

Survey on Strategic Games and Decision Making

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Abstract—Game theory is the study of how people interact and make decisions to handle competitive situations. It has mainly been developed to study decision making in complex situations. Humans routinely alter their behaviour in response to changes in their social and physical environment. As a consequence, the outcomes of decisions that depend on the behaviour of multiple decision makers are difficult to predict and require highly adaptive decision-making strategies. In addition to the decision makers may have preferences regarding consequences to other individuals and choose their actions to improve or reduce the well-being of others. Nash equilibrium is a fundamental concept in the theory of games and the most widely used method of predicting the outcome of a strategic interaction in the social sciences. A Nash Equilibrium exists when there is no unilateral profitable deviation from any of the players involved. On the other hand, no player in the game would take a different action as long as every other player remains the same.

Keywords—Game Theory, Nash Equilibrium, Rules of Dominance.

I. INTRODUCTION

GAME theory is a study of strategic decision-making. Precisely, it is the study of mathematical models of conflict and cooperation between intelligent rational decision-makers. A surrogate term suggested as a more descriptive name for the discipline is an interactive decisions. Game theory is broadly applied in economics, political science, and psychology, besides logic, computer science and biology [6]. The subject first addressed zero-sum games; corresponding one person's gains exactly equal net losses of the other participant or participants. Today, game theory applies to a wide range of behavioural relations and has developed into a term for the logical side of decision science, including both humans and non-humans. Game theory has some representation forms and important concepts such as Entry and Exit decisions.

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II. BACKGROUND

A. Representation of Game Theory

1) Extensive Form

The extensive form can be used to formalize games with a time sequencing of moves. Games here are played on trees. Here each vertex (or node) represents a point of choice for a player. The player is stated by a number listed by the vertex. The lines out of the vertex express a possible action for that player. The payoffs are detailed at the bottom of the tree. The extensive form can be seen as a tree with multiplayer decision. The extensive form is also capable of capturing games with simultaneous-moves and games with imperfect information. In order to denote it, either a dotted line connecting a number of vertices to denote them as a part of the same information set (i.e. to identify the current position of a player), or a continuous line is drawn to represent them.

2) Normal Form

The normal form that is also known as a strategic form is generally denoted with the help of a matrix which in turn denotes the players, strategies and payoffs. Typically a normal form can be described with the help of a function correlates each player's payoff with their respective combination of actions. Consider an example, in a game with two players; one decides to choose the row and other the columns. Each of the players has two strategies, which are described by the number of rows for player 1 and the number of columns. The interior of the matrix denotes the payoffs. The first number signifies the payoff for the row player; the second signifies the payoff for the column player. Consider a situation where Player 1 plays right and Player 2 plays left. Here the result is that Player 1 gets a payoff of 1, and Player 2 gets a payoff of 2 (i.e. each payoff uniquely implies the decision). Whenever a game is denoted in a normal form, it is automatically assumed that each player makes decisions contemporarily without the knowledge of others players' decisions. On the other hand whenever a player has some knowledge on the choices made by other players, the game is then denoted using the extensive form. By definition, every extensive-form of a game has an equivalent normal-form game; nonetheless the transformation to a normal form may result in an exponential inflate, making it a mathematically impractical one [6].

B. Entry and Exit Decisions

The manager of a firm is considering the possibility of entering a newly created market, where there is only one other firm operating in competence. The decision made by the manager will be solely based on the profitability existing in the market, which preferentially depends on the way in which

the official firm will react to the entry made. The official firm could be accommodating and let the entrant occupy the person's share in the existing market or the person could respond in an aggressive way, meeting the entrant with a strict price war. A further factor that affects the revenue stream is the investment that is put by the entering firm. The manager of the firm may invest in the latest technology and lower the person's operating costs or the person may go ahead with the existing technology and have higher operating costs [1]. So as to find the best alternative person, we go for Nash Equilibrium.

C. Nash Equilibrium

Nash equilibrium is a solution concept of a non-cooperative game involving two or higher number of players, where each player in the game is assumed to know the equilibrium strategies of the other players, and hence therefore no other player has anything to gain by changing their own strategy. Consider that each player in a game has chosen a strategy and here no other player can benefit by changing strategies while the other players keep unchanged, then the current set of strategy choices and the corresponding payoffs constitute Nash equilibrium.

Nash equilibrium is built on the idea that players have resolved all strategic uncertainty. This is often unrealistic, in particular in the laboratory. To accommodate this, solution concepts for games have been proposed that extend the spirit of Nash equilibrium. By modelling players' beliefs as measures that are not probability distributions, they aim to capture how players will behave when they entertain doubts about the solution [3]. This paper argues that for such a situation versions of Nash equilibrium are inappropriate. For, the first thing that players should lose confidence in is that some opponent will not play a particular best reply against the solution (one that the equilibrium beliefs exclude). Consequently, a solution under a "lack of confidence" must include all best replies against the solution. But this is the reverse inclusion as under Nash equilibrium.

D. Strategic Games

A strategic game is a model of interacting decision-makers. In recognition of the interaction, refer to the decision-makers as players. Each player has a set of respective actions. The proposed model identifies the interaction between the players by allowing each player to be affected by the decision of all players, not only the player's own decision. More precisely, each player has preferences about the action profile.

A strategic game with statistical preferences consists of

- 1) A set of players
- 2) For each player, a set of actions
- 3) For each player, preferences that are over the set of action profiles.

With the view to calculate performance of an individual in achieving the threshold criteria in which success or failure can be predicted using Rules of dominance [2].

III. RELATED WORK

A. Game Theory in Business Applications

Advances in Information Technology (IT) and e-commerce further enrich and broaden these intercommunications, by improving the degree of communication between different parties involved in commerce. Globalization has bought the entire world a playground for many firms, thus increasing the advancement in these interactions. Given that each firm is part of a complex web of communication, any business conclusion or action implemented by the firm impacts multiple entities that interact with or within that firm. Not paying proper attention to these interactions may also lead to unexpected and potentially very undesirable outcomes [6]. Each decision maker is a player in the game of business, when making a decision or choosing a strategy must consider the potential choices of other players, remembering that while making their decisions, other players are likely to give some thinking on and take into account the strategy of their own as well. Most firms exactly consider other player's actions, especially competitors, while choose their own.

B. Applying Game Theory to System Design

Applying techniques from game theory helps to formulate and analyse solutions to two systems problems: discouraging selfishness in multi-hop wireless networks and enabling cooperation among ISPs in the World Wide Web. It proved difficult to do so. Here this concept reports on our experiences and explains the issues encountered. It outlines the ways in which the genuine use of results from traditional game theory did not fit well with the requirements specified in our problems. It additionally identifies a required characteristic of the solutions did eventually adopt that distinguishes them from those available using game theoretic approaches [7].

C. Multi Hop Wireless Networks

The nodes of emerging multi-hop wireless networks, such as community meshes, may belong to different users. When the source and the destination nodes for a packet are not within direct transmission range of each other, they must rely on intermediate nodes to forward packets between them. While packet forwarding improves connectivity in the network, benefiting all nodes in the long-run, it is not individually rational because of the cost to the forwarder in terms of energy and bandwidth. Initially, the problem was considered as a mechanism design exercise and to find a provably strategy-proof solution [8]. However, eventually the problems described in subsequent sections motivated us to abandon even the attempt to formally model the problem, and used an informal approach to finding and validating a solution. Catch uses anonymous messages, where the identity of the sender is hidden, to discover the true network connectivity even though a cheating node may try to hide links to reduce its forwarding obligations [10].

The insight here is that the cheating node would want to be connected to at least one other node in the network, and because it cannot infer the sender of an anonymous message, it is forced to acknowledge connectivity to all of its neighbours.

Nodes also implement a kind of watchdog, monitoring the behaviour of their neighbours to verify that they correctly forward packets. If cheating is detected, all neighbours of the cheater (identified in the topology discovery step) are notified [5]. Each then isolates the cheater, which effectively cuts off its network connectivity. Thus, use the fear of being disconnected as a disincentive against cheating. The design of Catch reflects trade-offs that make its implementation possible and effective in the heterogeneous settings wanted to address, while in turn sacrificing the absolute guarantee that there could be no situations under which a node's selfish benefit might be maximized by operating in violation of the desired social goal.

D. ISP Route Negotiation

ISPs are competing, autonomous entities, but they must cooperate by delivering packets to each other so that the packets are able to reach their ultimate destinations. Currently, ISPs make unilateral decisions about routing; including which peering link is used to send a packet to the downstream ISP. Unsurprisingly, the ISPs' routing decisions are driven by self-interest, and often do not take global consequences into account [6]. The multi-hop wireless networks ISP negotiation problems are our first attempt to apply game theory to systems problems. When began our work, hoped that game theory would help us better understand the problems, suggest solutions, and aid in analysing the properties of the solutions

Game theory did help us to some extent and certain aspects of our solutions are derived from common game theoretic concepts. But for most part, have not yet succeeded and the previous two sections discuss several contributing factors [11]. Despite these difficulties, remain optimistic that the application of game theory to systems problems will be of great benefit in the future, hope that identification of these issues will lead to formulations of the theory that place more emphasis on systems factors such as highly skewed workloads and abilities of players, uncertainty, and implementation cost, competitive concerns and so forth. Human experience will also be useful to systems designers looking to use game theory as part of their design [9].

Perhaps from the made a fundamental mistake in approach, had in mind a set of goals and set out to build something to achieve them and also had in mind that game theory would help with this in a constructive sense, ideally leading us to clever solutions, and at least providing prior results on which to build. In retrospect, however, it seems more likely that our original goals were provably unrealizable than buildable, and should have looked to game theory to show that, and (if indeed necessary) to navigate towards achievable ones [12]. In order to go for the best strategic player among the available players the concept of rules of dominance is used.

IV. RULES OF DOMINANCE

The principle of dominance also known as dominant strategy or dominance method states that if one strategy of a player dominates over the other strategy in all conditions then the later strategy can be ignored. A strategy dominates over

the other only if it is preferable over other in all conditions. The concept of dominance is especially useful for the evaluation of two-person zero-sum games where a saddle point does not exist

A. Steps in Rules of Dominance

- 1) Used to evaluate individual performance based on criteria
- 2) Strategies are applied to payoff matrix
- 3) In this, one person's good performance will become other person's drawback
- 4) Payoff matrix of B equals Transpose of payoff matrix of A

B. Dominance Rules

- 1) Consider rows as r and column as c
- 2) If a row r_i is dominated by r_j (i.e. $r_i < r_j$), then eliminate r_i
- 3) If a column c_i is dominated by c_j (i.e. $c_i < c_j$), then eliminate c_j
- 4) If primary eliminations are not possible, average eliminations are used

C. Payoff Matrix

An $m \times n$ matrix which gives the possible outcome of a two best performing individuals from the matrix where there are m individuals and n qualities which are obtained from various sources. The analysis of the matrix in order to determine optimal strategies is the aim of game theory [4], [5]. The so-called "augmented" payoff matrix is defined as follows:

$$G = \begin{matrix} & p_0 & p_1 & p_2 & \dots & p_n & p_{n+1} & p_{n+2} & \dots & p_{n+m} \\ 5 & 5 & 4 & 8 & \dots & 9 & 8 & 9 & 7 & \\ & 5 & a_{11} & a_{12} & \dots & a_{1n} & 4 & 5 & 6 & \\ 7 & a_{21} & a_{22} & \dots & a_{2n} & 3 & 7 & 8 & & \\ \vdots & \vdots & \vdots & \dots & \vdots & \vdots & \vdots & \vdots & \vdots & \\ 3 & a_{m1} & a_{m2} & \dots & a_{mn} & 4 & 6 & 9 & & \end{matrix}$$

D. Algebraic Method

Consider payoff is $[a_{ij}]_{m \times n}$. Let (p_1, p_2, \dots, p_m) be the probability for person A with strategies (A_1, A_2, \dots, A_m) . Let (q_1, q_2, \dots, q_n) be the probability for person B with strategies (B_1, B_2, \dots, B_n) . V is the outcome of the assigned task [4]. The expected gain to person A, when person B selects B_1, B_2, \dots, B_n one by one by left-hand side using an equation

$$\begin{aligned} a_{11}p_1 + a_{21}p_2 + \dots + a_{m1}p_m &\geq V \\ a_{12}p_1 + a_{22}p_2 + \dots + a_{m2}p_m &\geq V \\ &\dots \\ &\dots \\ &\dots \\ a_{1n}p_1 + a_{2n}p_2 + \dots + a_{mn}p_m &\geq V \\ p_1 + p_2 + \dots + p_m &= 1 \text{ and } p_i \geq 0 \end{aligned}$$

Similarly, the expected loss to person B, when person A strategies A_i, A_j, \dots, A_m one by one can also be determined.

V. CONCLUSION

The concept of Game Theory has been analysed in order to obtain the strategies that a player would choose with respect to another player which enables the identification of the

capability of each player with their decision making and outcomes. Game theory obtains only the strategies of each player in a work. Hence, Rules of Dominance is used to obtain the best strategy among the available players. Hence our work is towards identifying the best performing individual in a team which will help in identifying a member who worked more towards the development of a project. This helps in granting rewards and performance appraisals of an employee in an efficient manner.

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