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Swarm Navigation in a Complex Environment

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Abstract**—**This paper proposes a solution to the motion planning and control problem of car-like mobile robots which is required to move safely to a designated target in a priori known workspace cluttered with swarm of boids exhibiting collective emergent behaviors. A generalized algorithm for target convergence and swarm avoidance is proposed that will work for any number of swarms. The control laws proposed in this paper also ensures practical stability of the system. The effectiveness of the proposed control laws are demonstrated via computer simulations of an emergent behavior.

Keywords**—**Swarm, practical stability, motion planning, emergent.

I. Introduction

TRAJECTORY planning and control of holonomic and nonholonomic systems has been an active area of research for more than two decades now. Basically, it involves finding a feasible trajectory from some initial configuration to a desired one while satisfying the velocity constraints of the system. In recent years, with the rapid advances in sensing, communication, computation, and actuation capabilities, groups or swarms are expected to cooperatively perform dangerous or explorative tasks in a broad range of potential applications. As highlighted by Latombe [1], motion planning is "eminently necessary, since, by definition, a robot accomplishes tasks by moving in the real world". The essence of *robot motion planning problem* can be formulated as a two-dimensional problem and is captured in the following classic definition (adopted from [2]):

Definition 1: Given a robot and a description of its workspace, propose a path that the robot can follow. In particular, if the workspace is cluttered with solid objects, propose a collision-free path that can lead the mobile robot from the desired starting point to the desired goal or target.

Devising motion planning algorithms for multi-agents sharing a common workspace is inherently difficult. This is a result of the environment being no longer static but dynamic. Static environments have provided excellent breeding grounds for high-powered algorithms so far. However, more recently there has been a shift of emphasis to include dynamic environments due to its applications in the real world. The dynamic environment is composed of both the stationary and the unpredictable (or predictable) dynamic obstacles [3]. These dynamic obstacles can incorporate the mobile robots themselves as well as other moving solid objects or obstacles in the environment. Thus, fundamental to the motion planning problem of multi-agents is the need to control and plan the motions of the agents that would yield inter-agent and agent to obstacle collision avoidances. Numerous papers have discussed this problem, some of which includes methods such as discretization of the configuration time-space using sequential space slicing [4], sheared cylindrical representations of moving obstacles and generating optimal tangential paths to the goals [5], hybrid systems [6], threaded petri nets [7], plan-merging [8], negotiations [9], online artificial potential fields strategy [1], [10], decomposition of the problem into path planning and velocity planning sub-problems [11] and a Lyapunov based control scheme for various nonholonomic multi-agents [2], [3], [12], to name a few.

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This article explores the challenging but indispensable area of multi-agent research. We will consider multiple vehicles and dynamic environments. The other novel aspects of this article are the moving obstacles, that is, the swarm of boids. Hence, there will be a number of car-like robots moving between start and goal configurations in a constrained environment.

II. System Modelling

In this section, we shall model a rear driven car-like vehicle and a general 2-dimensional swarm. Both the models will be used to illustrate via the Lyapunov based control scheme the effectiveness of the system models.

A. Vehicle Model - The Kinematics and Dynamics of the Car-like Robot

In this subsection, the kinematics and the dynamics of a car-like system will be described. The vehicle model consists of a rear wheel driven car-like vehicle, whereby engine power is applied to the rear wheels (see Fig. 1). Although polar coordinates are more popular with moving obstacle [13], we utilize the Cartesian coordinate system since it does not inject undesired singularities into the navigation problem [2].

Definition 2: The k th nonholonomic car-like mobile robot is a circular disk with rv_k and is positioned at center (xv_k, yv_k). In addition, the k th car-like robot is the set

$$A_k = \left\{ (z_1, z_2) \in \mathbf{R}^2 : (z_1 - xv_k)^2 + (z_2 - yv_k)^2 \le rv_k \right\},$$

for $k \in \{1, ..., m\}, m \in \mathbf{N}$, where A_k embodies a rear-wheel driven and front-wheel steered car-like vehicle.

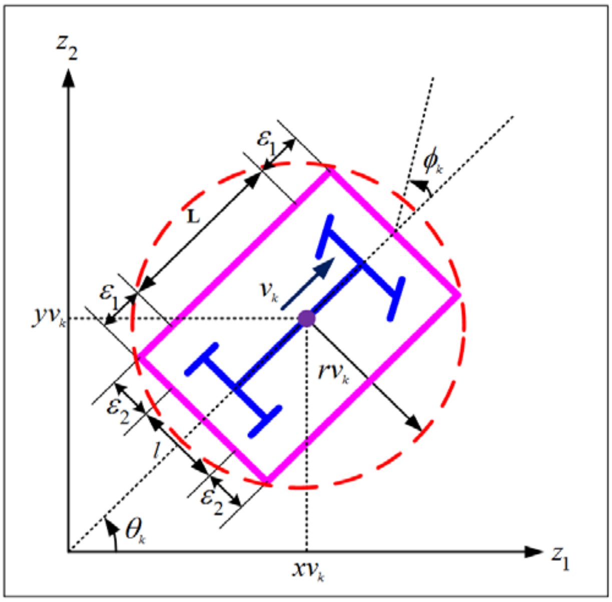

Fig. 1 A rear wheel driven vehicle with front wheel steering and steering angle ϕ_k.

Inclusion of the dynamics will then be producing a trajectory in the state-space. Thus, if m_k is the mass of the vehicle, F_k the force along the axis of the vehicle, Γ_k the torque about a vertical axis at (xv_k, yv_k) and I_k the moment of inertia of the vehicle, then dynamic model of the vehicle is

$$\left.\begin{aligned} x\dot{v}_k &= v_k\cos\theta_k - \frac{L}{2}w_k\sin\theta_k,\\ y\dot{v}_k &= v_k\sin\theta_k + \frac{L}{2}w_k\cos\theta_k,\\ \dot{\theta}_k &= w_k,\\ \dot{v}_k &= \sigma_{k1} := F_k / m_k,\\ \dot{w}_k &= \sigma_{k2} := \Gamma_k / I_k \end{aligned}\right\} \qquad (1)$$

where the variable θ_k gives the vehicles's orientation with respect to the main axes, v_k and w_k are the translational and rotational velocities, respectively, while σ_{k1} and σ_{k2} are, respectively, the instantaneous translational and rotational accelerations.

Referring to Fig. 1, to ensure that the entire vehicle safely steers past an obstacle, the planar vehicle will be enclosed by the smallest circle possible. L and l are, respectively, the length and width of the vehicle, then given the *clearance parameters* ε_1 and ε_2, enclose the vehicle by a protective circular region centered at (xv_k, yv_k), with radius

$$rv_k := \sqrt{(2\varepsilon_1 + L)^2 + (2\varepsilon_2 + l)^2}\,/\,2.$$

Assumption 1: The instantaneous accelerations σ_{k1} and σ_{k2} can move the car-like robot of A_k to its designated target and attain the desired final orientation.

B. A Two Dimensional Swarm Model

Following the nomenclature of Reynolds [15], each member of the flock is denoted as a boid. We shall construct a model of a swarm with m individuals moving with the velocity of the swarm's centroid. Following previous work such as those of [16] and [17], we consider the individuals as point masses.

At time $t \ge 0$, let $(xb_i(t), yb_i(t))$, $i = 1,\ldots,n$ be the planar position of the i th individual, which we shall define as a point mass residing in a disk of radius $rb_i > 0$,

$$B_i = \left\{(z_1, z_2) \in \mathbf{R}^2 : (z_1 - xb_i)^2 + (z_2 - yb_i)^2 \le rb_i^2\right\}. \qquad (2)$$

At time $t \ge 0$, let $(vb_i(t), wb_i(t)) := (\dot{x}b_i(t), \dot{y}b_i(t))$ be its instantaneous velocity of the i th point mass. Using the above notations, we thus have a system of first order ODE's for the i th individual, assuming the initial conditions at $t = t_0 \ge 0$:

$$\left.\begin{aligned} \dot{x}b_i &= vb_i(t),\\ \dot{y}b_i &= wb_i(t),\\ xb_{i0} &:= xb_i(t_0),\ yb_{i0} = yb_i(y_0). \end{aligned}\right\} \qquad (3)$$

If $g_i(x) := (vb_i, wb_i) \in \mathbf{R}^2$ and $G(x) := (g_1(x),\ldots,g_n(x)) \in \mathbf{R}^{2n}$, then our swarm system of m individuals is

$$\dot{\mathbf{x}} = G(x),\ x_0 = x(t_0). \qquad (4)$$

Definition 3: System (4) is said to be

(S1) *practically stable* if given (λ, A) with $0 < \lambda < A$, we have $\|x_0 - x^*\| < \lambda$ implies that $\|x(t) - x^*(t)\| < A,\ t \ge t_0$ for some $t_0 \in \mathbf{R}_+$;

(S2) *uniformly practically stable* if *(S1)* holds for every $t_0 \in \mathbf{R}_+$.

The following comparison principle for practical stability is also adapted from [18] for system (4), where, $K = \{a \in C[\mathbf{R}_+, \mathbf{R}_+] : a(u)$ is strictly increasing in u and $a(u) \to \infty$ as $u \to \infty$, $S(\rho) = \left\{x \in \mathbf{R}^{2n} : \|x - x^*\| < \rho\right\}$, and, for any Lyapunov-like function $V \in C\left[\mathbf{R}_+ \times \mathbf{R}^{2n}, \mathbf{R}_+\right]$,

$$D^+V(t,x) := \limsup_{h\to 0^+}\frac{V(t+h, x+hG(x)) - V(t,x)}{h},$$

for $(t,x) \in \mathbf{R}_+ \times \mathbf{R}^{2n}$, noting that if $V \in C^1[\mathbf{R}_+ \times \mathbf{R}_{2n}, \mathbf{R}_+]$, then $D^+V(t,x) = V'(t,x)$, where $V'(t,x) = V_t(t,x) + V_x(t,x)G(x)$.

Theorem 1: Lakshmikantham, Leela and Martynyuk [18]. Assume that

1. λ and A are given such that $0 < \lambda < A$;
2. $V \in C\left[\mathbf{R}_+ \times \mathbf{R}^{2n}, \mathbf{R}_+\right]$ and $V(t,x)$ is locally Lipschitzian in x;
3. for $(t,x) \in \mathbf{R}_+ \times S(A)$, $b_1\left(\left\|x - x^*\right\|\right) \le V(t,x) \le b_2\left(\left\|x - x^*\right\|\right)$, $b_1, b_2 \in K$ and $D^+V(t,x) \le q(t, V(t,x)), q \in C\left[\mathbf{R}_+^2, \mathbf{R}\right]$;
4. $b_2(\lambda) < b_1(A)$ holds.

Then the practical stability properties of the scalar differential equation

$$z'(t) = q(t,z),\ z(t_0) = z_0 \ge 0,$$

imply the corresponding practical stability properties of system (4).

III. Deployment of the Lyapunov-based Control Scheme

The principal control objective of this section is to utilize the Lyapunov-based control scheme to design the translational acceleration σ_{k1} and the rotational acceleration σ_{k2} such that the car-like vehicle, represented by system (1), will navigate safely among obstacles, reach a neighborhood of its destination whilst respecting kinodynamic constraints.

A. Details of the Vehicular Agents

1) Target of the Vehicle:

Now, in the target-attraction component of the Lyapunov-like function, intuitively, we want to have a kind of a yardstick that measures, at time $t \ge 0$, the midpoint position of A_k from its destination (p_{k1}, p_{k2}) and the rate at which it approaches or moves away from (p_{k1}, p_{k2}). A choice of probable target attractive functions that could accomplish this, on suppressing t, is

$$V_k(x) = \frac{1}{2}\left[(xv_k - p_{k1})^2 + (yv_k - p_{k2})^2 + v_k^2 + w_k^2\right].$$

2) Convergence of the Vehicle (Car-like robot):

We need to guarantee the convergence of the car-like robot to its prescribed target and ensure that the nonlinear controllers vanish at the target configuration. We adopt a new attractive function whose role is purely mathematical, and hence auxiliary. This function will be multiplied to each of the obstacle avoidance functions. This strategy implicitly guarantees that the goal configuration is a *global minimum* of the total potential. Thus an appropriate auxiliary function is defined as follows:

$$G_k(x) = \frac{1}{2}\left[(xv_k - p_{k1})^2 + (yv_k - p_{k2})^2 + (\theta_k - p_{k3})^2\right].$$

3) Kinodynamic Constraints:

The kinodynamic planning problem involves synthesizing a robots motion subject to kinematic constraints, such as any fixed or moving obstacle in the workspace and dynamic constraints, such as modulus bound on velocity.

Workspace: Boundary Limitations:

The boundaries of the workspace are considered as fixed obstacles, which have to be avoided by each articulated body at every time $t \ge 0$ so that the robot is confined within the workspace. Accordingly, for the avoidance we construct the following obstacle avoidance functions for the avoidance of the left, lower, right and upper boundaries, respectively, as follows:

$$WV_{k1} = xv_k - rv_k,$$
$$WV_{k2} = yv_k - rv_k,$$
$$WV_{k3} = b_1 - (xv_k - rv_k),$$
$$WV_{k4} = b_2 - (yv_k - rv_k).$$

Each of these is positive within the rectangle. That is, $WV_{k1}, WV_{k3} > 0$, for all $xv_k \in (rv_k, b_1 - rv_k)$ and $WV_{k2}, WV_{k4} > 0$ for all $yv_k \in (rv_k, b_2 - rv_k)$.

Modulus Bound on Velocities:

From a practical viewpoint, the translational speed and the steering angle of a car-like system are limited. If $v_{\max} > 0$ is the maximum speed, and $\phi_{\max}$ is the maximum steering angle satisfying $0 < \phi_{\max} < \pi/2$ then, as shown in [19], the additional constraints imposed on the translational and the rotational velocities are:

i. $|v_k| < v_{\max}$, where $v_{\max}$ is the *maximal achievable speed* of the mobile robot;

ii. $|w_k| \le \frac{|v_k|}{|\rho_{\min}|} < \frac{v_{\max}}{|\rho_{\min}|}$ where $\rho_{\min}$ is known as the *minimum turning radius* and is given as $\rho_{\min} = \frac{P}{\tan\phi_{\max}}$.

For the avoidance, we design the following obstacle avoidance functions:

$$U_{k1}(x) = \frac{1}{2}(v_{\max} - v_k)(v_{\max} + v_k),$$
$$U_{k2}(x) = \frac{1}{2}\left(\frac{v_{\max}}{|\rho_{\min}|} - w_k\right)\left(\frac{v_{\max}}{|\rho_{\min}|} + w_k\right),$$

for $k = 1, \dots, n$, which would guarantee the adherence to the limitations placed upon translational velocity v_k and the steering angle ϕ_k, respectively.

4) Inter-individual Collision Avoidance for the Carlike Mobile Robots:

In practice, the control algorithms must generate feasible trajectories based upon real-time perceptual information. A moving car-like mobile robot itself becomes a moving obstacle for all the other car-like mobile robots in the workspace. First, we make the following assumption:

Assumption 1: Due to the deterministic nature of our kinodynamic system, there is a prior knowledge of the directions of motion and the instantaneous velocities of the car like robots available to the system.

For car A_k to avoid car A_l, we design repulsive potential field functions with the associated obstacle avoidance function of the form

$$M_{kl}(x)=\frac{1}{2}\left[\left(xv_k-xv_l\right)^2+\left(yv_k-yv_l\right)^2-\left(rv_k+rv_l\right)^2\right],$$

for $k,l=1,...,m,\ l\neq k.$

B. Details of the Leader-less Swarm

For the attraction of the swarm to the centroid and for the inter-individual avoidance of the swarm, the functions are:

1) Attraction to the Centroid:

To ensure that the individuals of the swarm are attracted towards each other and also form a cohesive group by having a measurement of the distance from the *i*th individual to the swarm centroid, we use the following attraction function:

$$R_i(x)=\frac{1}{2}\left[\left(xb_i-\frac{1}{n}\sum_{j=1}^{n}xb_j\right)^2+\left(yb_i-\frac{1}{n}\sum_{j=1}^{n}yb_j\right)^2\right].$$

2) Avoidance of the Boundaries of the Workspace: This subsection adopts the planar workspace WS designed in the previous section. For the avoidance of the left, upper, right and lower boundaries, the following functions are utilized, respectively:

$$\begin{aligned} WB_{i1}&=xb_i-rb_i,\\ WB_{i2}&=yb_i-rb_i,\\ WB_{i3}&=b_1-\left(xb_i-rb_i\right),\\ WB_{i4}&=b_2-\left(yb_i-rb_i\right), \end{aligned}$$

where $WS:=\left\{(z_1,z_2)\in\mathbf{R}^2:0\le z_1\le b_1, 0\le z_2\le b_2\right\}$ and noting that they are all positive within the workspace.

3) Inter-individual Collision Avoidance:

For the boids to avoid each other, we design repulsive potential field functions of the form

$$Q_{ij}(x)=\frac{1}{2}\left[\left(xb_i-xb_j\right)^2+\left(yb_i-yb_j\right)^2-\left(rb_i+rb_j\right)^2\right],$$

for $i,j=1,...,n,\ \ j\neq i$. The function is an Euclidean measure of the distance between the individual boids, and will appear in the denominator of an appropriate term in the candidate Lyapunov-like function to be proposed.

4) Avoidance of Vehicular Agents by the Boids:

In practice, effective avoidance of moving obstacles is etiquette for mobile robots. Hence, avoidance of the moving swarms is another addition to the multitasking problem in this paper. Here, the car-like mobile robots becomes the moving obstacles for the swarm of boids in the workspace. This is a one-way collision avoidance whereby the swarm of boids avoids the car-like mobile robots. For the boids to avoid the vehicular agents, we design repulsive potential field function of the form

$$S_{ik}(x)=\frac{1}{2}\left[\left(xb_i-xv_k\right)^2+\left(yb_i-yv_k\right)^2-\left(rb_i+rv_k\right)^2\right],$$

where $k=1,...,m$ and $i=1,...,n.$

IV. Design of Nonlinear Controllers

This section will represent a Lyapunov-like function candidate and the nonlinear control laws for systems (1) and (3) will be designed. In parallel, we will consider the stability analysis pertaining to the dynamic system.

A. Lyapunov Function

As per the LbCS, we combine all the attractive and repulsive potential field functions, and introducing *tuning parameters* $\gamma_i>0,\ \eta_{is}>0,\ \beta_{ij}>0,\ \sigma_{ik}>0,\ \tau_{ks}>0,\ \varphi_{kl}>0$ and $\xi_{ku}>0$ for $i,j,k,l,m,n,s,u\in\mathbf{N}$ we define a Lyapunov-like function candidate for systems (1) and (3) as

$$\begin{aligned} L(x)=&\sum_{i=1}^{n}\left[\gamma_i R_i(x)+R_i(x)\left(\sum_{s=1}^{4}\frac{\eta_{is}}{WB_{is}(x)}+\sum_{\substack{j=1\\ j\neq i}}^{n}\frac{\beta_{ij}}{Q_{ij}(x)}+\sum_{k=1}^{m}\frac{\sigma_{ik}}{S_{ik}(x)}\right)\right]\\ &+\sum_{k=1}^{m}\left[V_k(x)+G_k(x)\left(\sum_{s=1}^{4}\frac{\tau_{ks}}{WV_{ks}(x)}+\sum_{\substack{l=1\\ l\neq k}}^{m}\frac{\varphi_{kl}}{M_{kl}(x)}+\sum_{u=1}^{2}\frac{\xi_{ku}}{U_{ku}(x)}\right)\right] \end{aligned}$$

B. Controller Design

To extract the control laws for the kinodynamic system, we differentiate the various components of $L(x)$ separately with respect to t along a solution of systems (1) and (3), carry out the necessary substitutions and upon suppressing x, we have the following for the swarm of boids and the vehicular agents:

1) Swarm of boids: Upon suppressing x and for $i=1,...,n$, we have

$$Lx_i = \left(\gamma_i + \sum_{\substack{j=1\\ j\neq i}}^{n} \frac{\beta_{ij}}{Q_{ij}(x)}\right)\left(xb_i - \frac{1}{n}\sum_{j=1}^{n} xb_j\right) + R_i\left(\frac{\eta_{i3}}{WB_{i3}^2} - \frac{\eta_{i1}}{WB_{i1}^2}\right)$$
$$-2R_i \sum_{\substack{j=1\\ j\neq i}}^{n} \frac{\beta_{ij}}{Q_{ij}^2(x)}\left(xb_i - xb_j\right) - R_i \sum_{k=1}^{m} \frac{\sigma_{ik}}{S_{ik}^2}\left(xb_i - xv_k\right),$$
$$Ly_i = \left(\gamma_i + \sum_{\substack{j=1\\ j\neq i}}^{n} \frac{\beta_{ij}}{Q_{ij}(x)}\right)\left(yb_i - \frac{1}{n}\sum_{j=1}^{n} yb_j\right) + R_i\left(\frac{\eta_{i4}}{WB_{i4}^2} - \frac{\eta_{i2}}{WB_{i2}^2}\right)$$
$$-2R_i \sum_{\substack{j=1\\ j\neq i}}^{n} \frac{\beta_{ij}}{Q_{ij}^2(x)}\left(yb_i - yb_j\right) - R_i \sum_{k=1}^{m} \frac{\sigma_{ik}}{S_{ik}^2}\left(yb_i - yv_k\right).$$

Next, given the convergence parameters $\alpha_{i1}, \alpha_{i2} > 0$, the nonlinear velocity controllers for the swarm of boids is:

$$\begin{aligned} vb_i &= -\alpha_{i1} Lx_i, \\ wb_i &= -\alpha_{i2} Ly_i, \end{aligned} \tag{5}$$

where $i = 1, \ldots, n.$

2) Vehicular Agents: Upon suppressing x and for $k = 1, \ldots, m$, we have

$$f_{k1} = \left(1 + \sum_{s=1}^{4} \frac{\tau_{ks}}{WV_{ks}(x)} + \sum_{\substack{l=1\\ l\neq k}}^{m} \frac{\varphi_{kl}}{M_{kl}(x)} + \sum_{u=1}^{2} \frac{\xi_{ku}}{U_{ku}(x)}\right)\left(xv_k - p_{k1}\right)$$
$$+G_k\left(\frac{\tau_{k3}}{WV_{k3}^2} - \frac{\tau_{k1}}{WV_{k1}^2}\right) - G_k \sum_{\substack{l=1\\ l\neq k}}^{m} \frac{\varphi_{kl}}{M_{kl}^2(x)}\left(xv_k - xv_l\right)$$
$$+G_l \sum_{\substack{l=1\\ l\neq k}}^{m} \frac{\varphi_{kl}}{M_{kl}^2(x)}\left(xv_l - xv_k\right),$$
$$f_{k2} = \left(1 + \sum_{s=1}^{4} \frac{\tau_{ks}}{WV_{ks}(x)} + \sum_{\substack{l=1\\ l\neq k}}^{m} \frac{\varphi_{kl}}{M_{kl}(x)} + \sum_{u=1}^{2} \frac{\xi_{ku}}{U_{ku}(x)}\right)\left(yv_k - p_{k2}\right)$$
$$+G_k\left(\frac{\tau_{k4}}{WV_{k4}^2} - \frac{\tau_{k2}}{WV_{k2}^2}\right) - G_k \sum_{\substack{l=1\\ l\neq k}}^{m} \frac{\varphi_{kl}}{M_{kl}^2(x)}\left(yv_k - yv_l\right)$$
$$+G_l \sum_{\substack{l=1\\ l\neq k}}^{m} \frac{\varphi_{kl}}{M_{kl}^2(x)}\left(yv_l - yv_k\right),$$
$$g_{k1} = 1 + G_k \frac{\xi_{k1}}{U_{k1}^2},$$
$$g_{k2} = 1 + G_k \frac{\xi_{k2}}{U_{k2}^2}.$$

Next, given *convergence parameters* $\delta_{k1}, \delta_{k2} > 0$, the translational and rotational speeds are given the following forms:

$$-\delta_{k1} \times v_k = \left(f_{k1}(x)\cos\theta_k + f_{k2}(x)\sin\theta_k\right) + g_{k1}(x)u_{k1}$$
$$-\delta_{k2} \times w_k = \frac{L}{2}\left(f_{k2}(x)\cos\theta_k - f_{k1}(x)\sin\theta_k\right) + g_{k2}(x)u_{k2}$$

where $k = 1, \ldots, m$ and L is the length of the k th car.

Hence, along a trajectory of system (1)

$$\dot{L}(x) = -\sum_{k=1}^{m}\left(\delta_{k1}v_k^2 + \delta_{k2}w_k^2\right) \le 0 \tag{6}$$

provided that the state feedback nonlinear navigation laws governing the k th car are of the form

$$\begin{aligned} u_{k1} &= -\left(\delta_{k1}v_k + f_{k1}\cos\theta_k + f_{k2}\sin\theta_k\right)/g_{k1}, \\ u_{k1} &= -\left[\delta_{k2}v_k + \frac{L}{2}\left(f_{k2}\cos\theta_k - f_{k1}\sin\theta_k\right)\right]/g_{k2}. \end{aligned} \tag{7}$$

Note that $\dot{L}(x) \le 0$ for all $x \in D(L(x))$.

V. Stability Analysis

Theorem 2: System (1) and (3) is uniformly practically stable.

Proof: Since

$$\dot{L}(x(t)) \le 0,$$

we have

$$0 \le L(x(t)) \le L(x(t_0)) \;\; \forall \;\; t \ge t_0 \ge 0. \tag{8}$$

Accordingly, for comparative analysis, it is sufficient to consider the practical stability of the scalar differential equation

$$z'(t) = 0, \;\; z(t_0) =: z_0, \;\; t_0 \ge 0. \tag{9}$$

The solution is

$$z(t; t_0, z_0) = z_0,$$

so that relative to every point $z^* \in \mathbf{R}$, we have

$$z(t; t_0, z_0 - z^*) = z_0 - z^*,$$

so that for any given number $P_0 > 0,$

$$\left|z(t; t_0, z_0 - z^*)\right| \le \left|z_0 - z^*\right| + P_0.$$

We shall next show that by applying Theorem 1, we can simultaneously derive the explicit form of $P_0 > 0,$ with which it is easy to see that (S2) holds for equation (9) if

$$A = A(\lambda) := \lambda + P_0.$$

To apply Theorem 1, we restrict our domain to $D(L(x))$ over which we see that $L(x) \in C\left[D(L(x)), \mathbf{R}_+\right],$

and note that $L(x)$ is locally Lipschitzian in $D(L(x))$ since $dL/dt \le 0$ in $D(L(x))$. Re-defining $S(\rho)$ as $S(\rho) = \{x \in D(L(x)): \ \|x - x^*\| < \rho\}$, we get

$$S(A) = \left\{x \in D(L(x)) : \|x - x^*\| < \lambda + P_0\right\}.$$

Recalling that $\gamma_i > 0,\ i \in \mathbf{N}$, we let

$$\gamma_{\min} := \min \gamma_i,\ i \in \mathbf{N} \ \text{ and } \ \gamma_{\max} := \max \gamma_i,\ i \in \mathbf{N}.$$

Further, let

$$b_1\left(\|x - x^*\|\right) := \frac{1}{2}\gamma_{\min}\|x - x^*\|^2$$

and

$$b_2\left(\|x - x^*\|\right) := \frac{1}{2}\gamma_{\max}\left[\|x - x^*\| + L(x_0)\right]^2,$$

noting that $b_1, b_2 \in K$. Then assuming $P_0 > 0$ is given, we easily see that, with (8), we have $b_1\left(\|x - x^*\|\right) \le L(x) \le b_2\left(\|x - x^*\|\right)$ for $x \in S(A)$, since

$$\sum_{i=1}^{n} R_i(x) = \frac{1}{2}\left[\left(xb_i - \frac{1}{n}\sum_{j=1}^{n} xb_j\right)^2 + \left(yb_i - \frac{1}{n}\sum_{j=1}^{n} yb_j\right)^2\right]$$

$$= \frac{1}{2}\|x = x^*\|.$$

Indeed, the inequality $b_2(\lambda) < b_1(\lambda)$ yields

$$\frac{1}{2}\gamma_{\max}\left[\lambda + L(x_0)\right]^2 < \frac{1}{2}\gamma_{\min}\left[\lambda + P_0\right]^2,$$

which holds if we choose

$$P_0 > \left[\left(\sqrt{\frac{\gamma_{\max}}{\gamma_{\min}}} - 1\right) + \left(\sqrt{\frac{\gamma_{\max}}{\gamma_{\min}}}\ L(x_0)\right)\right].$$

Since $\frac{\gamma_{\max}}{\gamma_{\min}} \ge 1$ for any $\gamma_{\max}$, $\gamma_{\min} > 0$, and because of (8), it is clear that P_0 exists and $P_0 > 0$. Thus, with $q(t, z) = 0$, we conclude the proof of Theorem 2.

VI. Simulation

This section demonstrates the simulation results for the car-like mobile robots navigating in a well-defined workspace cluttered with moving obstacles. The stability results obtained from the Lyapunov-like function will be verified numerically.

In this scenario, the car-like mobile robots move from an initial configuration to the target position whilst avoiding each other and the swarm of boids on its way to its target. This scenario could be modeled as a swarm of bees or pigeons following a car from one destination to another.

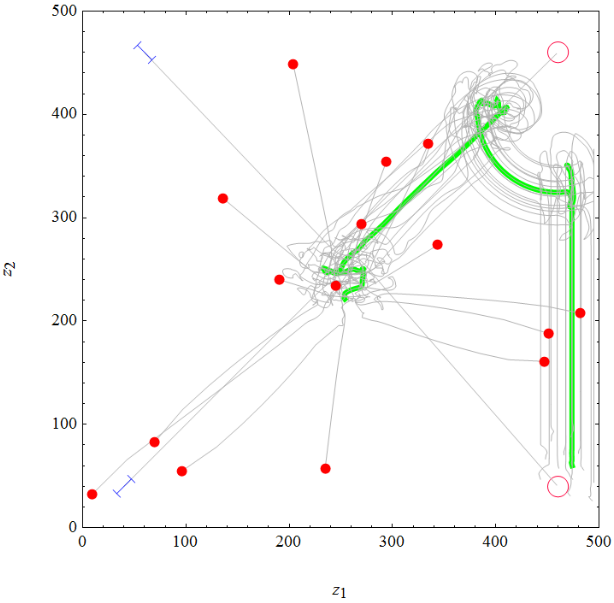

Fig. 2 The initial position of the car-like mobile robot and the swarm of boids

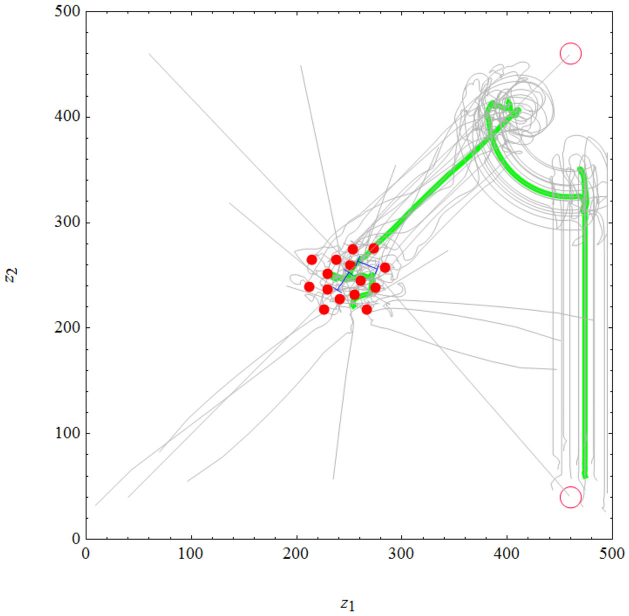

Fig. 3 The car-like mobile robot avoiding the swarm of boids at $t = 100$ units

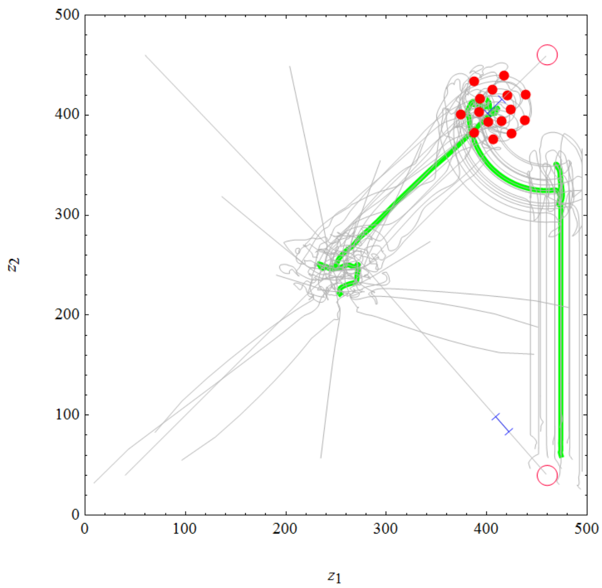

Fig. 4 The car-like mobile robot avoiding the swarm of boids at $t = 400$ units

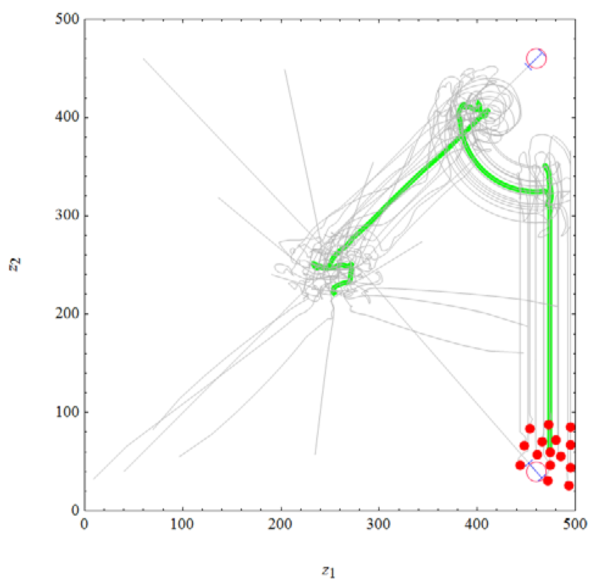

Fig. 5 The car-like mobile robot avoiding the swarm of boids at $t = 500$ units

VII. Conclusion

The paper essays a simple approach for solving the motion planning and control problem of car-like mobile robots. A target convergence and swarm avoidance scheme is developed and the control laws are designed using the Lyapunov-based control scheme so that the car-like mobile robots converge to their respective targets while avoiding collisions with a swarm of boids along their paths.

The nonlinear control laws presented in this paper guarantees practical stability of the system. This has been proved using the Lakshmikantham, Leela and Martynyuk method [18]. The practical stability of the system has been verified numerically via computer simulations. To the author's knowledge, this is the first time the swarm of boids has been considered together with the car-like mobile robots.

Future work will consider the introduction of multi-shaped fixed obstacles into the workspace.

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