when in fact there is only one

true change point. In Simulation number 2 signal is plotted in Fig. 4. In Fig. 5 distribution of change point location is shown. Simulation results confirm that, when the noise is not too correlated in each segmentation several change points exist. Generally, variations in the generative parameters in signal are often a crucial aspect of real world signal which encompass of many separate parameter regimes. An inability to react to regime changes can have an adverse effect on predictive performance.

In simulation number 2 there are some points that have a low probability of being exact change point that this is an artifact of algorithm and if the correlation between events is low then this local maxima of posterior probabilities is more obvious. If this happens, some non-change points will accumulate significant probabilities. In most conditions however, these distributions will have lower probabilities than the true change points. In Simulation number 3 signals is plotted in Fig. 6 and in Fig. 7 distribution of change point location is shown. In this condition the true change point is more evident localized from distribution because of lower variance in second segmentation of the signal.

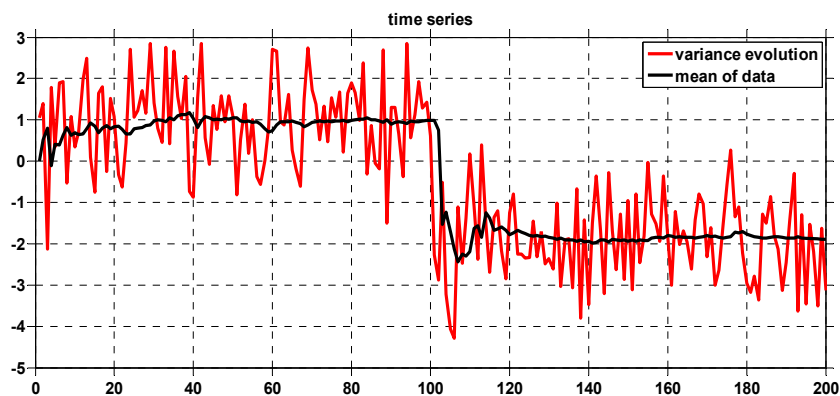


Fig. 2 Change point detection for artificial data with one built-in change point

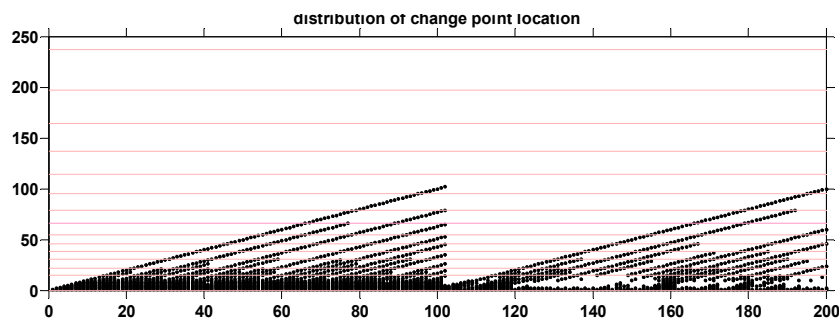


Fig. 3 The run-length distribution in data or distribution of change point location

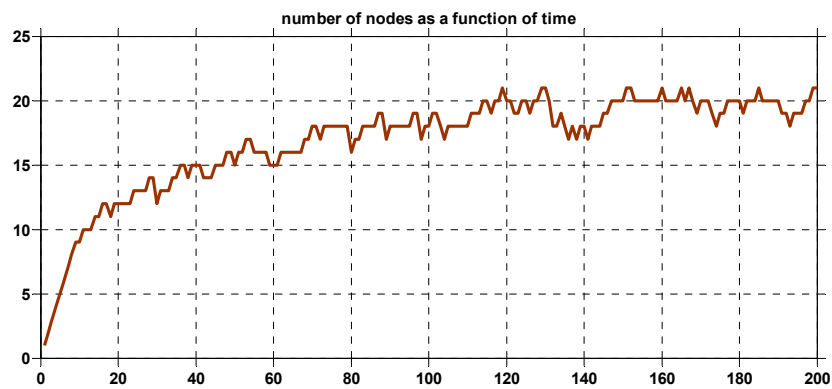


Fig. 4 Numbers of nodes in run-length distribution as a function of time

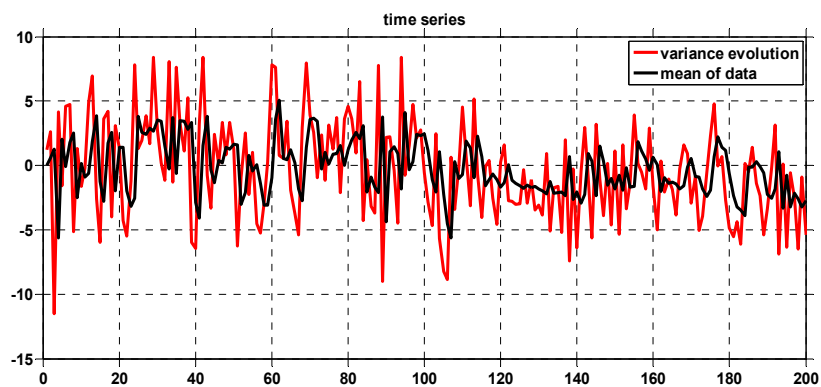


Fig. 4 Change point detection for artificial data with one built-in change point and more variance

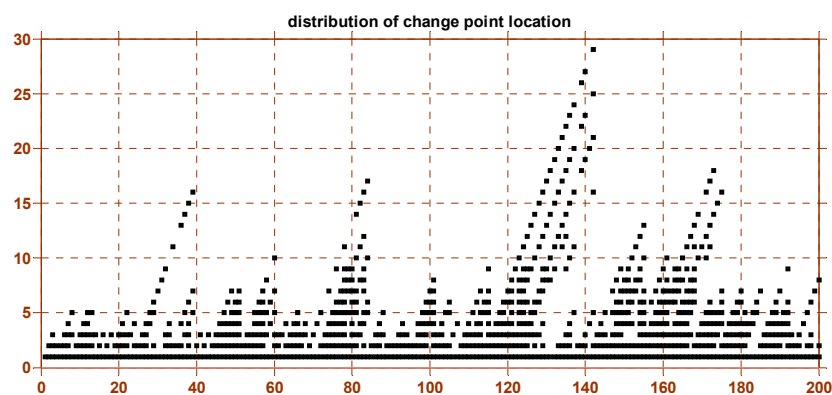


Fig. 5 The run-length distribution in data or distribution of change point location

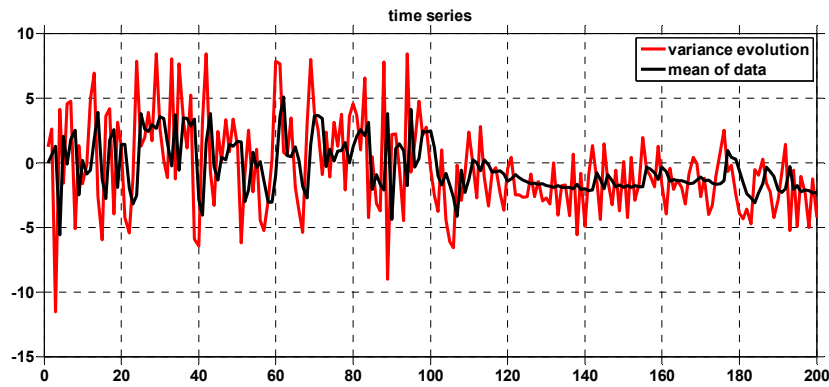


Fig. 6 Change point detection for artificial data with one built-in change point and complex statistical characteristics

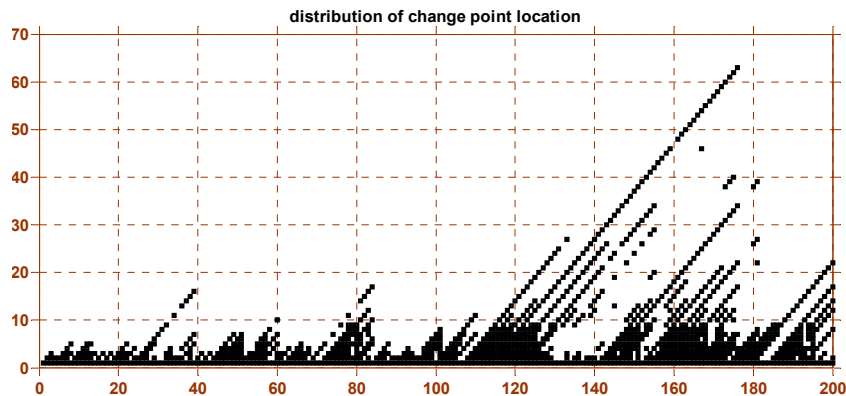


Fig. 7 The run-length distribution in data or distribution of change point location

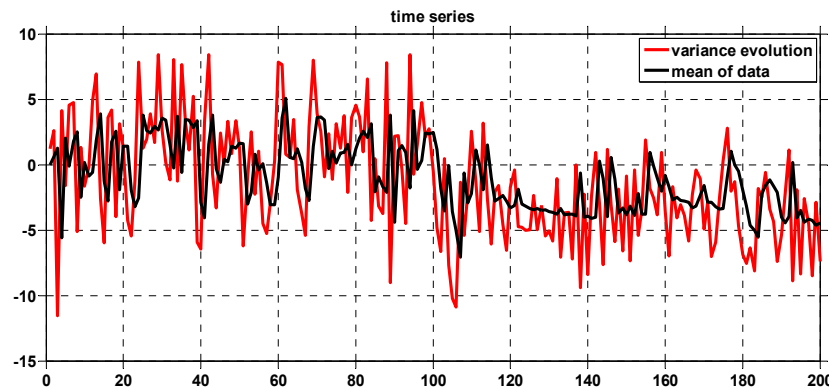


Fig. 8 Change point detection for artificial data with one built-in change point

In simulations 4 and 5 firstly mean value of data is changed and has more reduction in change point locations. Signals for Simulations 3 and 4 are plotted in Figs. 8 and 10. In Figs. 9 and 11 distribution of change point location is plotted. In these simulations many of the points around the accurate change points have slight probability values and firstly it proves that mean value variation has more effect to detection of proper change point and then for better constructing iteration of algorithm we can combine all of the close change points into one. For instance, if a time point has a higher posterior probability of being a change point than the time points

closely before and after it, algorithm can consider it as a change point. This grouping algorithm is so important for conditions when the mean value of data has more variations from variance or when we have more knowledge about data we can use another grouping method. This grouping includes creation a window scale of a certain length around each peak in all groups. Also, models of complex systems are generally not precise enough to predict consistently where critical thresholds may occur. Most of the time, the dynamic of systems near a critical point have common properties. In models, critical thresholds in modeling for such transitions

relate to bifurcations points in terms of equation in dynamical system.

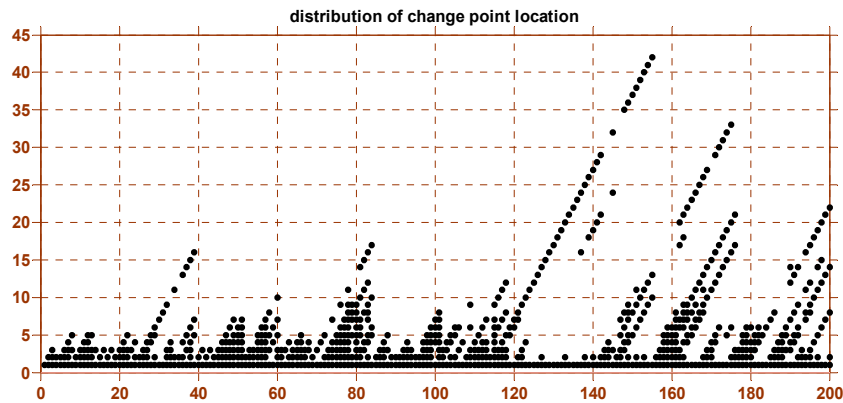


Fig. 9 The run-length distribution in data or distribution of change point location

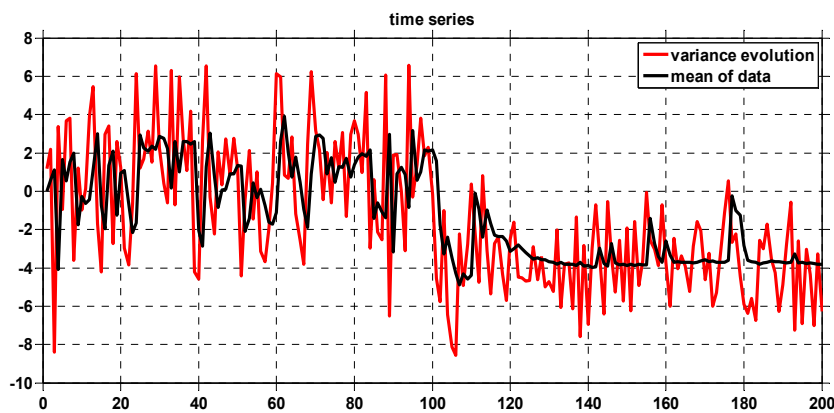


Fig. 10 Change point detection for artificial data with one built-in change point

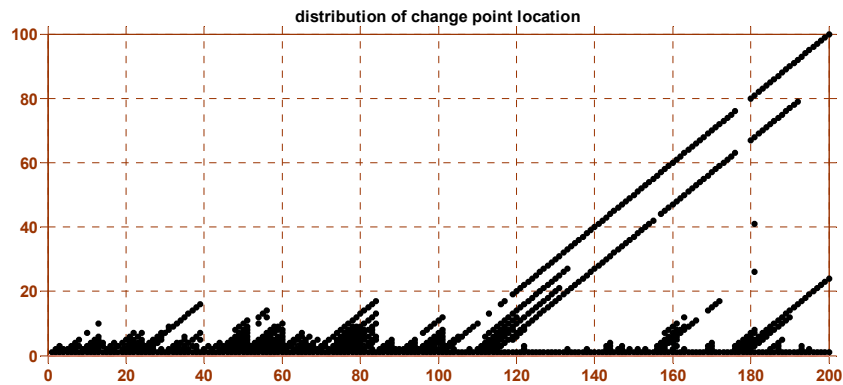


Fig. 11 The run-length distribution in data or distribution of change point location

#### VI. INVESTIGATION OF A MULTIPLE FLUCTUATING POSITION OF CHANGE-POINT IN SIGNAL

Generally, variation of regime in data is important for convergence in signal characteristics. Simulation setup for these conditions is presented in Table II. Signals for simulations 1 and 2 are plotted in Figs. 12 and 15. In Figs. 13 and 16 distribution of change point location are plotted and numbers of nodes in run-length distribution as a function of

time are plotted in Fig. 14. The data are simulated for conditions when system is in bifurcation case. This is evident from results that in these conditions many change points is taken place especially in simulation 2 that data have more variance in segmentations. Generally, when systems pass, bifurcations are referred to as critical transitions. At bifurcation points the leading eigenvalue of system that characterizes the rates of change around the equilibrium

becomes zero. This infers that as the system is near such critical points, it becomes gradually slow in recovering from small perturbations. It can be revealed that when a bifurcation is taken place in segmentation of signal, some important

changes in the pattern of signal are predictable to occur. Also, increasing variance is another source of changing and detecting change points in algorithm.

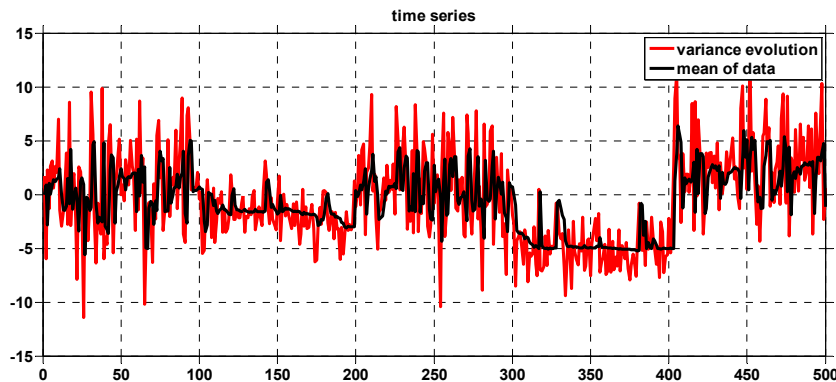


Fig. 12 Change point detection for artificial data with multiple built-in change point

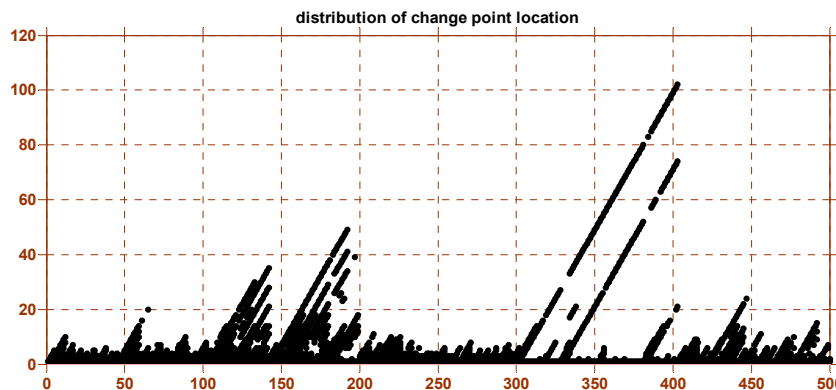


Fig. 13 The run-length distribution in data or distribution of change point location

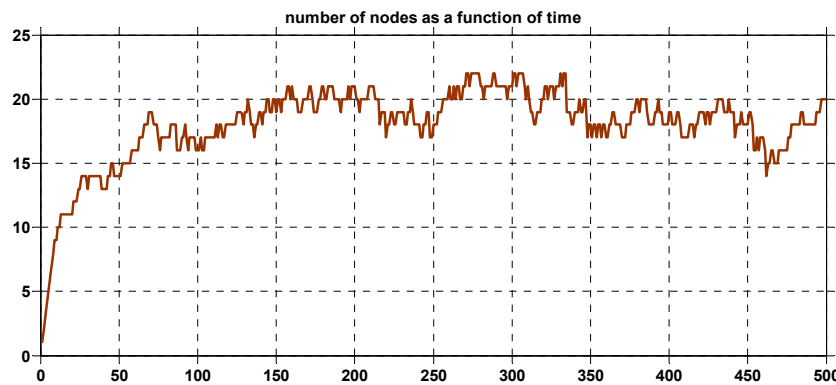


Fig. 14 Numbers of nodes in run-length distribution as a function of time

## VII. CONCLUSION

In this paper a Bayesian approach for detecting of abrupt change in temporal zone of signal is investigated. The location of abrupt changes, in different simulations, is computed for showing the importance influence of the statistical

characteristic of signal. And one of the main contributions of paper is related to the estimation of behavior rate in signal and identification of abrupt change location depends to regions of high density in signal and algorithm must be designed better in this condition for obtaining these parameters.



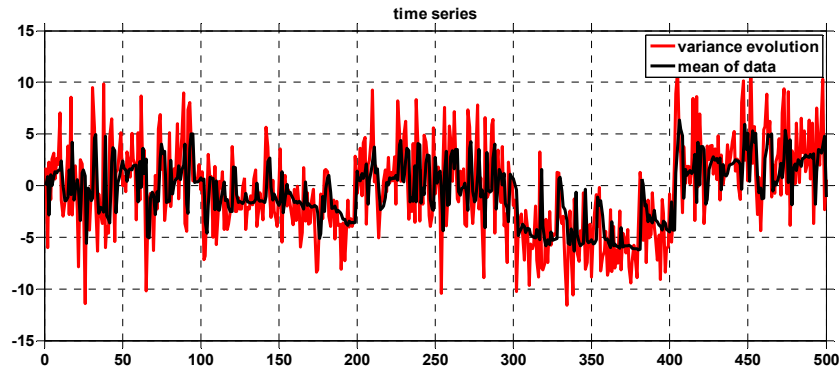


Fig. 15 Change point detection for artificial data with multiple built-in change point

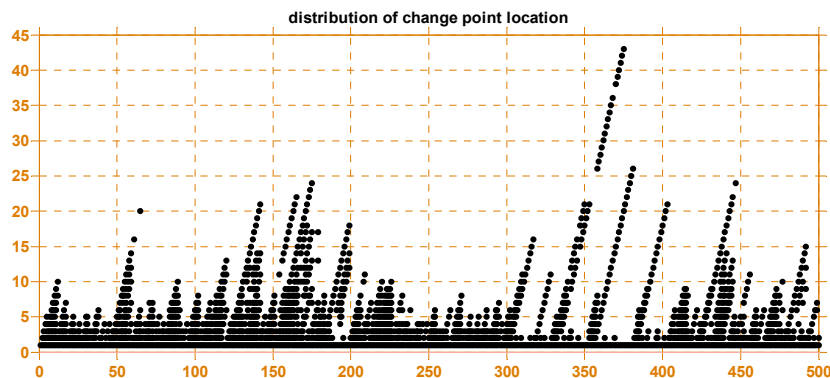


Fig. 16 The run-length distribution in data or distribution of change point location

## REFERENCES

- [1] Zhou F, De la Torre F, Hodgins JK. Hierarchical aligned cluster analysis for temporal clustering of human motion. *IEEE Transactions on Pattern Analysis and Machine Intelligence*. 2012 Jun 26;35(3):582-96.
- [2] Asadi F, Khorram M, Moosavian SA. CPG-based gait transition of a quadruped robot. In 2015 3rd RSI International Conference on Robotics and Mechatronics (ICROM) 2015 Oct 7 (pp. 210-215). IEEE.
- [3] Asadi F, Khorram M, Moosavian SA. CPG-based gait planning of a quadruped robot for crossing obstacles. In 2015 3rd RSI International Conference on Robotics and Mechatronics (ICROM) 2015 Oct 7 (pp. 216-222). IEEE.
- [4] Asadi F, Mollakazemi MJ. Investigation on Performance of Change Point Algorithm in Time Series Dynamical Regimes and Effect of Data Characteristics. *World Academy of Science, Engineering and Technology, International Journal of Mechanical, Aerospace, Industrial, Mechatronic and Manufacturing Engineering*. 2015 Jan 1;8(10):1787-93.
- [5] Angela JY, Dayan P. Uncertainty, neuromodulation, and attention. *Neuron*. 2005 May 19;46(4):681-92.
- [6] Behrens TE, Woolrich MW, Walton ME, Rushworth MF. Learning the value of information in an uncertain world. *Nature neuroscience*. 2007 Sep;10(9):1214-21.
- [7] Xuan X, Murphy K. Modeling changing dependency structure in multivariate time series. In *Proceedings of the 24th international conference on Machine learning 2007 Jun 20* (pp. 1055-1062).
- [8] Naeem MT, Jazayeri SA, Rezamahdi N, Toosi KN. Failure analysis of gas Matteson DS, James NA. A nonparametric approach for multiple change point analysis of multivariate data. *Journal of the American Statistical Association*. 2014 Jan 2;109(505):334-45.
- [9] Asadi F, Mollakazemi MJ, Uzelac IL, Moosavian SA. A novel method for arterial blood pressure pulse detection based on a new coupling strategy and discrete wavelet transform. In 2015 Computing in Cardiology Conference (CinC) 2015 Sep 6 (pp. 1081-1084). IEEE.
- [10] Asadi F, Mollakazemi MJ, Ataybi SA, Uzelac IL, Ghaffari A. Cardiac arrhythmia recognition with robust discrete wavelet-based and geometrical feature extraction via classifiers of SVM and MLP-BP and PNN neural networks. In 2015 Computing in Cardiology Conference (CinC) 2015 Sep 6 (pp. 933-936). IEEE.
- [11] Xuan X, Murphy K. Modeling changing dependency structure in multivariate time series. In *Proceedings of the 24th international conference on Machine learning 2007 Jun 20* (pp. 1055-1062).
- [12] Wilson RC, Nassar MR, Gold JI. Bayesian online learning of the hazard rate in change-point problems. *Neural computation*. 2010 Sep;22(9):2452-76.
- [13] Younes L, Albert M, Miller MI, BIOCARD Research Team. Inferring changepoint times of medial temporal lobe morphometric change in preclinical Alzheimer's disease. *NeuroImage: Clinical*. 2014 Jan 1;5:178-87.
- [14] Fox E, Sudderth EB, Jordan MI, Willsky AS. Bayesian nonparametric inference of switching dynamic linear models. *IEEE Transactions on Signal Processing*. 2011 Jan 6;59(4):1569-85.
- [15] Wilson RC, Nassar MR, Gold JI. Bayesian online learning of the hazard rate in change-point problems. *Neural computation*. 2010 Sep;22(9):2452-76.
- [16] Schölkopf B, Platt JC, Shawe-Taylor J, Smola AJ, Williamson RC. Estimating the support of a high-dimensional distribution. *Neural computation*. 2001 Jul 1;13(7):1443-71.
- [17] LaGraff JE, Ashpis DE, Oldfield ML, Gostelow JP. Minnowbrook V: Schölkopf B, Smola AJ, Bach F. *Learning with kernels: support vector machines, regularization, optimization, and beyond*. MIT press; 2002.
- [18] Kim J, Cheon S, Jin Z. Bayesian multiple change-points estimation for hazard with censored survival data from exponential distributions. *Journal of the Korean Statistical Society*. 2020 Jan 1:1-8.
- [19] Fan Y, Lu X. An online Bayesian approach to change-point detection for categorical data. *Knowledge-Based Systems*. 2020 Mar 29:105792.
- [20] Li Q, Yao K, Zhang X. A change-point detection and clustering method in the recurrent-event context. *Journal of Statistical Computation and Simulation*. 2020 Apr 12;90(6):1131-49.
- [21] Mollakazemi MJ, Asadi F. Real Time Adaptive Obstacle Avoidance in Dynamic Environments with Different DS. *International Journal of Mechanical and Mechatronics Engineering*. 2015 Jan 1;8(10):1794-9.
- [22] Kim J, Cheon S, Jin Z. Bayesian multiple change-points estimation for



hazard with censored survival data from exponential distributions.  
Journal of the Korean Statistical Society. 2020 Jan 1:1-8.