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System Reduction Using Modified Pole Clustering and Modified Cauer Continued Fraction

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Abstract**—**A mixed method by combining modified pole clustering technique and modified cauer continued fraction is proposed for reducing the order of the large-scale dynamic systems. The denominator polynomial of the reduced order model is obtained by using modified pole clustering technique while the coefficients of the numerator are obtained by modified cauer continued fraction. This method generated 'k' number of reduced order models for kth order reduction. The superiority of the proposed method has been elaborated through numerical example taken from the literature and compared with few existing order reduction methods.

Keywords**—**Modified Pole Clustering, Modified Cauer Continued Fraction, Order Reduction, Stability, Transfer Function.

I. Introduction

REDUCTION of high order system to low order model has been an important research in the field of science and engineering for many years. Numerous methods of order reduction techniques for linear dynamic systems in the frequency domain are available the literature [1]-[4]. Further, some methods have also been recommended by combining the features of two different methods [5]-[7]. The clustering method suggested by Sinha and Pal [8] is being modified by Vishwakarma [9] which generate more effective cluster centre than cluster centre obtained [8]. In continued fractions [10] the reduction of a high-order system is based on expanding the given transfer function into a Cauer continued fraction about $s = 0$ and truncating it to find a lower order model. In some cases, it develops an unstable model.

To overcome this failing, Chuang [11] introduced a modification to the continued fraction expansion which combines expansions about $s = 0$ and $s = \infty$ alternately, to realize a good approximation to both the initial and steady state responses. Further Parthasarathy and J. [12] modified Cauer continued fraction (MCF) technique in order to retain the rank of the high-order system in the reduced order model.

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II. Statement of the Problem

Let the transfer function of high order original system of the order $'n'$ be

$$G(s) = \frac{N(s)}{D(s)} = \frac{a_0 + a_1 s + a_2 s^2 + ... + a_{n-1} s^{n-1}}{b_0 + b_1 s + b_2 s^2 + ... b_n s^n} \tag{1}$$

$$\left[G_n(s)\right] = \frac{1}{D_n(s)} \begin{bmatrix} a_{11}(s) & a_{12}(s) & \dots & a_{1p}(s) \\ a_{21}(s) & a_{22}(s) & \dots & a_{2p}(s) \\ \vdots & \vdots & & \\ a_{q1}(s) & a_{q2}(s) & \dots & a_{qp}(s) \end{bmatrix} \quad \vdots \tag{2}$$

p and q are the number of input and output variables respectively. Let the transfer function of the reduced single and multivariable system of the order 'k' be

$$R_k(s) = \frac{N_k(s)}{D_k(s)} = \frac{c_0 + c_1 s + c_2 s^2 + ... + c_{k-1} s^{k-1}}{d_0 + d_1 s + d_2 s^2 + ... d_k s^k} \tag{3}$$

$$\left[R_k(s)\right] = \frac{1}{D_k(s)} \begin{bmatrix} c_{11}(s) & c_{12}(s) & \dots & c_{1p}(s) \\ c_{21}(s) & c_{22}(s) & \dots & c_{2p}(s) \\ \vdots & \vdots & & \\ c_{q1}(s) & c_{q2}(s) & \dots & c_{qp}(s) \end{bmatrix} \quad \vdots \tag{4}$$

The objective of this paper is to realize the k^{th} order reduced models in the form of (3) and (4) from the original system (1) and (2) respectively, such that it retains the significant features of the original high order system.

III. Description of the Method

The reduction procedure for getting the k^{th} order reduced models consists of the following two steps:

Step. 1

Determination of the denominator polynomial for the k^{th} order reduced model using modified pole clustering. The following are the simplest rules for making cluster the poles of the original system.

A. Separate clusters should be made for real and complex poles.

B. Clusters of the poles in the left half s -plane should not contain any pole of the right half s -plane and vice-versa.

C. Poles on the jw axis have to be retained in the reduced order model.

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D. Poles at the origin have to be retained in the reduced order model.

The iterative algorithm for synthesizing the reduced denominator polynomial using modified pole clustering technique is stated as below:

Let there be r real poles in i^{th} cluster are $p_1, p_2, ... p_r$ where $|p_1| < |p_2| .... < |p_r|$ and then modified cluster centre p_{ai} can be obtained by using modified pole clustering.

Let $'m'$ pair of complex conjugate poles in the j_{th} cluster be

$$[(\alpha_1 + j\beta_1), (\alpha_2 + j\beta_2), ... (\alpha_m + j\beta_m)]$$

where

$$|\alpha_1| < |\alpha_2| .... < |\alpha_r|$$

Now using the same algorithm separately for real and imaginary parts of the complex conjugate poles, the modified cluster $c_j = \left[\sum_{i=0}^{r} \left(\frac{-1}{|p_i|} \right) \div r \right]^{-1}$ centre is obtained, which is written as

$$\phi = A_{ei} \pm jb_{ei} \text{ where } \phi = A_{ei} + jb_{ei} \quad and \quad \phi = A_{ei} - jb_{ei}$$

An interactive computer oriented algorithm is used to get modified cluster centre and is given as follows:

Step.I. Let $'r'$ real poles in a cluster be $|p_1| < |p_2| ... < |p_r|$.

Step.II. Set $j = 1$

Step.III. Find pole cluster centre $c_j = \left[\sum_{i=0}^{r} \left(\frac{-1}{|p_i|} \right) \div r \right]^{-1}$

Step.IV. Set $j = j + 1$

Step.V. Now find a modified cluster centre from $c_j = \left[\left(\frac{-1}{|p_1|} + \frac{-1}{|c_{j=1}|} \right) \div 2 \right]^{-1}$

Step.VI. Is $r = (j+1)$? , if No, and then go to *step-4* otherwise go to step-VII.

Step.VII. Take a modified cluster centre of the k_{th} – cluster as $p_{ak} = c_j$

For synthesizing the k^{th} order denominator polynomial, one of the following cases may occur:

Case. 1

If all modified cluster centres are real, then denominator polynomial of the k^{th} order reduced model can be obtained as

$$D_k(s) = (s - p_{e1})(s - p_{e2})...(s - p_{ek}) \tag{5}$$

where $p_{e1}, p_{e2} ... p_{ek}$ are $1^{st}, 2^{nd} ... k^{th}$ modified cluster centre respectively.

Case. 2

If all modified cluster centers are complex conjugate then the k^{th} order denominator polynomial is taken as

$$D_k(s) = (s - \phi_{e1})(s - \phi_{e2})...(s - \phi_{ek/2}) \tag{6}$$

Case. 3

If some cluster centers are real and some are complex conjugate. For example, two cluster centers are real and one pair of cluster centre is complex conjugate, then k^{th} order denominator can be obtained as

$$D_k(s) = (s - p_{e1})(s - p_{e2})...(s - p_{e(k-2)})(s - \phi_{e1})(s - \phi_{e2}) \tag{7}$$

hence the denominator polynomial $D_k(s)$ is obtained as

$$D_k(s) = d_o + d_1 s + d_2 s^2 + ... d_k s^k \tag{8}$$

Step. 2

Determination of the numerator of the reduced model using modified cauer continued fraction.

Step.I. By applying the algorithm [13], evaluate the first $'k'$ quotients $h_1, H_1, h_2, H_{2,}$ in the modified Cauer form of continued fraction.

Step.II. Now a modified Routh array for $k = 6$ is built as given below:

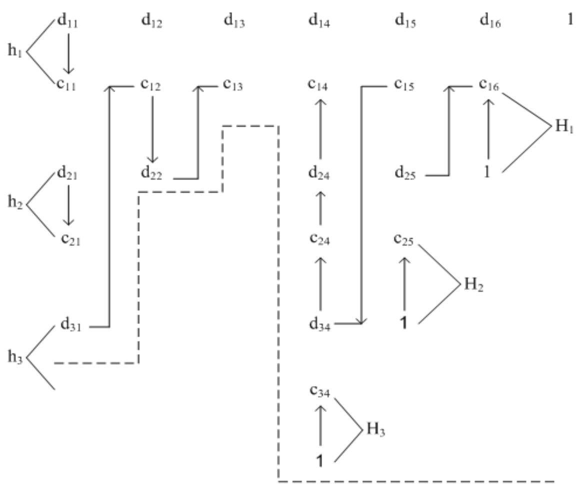

Fig. 1 Modified Cauer Continued Fraction

First two rows are formed from the coefficient of $R_k(s)$. The remaining entries in the array are obtained by algorithm [14].

IV. Numerical Examples

Two numerical examples are chosen from the literature to illustrate the algorithm of the proposed method. The examples are solved in details to get second order reduced model. An integral square error (ISE) are calculated between the transient

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parts of the original and reduced model using MATLAB to measure the goodness of the reduced order model i.e. lower ISE, closer the $[R_k(s)]$ to $[G_n(s)]$ which is given by

$$ISE = \int_0^\infty [y(t) - y_k(t)]^2 \, dt \tag{9}$$

Example 1

Consider a 4th order system taken from Mittal et al. [15].

$$G_4(s) = \frac{s^3 + 7s^2 + 24s + 24}{s^4 + 10s^3 + 35s^2 + 50s + 24}$$

The poles of the system are $-1, -2, -3, -4$

For getting 2^{nd} – order RM (Reduced Model), two possible clusters are $(-1, -2)$ and $(-3, -4)$.

Using the algorithm of step-1, two modified cluster centres are computed as

$p_{e1} = -1.066$ and $p_{e2} = -3.0967$. Hence, Denominator polynomial $D_2(s)$ is obtained as

$$D_2(s) = s^2 + 6.0118s + 5.0270$$

Now using step-2, Evaluate the MCF quotients by forming the array

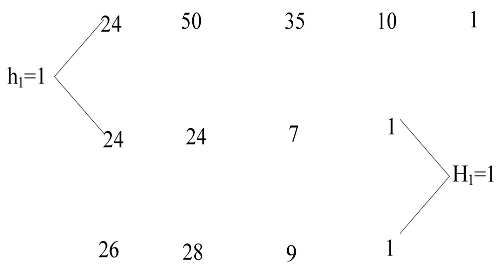

Fig. 2 (a) Routh Array

Now draw the modified Routh array

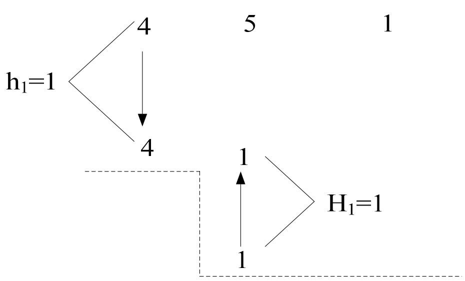

Fig. 2 (b) Modified Routh Array

Therefore 2^{nd} – order Reduced Model is obtained as

$$R_2(s) = \frac{s + 3.301}{s^2 + 4.1627s + 3.301}$$

The error index ISE is calculated between the reduced model and original system and shown in the Table I. The unit step and frequency response of the 2^{nd} – order reduced model i.e., $R_2(s)$ and original system $G_4(s)$ are plotted in Figs. 3 and 4 respectively.

TABLE I
COMPARISON OF REDUCED MODELS

Methods of order reduction	Reduced Models $R_2(s)$	ISE
Proposed Method	$\frac{s + 3.301}{s^2 + 4.1627s + 3.301}$	0.0058
Pal [16]	$\frac{16.008s + 24}{30s^2 + 42s + 24}$	0.011
Chidambara [17]	$\frac{-s + 2}{s^2 + 3s + 2}$	0.220
Shieh and Wei [18]	$\frac{s + 2.3014}{s^2 + 5.7946s + 2.3014}$	0.142

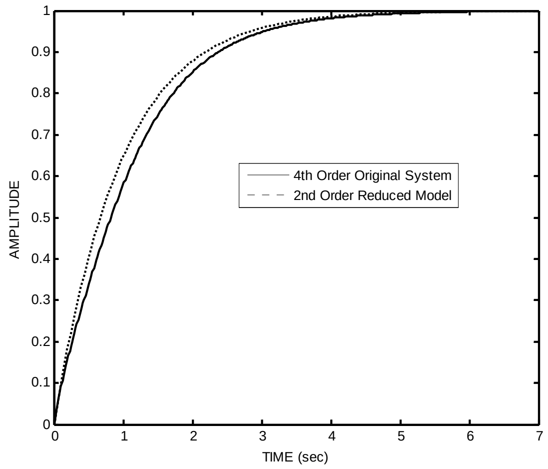

Fig. 3 Step response of original system and reduced model

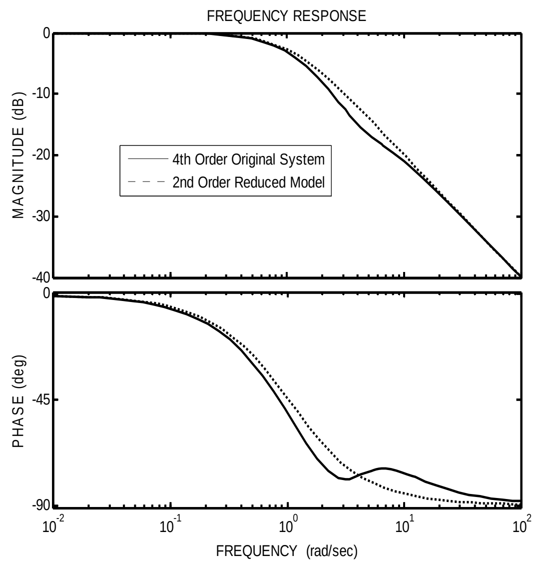

Fig. 4 Frequency response of original system and reduced model

Example 2

Consider a 6^{th} – order multivariable system from Bistriz and Shaked [19] described by the transfer function matrix as

$$[G(s)] = \begin{bmatrix} \frac{2(s+5)}{(s+1)(s+10)} & \frac{(s+4)}{(s+2)(s+5)} \\ \frac{(s+10)}{(s+1)(s+20)} & \frac{(s+6)}{(s+2)(s+3)} \end{bmatrix} = \frac{1}{D(s)}\begin{bmatrix} a_{11}(s) & a_{12}(s) \\ a_{21}(s) & a_{22}(s) \end{bmatrix}$$

where,

$$D(s) = s^6 + 41s^5 + 571s^4 + 3491s^3 + 10060s^2 + 13100s + 6000$$

$$a_{11}(s) = 2s^5 + 70s^4 + 762s^3 + 3610s^2 + 7700s + 6000$$

$$a_{12}(s) = s^5 + 38s^4 + 459s^3 + 2182s^2 + 4160s + 2400$$

$$a_{21}(s) = s^5 + 30s^4 + 331s^3 + 1650s^2 + 3700s + 3000$$

$$a_{22}(s) = s^5 + 42s^4 + 601s^3 + 3660s^2 + 9100s + 6000$$

The poles of the system are located at: $-1,-2,-3,-5,-10,-20$

Let the 2^{nd} – order model is required to be synthesized, therefore two clusters are $(-1,-2,-3)$ and $(-5,-10,-20)$.

By using the proposed method, the following two biased 2^{nd} – order models are obtained as

$$R_2(s) = \frac{1}{D_2(s)}\begin{bmatrix} 2s + 5.5442 & 1s + 2.2177 \\ 1s + 2.7721 & 1s + 5.5442 \end{bmatrix}$$

where

$$D_2(s) = s^2 + 6.3258s + 5.5442$$

The unit step response of the 2^{nd} – order reduced model i.e., $R_2(s)$ and original system $G_6(s)$ are plotted in Fig. 5. The error index ISE is calculated between the reduced model and original system and shown in the Table II.

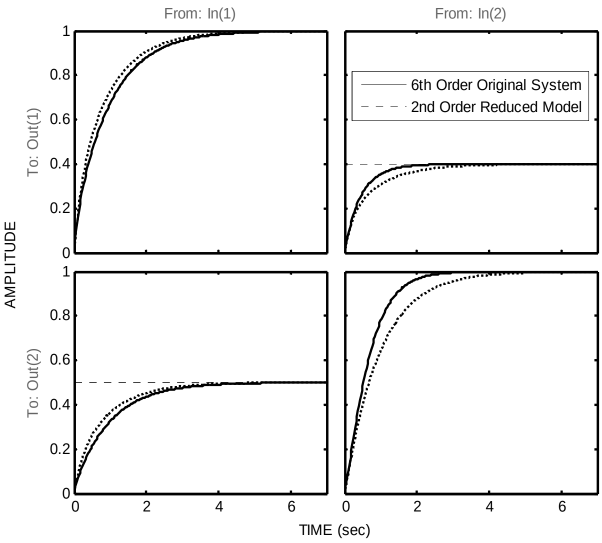

Fig. 5 Unit step response of original system and reduced model

TABLE II
COMPARISON OF REDUCED MODELS

$r_{ij} = \frac{a_{ij}(s)}{D_2(s)}$ $(i = 1,2;\ j = 1,2)$	**ISE by Proposed Method**	**ISE by Parmar [20]**	**ISE by Prasad [21]**
$r_{11}(s)$	0.00492	0.038713	0.135505
$r_{12}(s)$	0.00304	0.028153	0.002446
$r_{21}(s)$	0.00263	0.007419	0.040013
$r_{22}(s)$	0.02661	0.144096	0.067897

V. CONCLUSION

Mixed method combines the advantages of the modified pole clustering and the modified Cauer continued fraction method has been presented, to derive stable reduced order models for linear dynamic systems. In this method the poles of the reduced model are determined by iteration of pole cluster while the zeros are synthesized by using the modified Cauer continued fraction technique. The proposed method has been extended for order reduction of linear multivariable systems and also tried on two numerical examples having real poles only. The comparison between the proposed and the other well known existing order reduction techniques is also exposed as given in Tables I and II, from which it is understandable that the proposed method compares healthy with other techniques of model order reduction. The numerical examples illustrate the proposed method, which can be tried on many systems as it preserves both time domain and frequency domain characteristics of the original system.

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