

# Study of Photonic Crystal Band Gap and Hexagonal Microcavity Based on Elliptical Shaped Holes

A. Benmerkhi, A. Bounouioua, M. Bouchemat, T. Bouchemat

**Abstract**—In this paper, we present a numerical optical properties of a triangular periodic lattice of elliptical air holes. We report the influence of the ratio (semi-major axis length of elliptical hole to the filling ratio) on the photonic band gap. Then by using the finite difference time domain (FDTD) algorithm, the resonant wavelength of the point defect microcavities in a two-dimensional photonic crystal (PC) shifts towards the low wavelengths with significantly increased filling ratio. It can be noted that the Q factor is gradually changed to higher when the filling ratio increases. It is due to an increase in reflectivity of the PC mirror. Also we theoretically investigate the H1 cavity, where the value of semi-major axis ( $R_x$ ) of the six holes surrounding the cavity are fixed at  $0.5a$  and the  $R_x$  of the two edge air holes are fixed at the optimum value of  $0.52a$ . The highest Q factor of  $4.1359 \times 10^6$  is achieved at the resonant mode located at  $\lambda = 1.4970 \mu\text{m}$ .

**Keywords**—Photonic crystal, microcavity, filling ratio, elliptical holes.

## I. INTRODUCTION

PC is a periodic optical structure in which the refractive index changes periodically. In recent years, PC-based optical devices have received considerable attention due to their interesting characteristics, such as their small size, photonic band gap, negligible loss in micrometer range, extremely low group velocity, flexibility in shape and dimensions, improved frequency-dependent selection and applicability in photonic integrated circuits (PICs).

From the beginning of research on PCs, a major area of investigation concerned two dimensional (2D) PCs [1]. This was mainly caused by experimental reasons as the fabrication of 3D PCs appeared to be more difficult and cumbersome than that of 2D PCs. Additionally, the calculation of band structures for 2D PCs is less time consuming and a lot of interesting phenomena can already be studied in 2D PCs. With the characteristics of relatively easy fabrication processing and matured techniques, PC lasers [2], PC waveguides [3] and some other devices [4], [5] based on 2D PC have quickly been produced. Column dielectric rod and column hole structures are two fundamental 2D PC structures, and they have been fully investigated [6], [7]. However there are relatively few papers dedicated to investigations of the bandgap and the cavity characteristics of elliptical dielectric rod or hole structure. Kalra et al. [8] have studied theoretical bandgap structures in 2D square lattice elliptical dielectric rod PCs. In order to obtain the maximal bandgap width, one of the ellipse axes is fixed to  $0.3a$ , varying the other axis from  $0.1a$  to  $0.5a$

with a step length of  $0.01a$ . Benmerkhi et al. [9], [10] studied the effect of the size, shape of neighboring holes of the cavity on the Q-value and the sensitivity. They clearly observed that the transmission of the cavity with elliptical shape is very high than that with reduced holes. We note an increase in the sensitivity value of 16% which is achieved by replacing the six modified circular holes on nearest neighboring holes of the cavity with ellipses.

In this paper, we discuss the variation of photonic band gap size of the ellipticity of the constituent air holes for transverse electric (TE) polarizations. It is shown that the photonic band gap (PBG) width becomes wider by the increasing of filling ratio. We then study the effects of the variation of elliptical holes on the PC cavity. We observe that the increase in the filling ratio generates: a displacement resonant wavelength towards the low wavelengths. We theoretically investigate further increasing the cavity Q factor by tailoring the envelope function of the electric field profile through change of the size of two air holes near the cavity edges. For this cavity, the highest Q factor of  $4.1359 \times 10^6$  is achieved at the resonant mode located at  $\lambda = 1.4970 \mu\text{m}$  when  $R_x = 0.52a$ .

## II. STUDY ON MAXIMUM BAND GAP WITH ELLIPTICAL HOLES

In the designed structure shown in Fig. 1, 2D triangular PC of air holes patterned perpendicularly to a Indium phosphide (InP) based confining heterostructure with the effective RI  $n = 3.32$  ( $\epsilon = 11$ ) is selected. The lattice constant is " $a$ " while the air holes are the elliptical shape. Ellipse has the  $R_x$  and semi-minor axis ( $R_y$ ). Details on the sample fabrication are given in [11], [12].

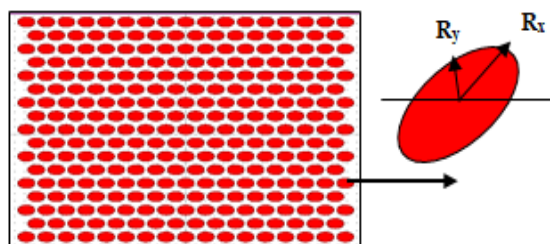


Fig. 1 Schematic diagram of 2D triangular lattice PC

The computational method used is based on a 2D FDTD method algorithm. Perfectly matched layer (PML) conditions have been considered in the calculations to ensure no back reflection in the limit of the analyzed region [13]. This crystal is light by a Gaussian wave under normal incidence with a polarized TE. The length of the PC is  $17a$  and the time step is

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chosen as 0.01. Note that it might be necessary to reduce the time step below the stability limit when simulating metals since the courant condition can change in this case.

TABLE I  
THE PBG WIDTH WITH DIFFERENT FILLING RATIOS

Filling ratio (f)	0.70	0.75	0.80
PBG ( $\mu\text{m}$ )	1.383-1.835	1.3171-1.8171	1.25-1.7921
PBG width ( $\mu\text{m}$ )	0.45	0.50	0.5421

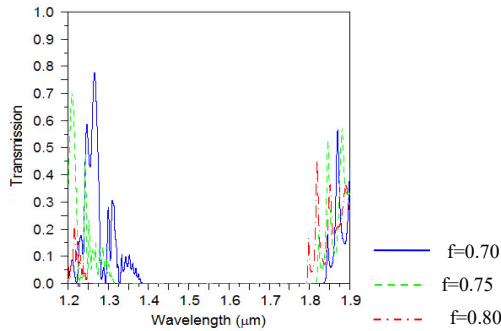


Fig. 2 Transmission spectrum with different filling ratio

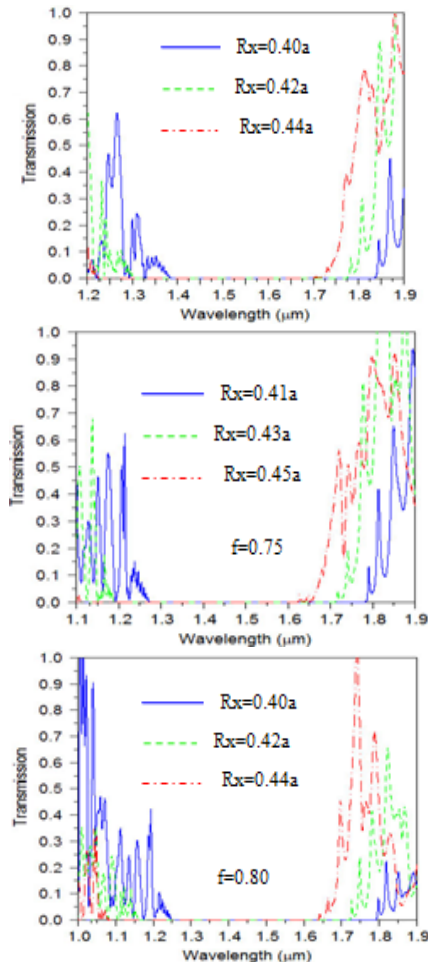


Fig. 3 Transmission spectra with variation of Rx to different filling ratio (f)

In general, the dispersion properties of 2D PCs are controlled by the dielectric contrast, the lattice type, i.e. square or triangular geometry, and the filling ratio. We begin our consideration by study the influence of the filling ratio on the PBG, we take the same structure where the Rx is equal to 0.4a. By comparing the various spectra, we observe that the PBG width becomes wider by the increasing of filling ratio for three different values of 0.7, 0.75, and 0.8 (see Table I). Fig. 2 shows that the air band edge (low wavelength) exhibits a large redshift than that of the dielectric band edge (high wavelength).

Next, we calculate the transmission spectra for different values of Rx and Ry while the filling ratio is fixed at three values of 0.7, 0.75 and 0.8. The corresponding transmissions are plotted in Fig. 3. The results expose that the width of the PBG does not change wider with the increasing of the value of Rx all the while.

From the transmission spectrum, we see that for  $f = 0.7$  the width of PBG with  $R_x = 0.42a$  is wider than that of  $R_x = 0.4a$  or  $0.44a$ . Also, the PBG width when  $R_x$  is equal to  $0.43a$  is wider than that of  $R_x = 0.41a$  or  $0.45a$  to the filling ratio  $f = 0.75$ , and for the filling ratio "f" is equal to 0.80, the PBG width of  $R_x = 0.42a$  is widest comparing with the width of  $R_x = 0.4a$  and  $0.44a$ . Therefore, the PBG width is not expanded with the increase of Rx. By comparing the various spectra, we observe that the increase of Rx generates a band gap shift towards the low wavelengths. It indicates that the high boundary wavelength decreases rapidly by the increasing of Rx for different values of "f".

### III. STUDY ON MAXIMUM QUALITY FACTOR CAVITY WITH ELLIPTICAL HOLES

Hexagonal cavities are carried by removing air holes of the PC in a hexagon. These cavities are commonly called "cavity H<sub>x</sub>" with X the number of holes removed by side of the hexagon. Removing one air hole at the center of the lattice creates a H<sub>1</sub> microcavity (see Fig. 4). We use FDTD solutions to study the influence of the filling ratio and the size of the elliptical air holes on the PC cavity. The lattice constant of PC is  $0.42\mu\text{m}$ . The quality factor is calculated using the 2D FDTD method, combined with fast harmonic analysis. The Q factor is defined as  $\lambda_0/\Delta\lambda$ , where  $\Delta\lambda$  is the full width at half-maximum (FWHM) of the resonator's Lorentzian response and  $\lambda_0$  is the resonance wavelength. First, we directly simulated the influence of the filling ratio on the H<sub>1</sub> microcavity, we consider the same structure where the  $R_x = 0.4a$  then we varied the filling ratio (f) for different values of 0.7, 0.75 and 0.80. For  $f = 0.7$ , Fig. 5 (a) shows transmission of the H<sub>1</sub> cavity where we observe a resonant mode  $\lambda_0 = 1.6298\mu\text{m}$  with the quality factor  $Q = 3.6844 \times 10^5$ , for  $f = 0.75$ , it is noted that the resonant mode shifts towards the wavelength  $\lambda_0 = 1.60097\mu\text{m}$ . We observe around this frequency an improvement of quality factor  $Q = 8.6259 \times 10^5$  (see the green line of Fig. 5 (a)) and for  $f = 0.8$ , we observe a resonant mode  $\lambda_0 = 1.5726\mu\text{m}$  and  $Q = 1.6729 \times 10^6$  (see the red line of Fig. 5 (a)). By comparing the various spectra, we observe that the increase in the filling ratio generates a displacement resonant

wavelength towards the low wavelengths; this is due to the reduction length of cavity and the quality factor is improved (see Fig. 5 (b)). It is because that the Q factor will be enhanced when the wall of the cavity is increased [14]. It further suggests that a higher Q value and lower transmission are achievable by an increase of "f". This result is in agreement with the traditional cavities.

After that we compute the quality factor for a variety the value of Rx and Ry while the filling ratio is fixed at two values of 0.7 and 0.75.

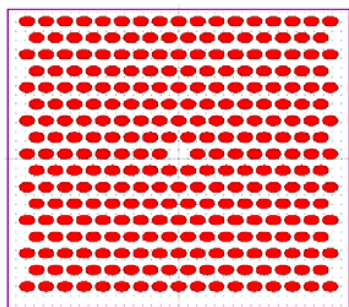


Fig. 4 Sight of top of the H<sub>1</sub> cavity made by removed one hole.

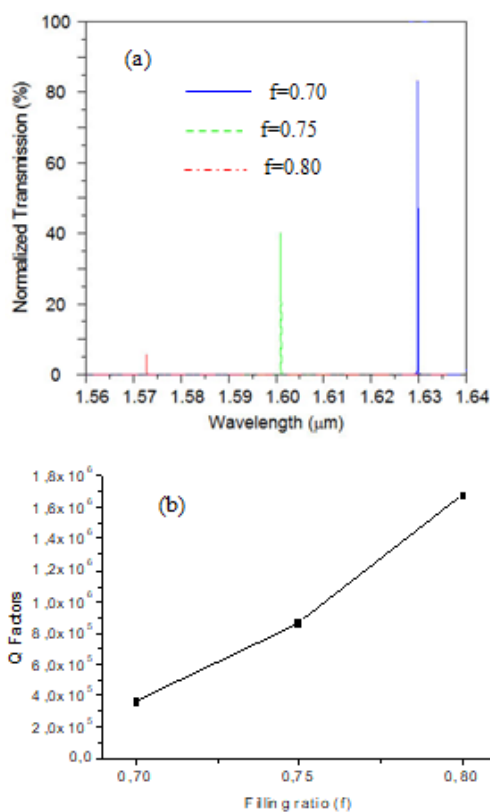


Fig. 5 (a) Transmission of the H<sub>1</sub> cavity in a triangular lattice for different filling ratio, (b) Quality factor Q as a function of the filling ratio on the H<sub>1</sub> microcavity

A plot of Q factors versus Rx for f = 0.7 and 0.75 are given in Fig. 6. The maximum quality factor that we calculated Q =

$3.5367 \times 10^6$  appears at Rx = 0.43a for f = 0.75 but the transmission is very low (see the green line in Fig. 7).

In order to acquire the high transmission efficiency and Q factor simultaneously of the cavity with Rx = 0.43a and f = 0.75, by reducing unwanted reflection due to mismatch and through minimization of propagation losses, we modified H<sub>1</sub> geometry, where the value of Rx of the six holes surrounding the cavity are fixed at 0.5a (indicated by the darker circle in insert of Fig. 7). The transmission spectrum obtained by this cavity is represented in Fig. 7, we clearly observe the resonance peak of this cavity shift towards the wavelength  $\lambda = 1.4999 \mu\text{m}$  with a quality factor Q =  $3.8 \times 10^6$ .

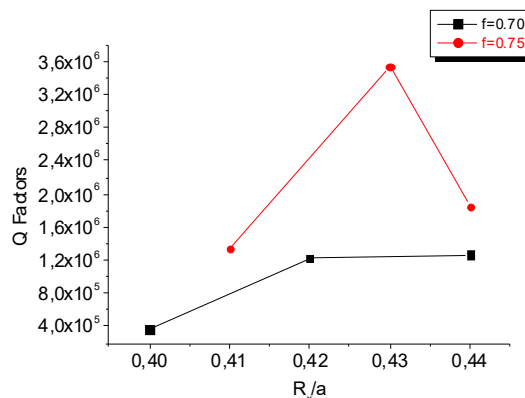


Fig. 6 Quality factor Q as a function of the Rx on the H<sub>1</sub> microcavity: f = 0.70 (black line), f = 0.75 (red line)

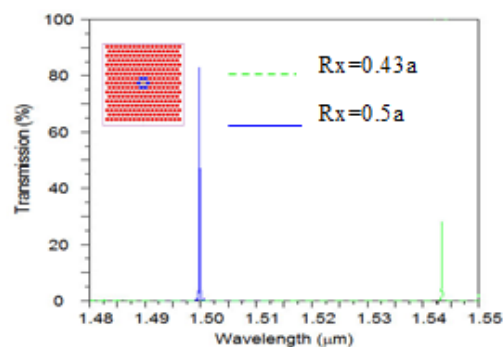


Fig. 7 Transmission of the H<sub>1</sub> cavity in a triangular lattice of the H<sub>1</sub> cavity where the value of Rx of the six holes surrounding the cavity are fixed at 0.5a (blue line) and for 0.43a (green line). In insert: Representation of the H<sub>1</sub> cavity where the value of Rx of the six holes surrounding the cavity are fixed at 0.5a

Next we increased the Rx (two edge air holes) in the range 0.5a–0.53a, (indicated by the green circle in the insert of Fig. 8 (b)).

The results for the quality factors and the size of single hole are plotted in Fig. 8 (a). The highest Q factor of  $4.1359 \times 10^6$  is achieved when Rx = 0.52a at the resonant mode located at  $\lambda = 1.4970 \mu\text{m}$  (see Fig. 8 (b)). Thus, we choose this value as the optimum result due to its high transmission efficiency and Q factor.

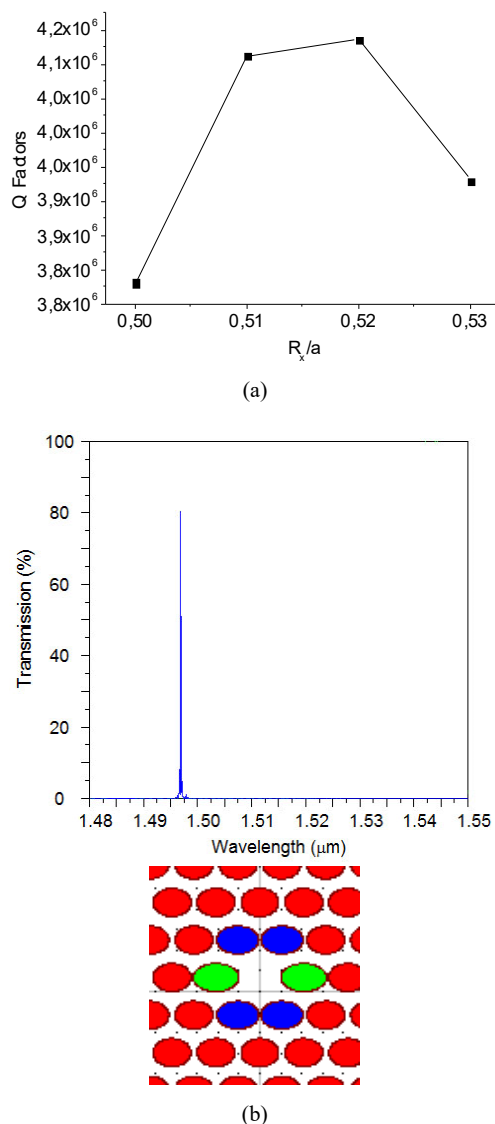


Fig. 8 (a) Quality factor  $Q$  as a function of the  $R_x$  of the two edge air holes, (b) Transmission of the H1 cavity in a triangular lattice of the H1 cavity where the value of  $R_x$  of the two edge air holes is fixed at 0.52a. In insert: representation of the H1 cavity

#### IV. CONCLUSION

First we studied the maximum PBG of 2D PC with elliptical air holes. We extended this study to the hexagonal cavity  $H_1$  for a lacunar defect; we have shown that the resonant modes of the cavity with 2D PCs are very sensitive to the variations of the geometrical parameters of the PC such as: the filling factor which influences the position of resonance wavelength.

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