

# Study on Cross-flow Heat Transfer in Fixed Bed

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**Abstract**—Radial flow reactor was focused for large scale methanol synthesis and in which the heat transfer type was cross-flow. The effects of operating conditions including the reactor inlet air temperature, the heating pipe temperature and the air flow rate on the cross-flow heat transfer was investigated and the results showed that the temperature profile of the area in front of the heating pipe was slightly affected by all the operating conditions. The main area whose temperature profile was influenced was the area behind the heating pipe. The heat transfer direction according to the air flow directions. In order to provide the basis for radial flow reactor design calculation, the dimensionless number group method was used for data fitting of the bed effective thermal conductivity and the wall heat transfer coefficient which was calculated by the mathematical model with the product of Reynolds number and Prandtl number. The comparison of experimental data and calculated value showed that the calculated value fit the experimental data very well and the formulas could be used for reactor designing calculation.

**Keywords**—Cross-flow, Heat transfer, Fixed bed, Mathematical model

## I. INTRODUCTION

METHANOL shows great advantages in properties, economy and its practical application as an alternative fuel [1]. The combination of low stock cost, single reactor series and low pressure synthesis technology would be the tactics of the methanol industry development [2] and the scale of methanol production was much more enlarged when the technology of methanol conversion to light olefins has received much attention to date from different aspects [3]. Large scale methanol synthesis is the trend of the methanol industry in the world and corresponding methanol synthesis reactor technology is necessary for large scale methanol production [4]. Radial flow reactor was a nice choice for large scale methanol synthesis in single reactor series for its unique character as low pressure drop, wide flow area and easy to be magnified simply by increase the axial height. As we know that methanol synthesis is a strong exothermic reaction and how to remove the reaction

heat from the reactor in time was most important problem during the reactor design. So the study on the heat transfer was one important base in the process of reactor designing.

Lots of researches had taken place on the heat transfer in fixed bed reactor. Wu [5] used the stable state method studied the effective conductivity coefficient on the Co-based catalyst bed with the air was static when the temperature ranged from 165~265°C and the pressure was atmosphere. Some people studied the effective conductivity coefficient with the air was static from both experiments and the theory since long ago such as Krisc and Kroll [6], Yagi and Kunii [7], Kunii and Smith [8], Schlunder [9], Butt [10], David [11], Zehner Schlunder [12], Krupiczka [13] and so on .

In the radial flow reactor which packed the cooling pipes in catalyst bed, the flow type of gas and the medium for heat removing was cross-flow and the heat transfer was different with it in either the fixed bed reactor with the gas was static or the flow type was cocurrent or countercurrent. When the heat transfer type was cross-flow, the heat transfer was investigated by Cheng and Hatton and the model of heat transfer was established. The experience interrelated formula was built by Nasr [14], [15] and Fand [16] after the experiment researches carried out on cross-flow heat transfer in fixed bed.

In order to provide basic basis for radial flow methanol synthesis reactor designing, the cross-flow heat transfer in the radial flow fixed bed reactor which was packed by the commercial C309 Cu-based methanol synthesis catalyst was investigated in this paper.

## II. EXPERIMENT

Fig. 1 demonstrates the experimental flow. Air was compressed by the air pump (1) and measured by the rotameter (3) after the water in air was removed when it pass through the dryer (2). The air was heated in preheater (4) and flow in the reactor which was half Z type radial flow reactor. The air flowed in radial from one side of reactor to another side. The reactor was packed by commercial catalyst with a heating pipe (7) located in the center of the catalyst bed instead of cooling pipe of actual reactor. The heat quantity of heating pipe was controlled by the temperature controller (5) and the temperature profile of catalyst bed was measured by a series of electric thermo-couples which were assembled in catalyst bed. In order to ensure the air distribution was equal when air flows through the catalyst bed in radial, multilayer porous plates were installed in the two sides of reactor inlet.

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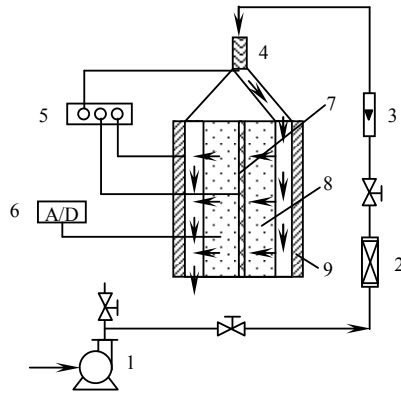


Fig. 1 Experimental flow sheet

- 1-Air pump; 2-Dryer; 3- Rotameter; 4-Preheater;  
5- Temperature controller; 6-Temperature demonstrator;  
7-Heating pipe; 8-Catalyst; 9- insulating layer

The experiment was carried out under the conditions as follow: heating pips temperature ranged from 200 to 260□, reactor inlet temperature ranged from 180 to 260□ and the flow volume velocity of air ranged from 3 to 7Nm<sup>3</sup>/h. The size of the catalyst bed was 200mm×200mm×200mm.

### III. MATHEMATICAL MODEL

The two dimension mathematical model of cross-flow heat transfer in fixed bed reactor with the gas flow was perpendicular to cooling pipes was established based on the hypotheses as follow: (1) air was ideal gas and the physical parameters of air was just related to temperature; (2) the temperature of heating pipe was equal in axial direction; (3) the axial direction heat conduction of catalyst bed according to heating pipe could be ignored; (4) the gas flow in reactor was plug-flow and the gas velocity distribution in catalyst bed could be ignored.

The mathematical model could be described as formula (1):

$$GC_p \frac{\partial T}{\partial x} = \lambda_b \frac{\partial^2 T}{\partial x^2} + \lambda_b \frac{\partial^2 T}{\partial y^2} \quad (1)$$

Where  $G$  was the mole flow rate;  $C_p$  was air constant pressure heat capacity;  $\lambda_b$  was the bed effective thermal conductivity.

The bound conditions of catalyst bed were:

$$x=0, T=T_{in}; \quad x=H, \frac{\partial T}{\partial x}=0;$$

$$y=0, \frac{\partial T}{\partial y}=0; \quad y=L/2, \frac{\partial T}{\partial y}=0.$$

The bound condition of heating pipe was:

$$y^2 + (x - \frac{H}{2})^2 = r_0^2, \quad \alpha_w (T_w - T) = -\lambda_b \frac{\partial T}{\partial N}$$

Where  $H$  was the height of catalyst bed;  $L$  was the width of catalyst bed;  $r_0$  was radius of heating pipe;  $\alpha_w$  was the wall heat transfer coefficient;  $T_w$  was the temperature of heating pipe.

### IV. TEMPERATURE PROFILE OF CATALYST BED

There were 20 sets of electric thermo-couples installed in the catalyst bed to investigate the temperature profile. In order to descript clearly, the coordinate system of catalyst bed was set as

fig. 2.

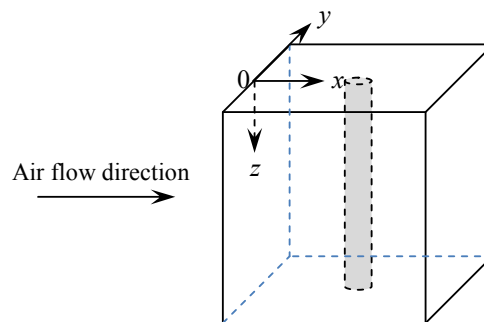
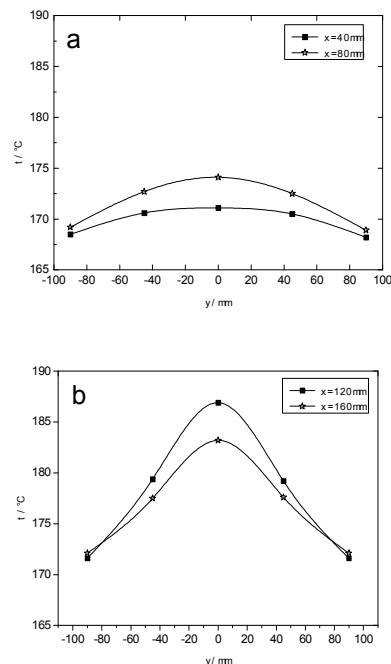


Fig. 2 Coordinate system of catalyst bed

There were 5 sets electric thermo-couples installed in the cross area (where  $z=100$ mm) of the catalyst bed which could point out the radial temperature distribution based on the hypotheses referred before. The temperature profile was shown as fig. 3 when the operating conditions were: the air flow rate was 3.0m<sup>3</sup>/h, the temperature of heating pipe was 200°C and the reactor inlet temperature was 165°C.

Fig. 3 Temperature profile in catalyst bed  
a-in front of the heating pipe; b-behind the heating pipe

From fig. 2, it could be known that the temperature of the area which was behind the heating pipe was higher than the temperature of the area which was in front of the heating pipe obviously. In the area which was in front of the heating pipe, the direction of heat transfer was opposite with the direction of flow and the quantity of heat transfer was small. In the area which was behind the heating pipe, the heat transfer direction was the same with the air flow direction which was beneficial to heat

transfer so the air was heated much more than the air in front of the heating pipe. The areas behind the heating pipe and the two sides of heating pipe were main heated areas and especially there was a relatively wide range behind the heating pipe. In front of the heating pipe, the air was little heated even it was very close to the heating pipe. Results showed that the heat transfer was according to the air flow direction and the air flow direction affect the temperature profile seriously in fixed bed reactor when the type of heat transfer was cross-flow heat transfer.

## V. EFFECTS OF OPERATING CONDITION ON TEMPERATURE PROFILE

### A. Effects of reactor inlet air temperature

The effects of reactor inlet air temperature on temperature profile were investigated when the temperature of heating pipe was 205°C and the air flow rate was 5.0m<sup>3</sup>/h. The temperature distribution of catalyst bed was shown as fig.4 when the reactor inlet temperature was 150°C, 160°C and 175°C respectively.

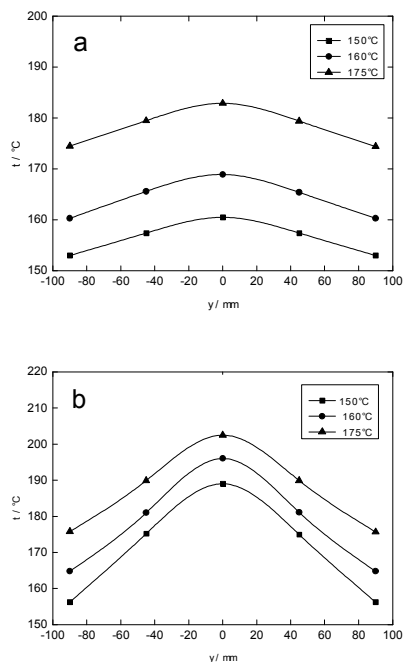


Fig. 4 Effects of reactor inlet air temperature a-in front of the heating pipe; b-behind the heating pipe

From fig. 4, it could be known that all the points of catalyst bed in front of the heating pipe increased almost equally with the reactor inlet air temperature increased which means that the reactor inlet air temperature affected the temperature distribution of the catalyst bed which in front of the heating pipe slightly. In the area which was behind the heating pipe, the increase degree of the temperature of the edge of the catalyst bed was higher than which of the center of the catalyst bed and the temperature curve became more gentle with the reactor inlet

air temperature increased which means that the reactor inlet air temperature affected the heat transfer degree a lot.

### B. Effects of the heating pipe temperature

The effects of the heating pipe temperature on the temperature profile of the catalyst bed were investigated when the reactor inlet air temperature was 150°C and the air flow rate was 5.0m<sup>3</sup>/h. The temperature trend curves were shown as fig.5 when the heating pipe temperature was 195°C, 200°C and 205°C respectively.

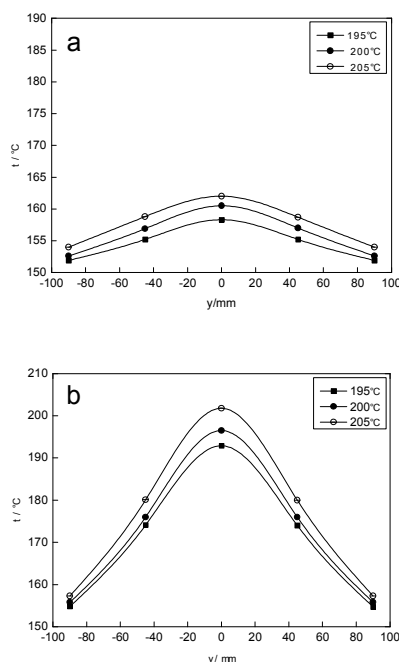


Fig. 5 Effects of the heating pipe temperature a-in front of the heating pipe; b-behind the heating pipe

Fig. 5 showed that the catalyst bed temperature increased with the heating pipe temperature increased, but there were some differences between the change trend of the area in front of the heating pipe and the area behind the heating pipe. In the area in front of the heating pipe, the temperature increased almost equally in all measured points which means that the heat transfer of the area in front of the heating pipe changed reposefully with the heating pipe temperature increased. In the area behind the heating pipe, the closer the measured point near the heating pipe the greater the temperature changed. So it was known that, to the temperature profile of the catalyst bed, the area behind the heating pipe was the main area which was influenced by the heating pipe temperature and the heat transfer direction according to the flow direction.

### C. Effects of the air flow rate

The effects of the air flow rate on the temperature profile of catalyst bed were investigated as the heating pipe temperature

was 205°C and the reactor inlet temperature was 175°C. The temperature change trend was shown as fig. 6 when the air flow rate ranged from 3.0 to 7.0Nm<sup>3</sup>/h.

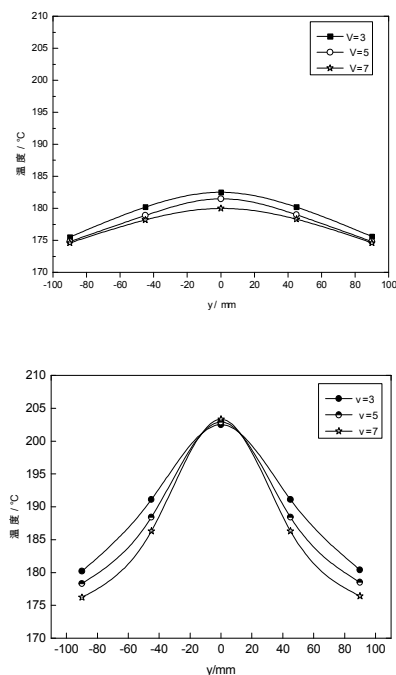


Fig. 6 Effects of the heating pipe temperature a-in front of the heating pipe; b-behind the heating pipe

In the area in front of the heating pipe, the closer the measured point near the heating pipe the greater the temperature changed and in the edge of catalyst bed, the temperature changed slightly with the air rate flow increased. So the temperature curve became more flat in the area in front of the heating pipe. In the area behind the heating pipe, the change of temperature was more complex. In the area which was close to the heating pipe, the temperature increased with the air flow rate increased but when the distance from the heating pipe increased to a certain degree, the temperature decreased with the air flow rate increased. The higher temperature area became narrow. In the area behind the heating pipe, the farther near the heating pipe the greater the temperature changed, which was opposite to the change trend of the area in front of the heating pipe.

From the researches above, the conclusions could be obtained that the area behind the heating pipe was the main area which was influenced by the operating conditions in fixed bed reactor when the heat transfer type was cross-flow. The heat transfer direction according to the air flow direction and there was almost no heat was transferred in the reverse direction of the air flow direction, which was the characteristics of the cross-flow heat transfer.

## VI. BED EFFECTIVE THERMAL CONDUCTIVITY AND WALL HEAT TRANSFER COEFFICIENT

In the mathematical model established, the bed effective thermal conductivity ( $\lambda_b$ ) and the wall heat transfer coefficient ( $\alpha_w$ ) were undetermined coefficients and the value of which were got during the solving process of mathematical model. The temperature profile of the catalyst bed which was obtained from experiment was used for the solving process.

Optimal method was used in the model solving process and the objective function was the sum of squares of the actual temperature and the calculated value of each measured point in the catalyst bed. The values of the bed effective thermal conductivity and the wall heat transfer coefficient were calculated when the objective function reached the minimum.

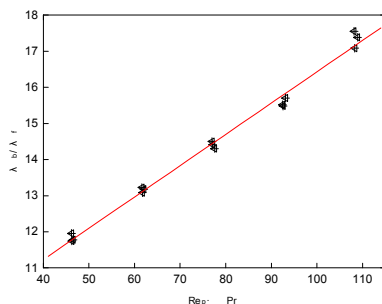
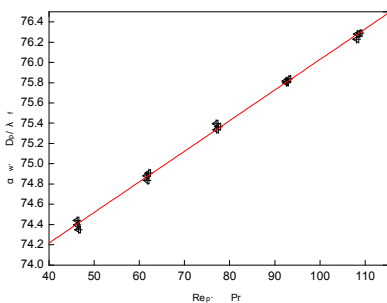
Table I showed the bed effective thermal conductivity ( $\lambda_b$ ) and the wall heat transfer coefficient ( $\alpha_w$ ) when the operation conditions as follow: the Reynolds number ( $Re_p$ ) ranged from 67 to 160, the temperature of heating pipe was 205°C, and the reactor inlet air temperature was 150°C, 175°C and 200°C respectively.

TABLE I  
VALUE OF  $\lambda_b$  AND  $\alpha_w$

| $Re_p$ | $t/^\circ\text{C}$ | $\lambda_b/\text{W}\cdot\text{m}^{-2}\cdot\text{k}^{-1}$ | $\alpha_w/\text{W}\cdot\text{m}^{-1}\cdot\text{k}^{-1}$ |
|--------|--------------------|--|---|
| 67.96  | 150                | 434.52   | 0.42  |
| 90.61  | 150                | 437.79   | 0.47  |
| 113.23 | 150                | 440.46   | 0.51  |
| 135.92 | 150                | 443.21   | 0.56  |
| 158.97 | 150                | 445.85   | 0.62  |
| 67.96  | 175                | 456.74   | 0.44  |
| 90.61  | 175                | 459.44   | 0.49  |
| 113.23 | 175                | 462.51   | 0.54  |
| 135.92 | 175                | 465.36   | 0.58  |
| 158.97 | 175                | 468.32   | 0.64  |
| 67.96  | 200                | 491.11   | 0.47  |
| 90.61  | 200                | 488.46   | 0.52  |
| 113.23 | 200                | 485.75   | 0.57  |
| 135.92 | 200                | 482.44   | 0.61  |
| 158.97 | 200                | 479.59   | 0.69  |

From table I, it could be known that both the bed effective thermal conductivity ( $\lambda_b$ ) and the wall heat transfer coefficient ( $\alpha_w$ ) increased with either the Reynolds number or the reactor inlet temperature increased.

The relationship between the bed effective thermal conductivity and the product of Reynolds number and Prandtl number ( $Pr$ ) was shown as fig. 6. The relationship between the wall heat transfer coefficient and the product of Reynolds number and Prandtl number was shown as fig. 7.

Fig. 7 Relationship between  $\lambda_b$  and  $Re_p \cdot Pr$ Fig. 8 Relationship between  $\alpha_w$  and  $Re_p \cdot Pr$ 

The dimensionless number group method was used for data fitting of the bed effective thermal conductivity and the wall heat transfer coefficient with the product of Reynolds number and Prandtl number and the results were:

$$\frac{\lambda_b}{\lambda_f} = 7.7644 + 0.0867 Re_p \cdot Pr \quad (2)$$

$$\frac{\alpha_w D_p}{\lambda_f} = 73.01 + 0.0302 Re_p \cdot Pr \quad (3)$$

The catalyst bed temperature profile was calculated when the operating condition was: the temperature of the heating pipe was 205°C, the reactor inlet air temperature was 175°C and the air flow rate was 3.0m<sup>3</sup>/h, the comparison of the actual temperature and the calculated value was shown as table II.

TABLE II  
COMPARISON OF EXPERIMENTAL DATA AND CALCULATED VALUE

| No. | Actual value/°C | Calculated value/°C | Absolute error/°C | Relative error /% |
|-----|-----------------|---------------------|-------------------|-------------------|
| 1   | 175.2           | 175.3               | -0.1              | -0.06             |
| 2   | 175.9           | 175.7               | 0.2               | 0.11              |
| 3   | 176.8           | 176.3               | 0.5               | 0.28              |
| 4   | 176.2           | 175.7               | 0.5               | 0.28              |
| 5   | 175.0           | 175.3               | -0.3              | -0.17             |
| 6   | 176.3           | 176.2               | 0.1               | 0.06              |
| 7   | 181.3           | 179.3               | 2.0               | 1.10              |
| 8   | 185.9           | 191.2               | -5.3              | -2.85             |
| 9   | 181.1           | 179.3               | 1.8               | 0.99              |
| 10  | 176.4           | 176.2               | 0.2               | 0.11              |
| 11  | 177.5           | 178.2               | -0.7              | -0.39             |
| 12  | 191.0           | 185.8               | 5.2               | 2.72              |
| 13  | 202.9           | 202.9               | 0.0               | 0.00              |
| 14  | 190.9           | 185.8               | 5.1               | 2.67              |
| 15  | 177.4           | 178.2               | -0.8              | -0.45             |
| 16  | 178.3           | 180.2               | -1.9              | -1.07             |
| 17  | 186.8           | 186.3               | 0.5               | 0.27              |

|    |       |       |      |       |
|----|-------|-------|------|-------|
| 18 | 192.5 | 193.7 | -1.2 | -0.62 |
| 19 | 186.9 | 186.3 | 0.6  | 0.32  |
| 20 | 178.1 | 180.2 | -2.1 | -1.18 |

Table II showed that the maximum difference in temperature of the actual value and calculated value was 5.3 and the absolute value of the max relative error was just 2.85%, which means that the calculated value fit the actual value well.

## VII. CONCLUSION

The cross-flow heat transfer was studied with the operating conditions as follow: the reactor inlet air temperature ranged from 150 to 175°C, the heating pipe temperature ranged from 195 to 205°C and the flow rate ranged from 3.0 to 7.0m<sup>3</sup>/h. the results showed that the area affected by the heat transfer was mainly behind the heating pipe. The process of cross-flow heat transfer was in close connection with the air flow direction, the heat transfer in the same direction as the air flows and there was almost no heat transfer in the reverse direction.

The mathematical model of cross-flow heat transfer was established, the experiment data was used for the solving process of the model and the bed effective thermal conductivity and the wall heat transfer coefficient were calculated during the solving process. The dimensionless number group method was used for data fitting of the bed effective thermal conductivity and the wall heat transfer coefficient with the product of Reynolds number and Prandtl number, and the comparison showed that the calculated value fit the experimental data very well.

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