Study of Adaptive Filtering Algorithms and the Equalization of Radio Mobile Channel

Said Elkassimi, Said Safi, B. Manaut

Abstract**—**This paper presented a study of three algorithms, the equalization algorithm to equalize the transmission channel with ZF and MMSE criteria, application of channel Bran A, and adaptive filtering algorithms LMS and RLS to estimate the parameters of the equalizer filter, i.e. move to the channel estimation and therefore reflect the temporal variations of the channel, and reduce the error in the transmitted signal. So far the performance of the algorithm equalizer with ZF and MMSE criteria both in the case without noise, a comparison of performance of the LMS and RLS algorithm.

Keywords**—**Adaptive filtering second equalizer, LMS, RLS Bran A, Proakis (B) MMSE, ZF.

I. Introduction

ADAPTIVE filtering [6] is based on finding optimal parameters by minimizing a performance criterion.

Frequently, this minimization is done by seeking the least squares. The performances of digital transmission system [3], [9] are expressed in terms of reliability. This may be achieved by:

- The coding of channel, or correct coding of error,
- Equalization, which allows to make the most the pass band of the channel offsetting receipt [8] the distortions introduced by the transmission medium, electronic equipment, etc...

There are two approaches:

- The adaptive approach to switch to the channel estimation [11] and therefore take into account the temporal variations of the channel,
- A suboptimal approach called LEVELS.

In this paper, we study the adaptive filtering algorithms such as LMS and RLS algorithms to estimate the coefficients of the FIR filter h_E in the noisy cases [4], and the equalization algorithm based with ZF and MMSE criteria [7].

II. Adaptive Equalization

The equalization approach has some drawbacks related to the need for accurate channel estimation and calculation of the correlation matrix of the received data and its inverse [5]. On the other hand, if the channel varies in time, this approach does not allow adjusting the coefficients of the equalizer [1]. In fact, the transversal equalizer on the MSE criterion is based on minimizing the function:

$$J(h_E) = E[(a_k - z_k)^2] \tag{1}$$

It is therefore necessary to calculate the gradient as:

$$\nabla J(h_E) = 2\left(R_y H_E - R_{ay}\right) = 0 \tag{2}$$

This leads to a complexity in costly analytical solution:

$$H_E = R_y^{-1}.R_{ay} \tag{3}$$

In the adaptive approach, one can dispense with the channel estimation and therefore take into account the temporal variations of the channel [10]:

A. LMS «Least Mean Square» Algorithm

In the implementation of the MSE criterion, an alternative to avoid reverse of R_y is to apply an iterative method to calculate the coefficients that minimize the cost function: $J(h_E)$.

From the values of $h_E(k-1)$ the values can be calculated from $h_E(k)$ using the algorithm of the gradient:

$$h_E(k) = h_E(k-1) + \mu(R_{ay} - R_y h_E(k-1) \tag{4}$$

With μ positive constant called the coefficient adaptation (replacingR_y^{-1}) for controlling the convergence. However, the calculation of $\nabla J(h_E^{(n)})$ always requires knowledge R_y and R_{ay} by using a training sequence.

It then modifies the algorithm by replacing the gradient by its estimated (LMS is a gradient algorithm called "stochastic" and not deterministic). Is replaced at each step R_y and R_{ay} estimated by $y_k\,.y_k^T$ and $a_k.Y_k$. The equation becomes:

$$h_E(k) = h_E(k-1) + \mu\left(a_k - y_k^T h_E(k-1)\right)y_k = h_E(k-1) + \mu(a_k - z_k)y_k = h_E(k-1)\mu e_k y_k \tag{5}$$

The error signal e_k represents the desired difference between the data at time k and the actual output $z(kT)$.

The LMS allows every moment to "update" the equalizer filter coefficients in proportion to the estimation error e_k.

In case of variations of the channel, the equalizer will be able to adapt more rapidly than the constant μ is greater. It can be assumed that an adequate value μ to ensure convergence in the case of channels with slow variations is: $\mu = \frac{0.2}{(P_s+P_n)(2N+1)}$ avec (2N+1) number of coefficients of the equalizer, P_s signal power and P_n noise power. We can summarize the LMS algorithm in the following diagram:

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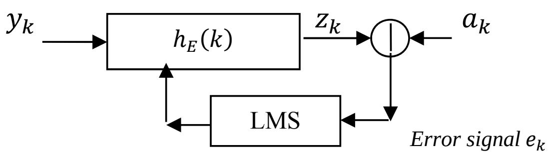

Fig. 1 LMS Algorithm diagram

B. RLS (Recursive Least Square) Algorithm

The basic algorithm of the stochastic gradient is LMS wherein the vector is approximated by a gradient from the estimation data. However, when the channel has a very even spread impulse response; the LMS converges very slowly due to a single parameter control (no adaptation). Can implement the algorithm Kalman/Godard [13] also known as recursive least squares algorithm (RLS) which has a good rate of growth, of course, priced at more calculations. This algorithm is defined by:

a) Calculating the error signal at time kT dependent coefficients at instant $(k-1)T$ previous:

$$e_k = a_k - z_k = a_k - y_k^T h_E(k-1) \tag{6}$$

b) Update the coefficients:

$$h_E(k) = h_k(k-1) + \mu P(k) y_k (a_k - z_k) \tag{7}$$

The difference from the LMS is within the term $P(k)$; is an estimate of R_y^{-1} obtained recursively:

$$P(k) = \frac{1}{1-\mu}\left(P(k-1) - \frac{\mu P(k-1)\, y_k\, y_k^T P(k-1)}{1-\mu+\mu\, y_k^T\, P(k-1)\, y_k}\right) \tag{8}$$

The term $P(k)$ makes optimum use of the various coefficients which explains the superiority of the RLS algorithm in terms of speed of convergence.

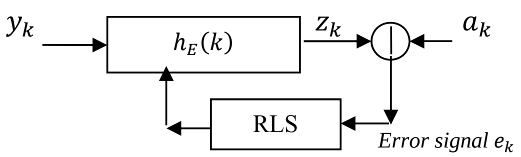

Fig. 2 RLS Algorithm diagram

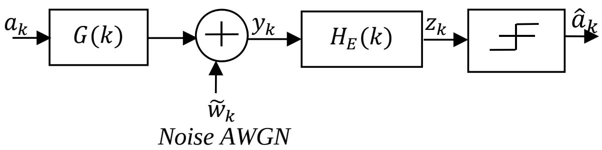

Fig. 3 Equalization Algorithm diagram

III. Equalization Algorithm

Samples received are written by:

$$y_k = \sum_n a_n g_{k-n} + \widetilde{w}_k = a_k g_0 \sum_{n \neq k} a_n g_{k-n} + \widetilde{w}_k \tag{9}$$

or $\widetilde{w}_k$ is a sample of additive Gaussian noise centered (AWGN) of variance $\sigma^2 = E(|\widetilde{w}_k|^2)$.

The general idea is to apply an equalizer filter $H_E(k)$ to the samples y_k compensate for the equivalent channel $G(k)$ (Fig.3).

The problem is: what criteria to choose $H_E(k)$?

Consider a transverse filter for $(2N+1)$ coefficients, transverse equalizers [12] are the easiest to implement. Indeed, this is simply to use a digital finite impulse response filter [9] for which the methods of calculation and implementation are well known.

$$z(k) \quad = \sum_{n=-N}^{+N} y(-n) h_{E,n} \tag{10}$$

k represents the time flowing from -2N à 2N for (2N+1) input samples. We can write the relation of convolution matrix form: $Z = Y.H_E$. With:

$$Z = \begin{pmatrix} z(-2N) \\ z(-2N+1) \\ \vdots \\ z(0) \\ \vdots \\ z(2N) \end{pmatrix} \tag{11}$$

column vector of dimension (4N+1)

$$H_E = \begin{pmatrix} h_{E,-N} \\ h_{E,-N+1} \\ \vdots \\ h_{E,0} \\ \vdots \\ h_{E,N} \end{pmatrix} \tag{12}$$

column vector of dimension (2N+1)

$$Y = \begin{pmatrix} y(-N) & 0 & 0 & \cdots & \cdots & \cdots & 0 \\ y(-N+1) & y(-N) & 0 & 0 & \cdots & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ y(N) & y(N-1) & y(N-2) & \cdots & \cdots & y(-N+1) & y(-N) \\ 0 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \vdots & \vdots & \vdots & \vdots & 0 & y(N-1) \\ 0 & 0 & \cdots & \cdots & \cdots & & y(N) \end{pmatrix} \tag{13}$$

Is a matrix of dimension $(4N+1) \times (2N+1)$ the purpose of the equalization algorithm [7] is to determine the coefficients $\{h_{E,n}\}$ to minimize the error probability Pe, and remove the IES; this algorithm is based on the criteria «Zéro-Forcing» (ZF) and «Minimum Mean Square Error» (MMSE).

IV. Simulation and Comparison

A. Performance of the Algorithm of the Equalizer

1. The Equalization Algorithm Based on ZF Criterion

ZF criterion is applied with equalization, for comparing the output of the equalizer, with the channel Bran A [3] response, in the environment noise [10 and 30 dB]

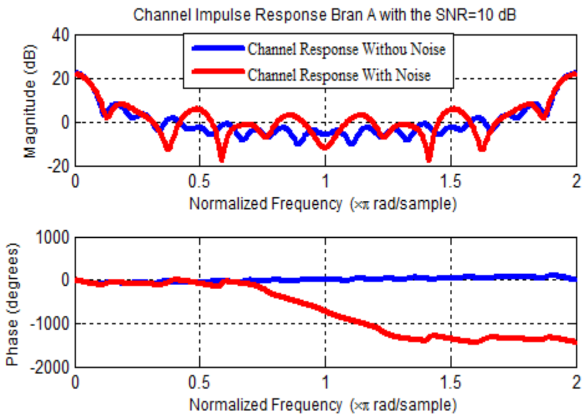

Fig. 4 Channel impulse response Bran A, with the SNR=10 dB

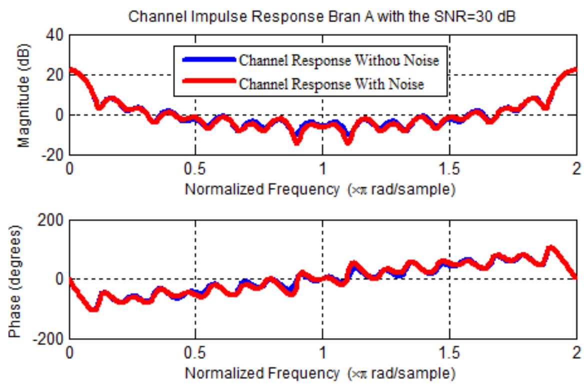

Fig. 5 Channel impulse response Bran A, with the SNR=30 dB

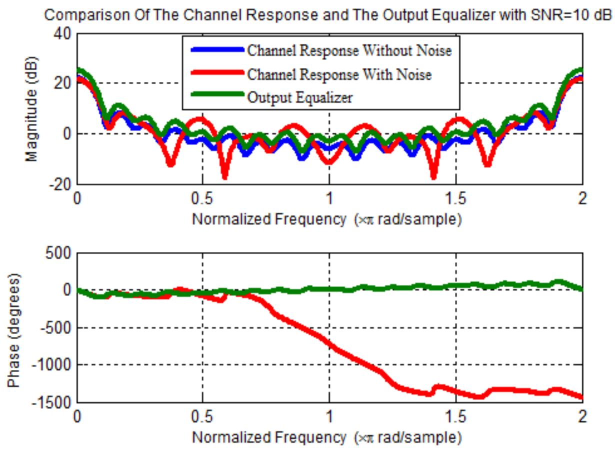

Fig. 6 Comparison of the channel response Bran A; and the sortie equalizer with the ZF criterion in the SNR=10 dB cases

From results obtained it can be seen that the algorithm of the equalizer with ZF criterion gives a satisfactory equalization Bran A channel, consequently, it reduces the effect of noise.

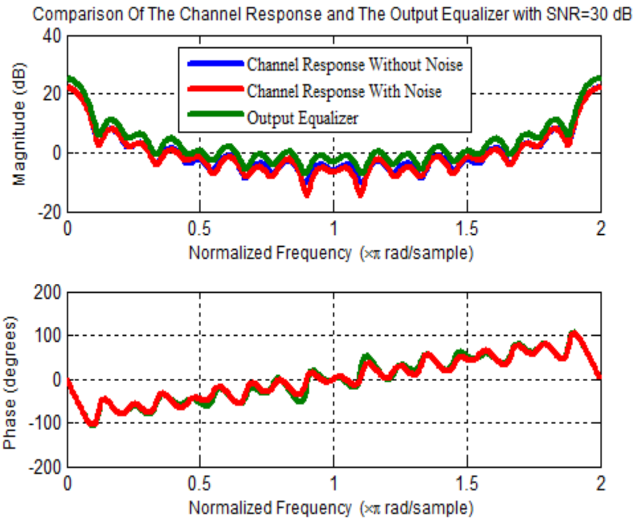

Fig. 7 Comparison of the channel response Bran A; and the sortie equalizer with the ZF criterion in the SNR=30 dB cases

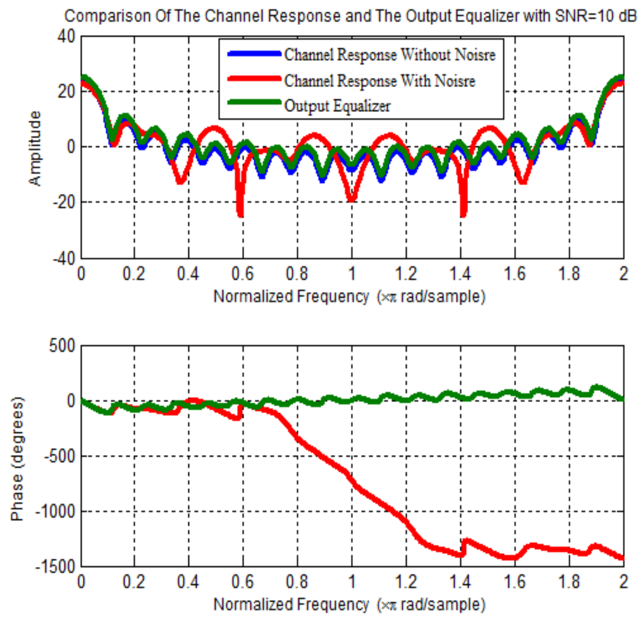

Fig. 8 Comparison of the channel response Bran A; and the sortie equalizer with the MMSE criterion in the SNR=10 dB cases

B. The Equalization Algorithm Based On MMSE Criterion

We test the performance of the algorithm equalizer with MMSE criterion, with and without noise, to the Bran A channel; values of the SNR by 10 and 30 dB.

The algorithm equalization with the MMSE criteria; gives a good equalization of Bran A channel; then the criterion of Mean Square Error (MMSE) criterion is a more robust with respect to noise. It enables a compromise between reducing noise and the interference between the symbols (IES) (Fig. 8).

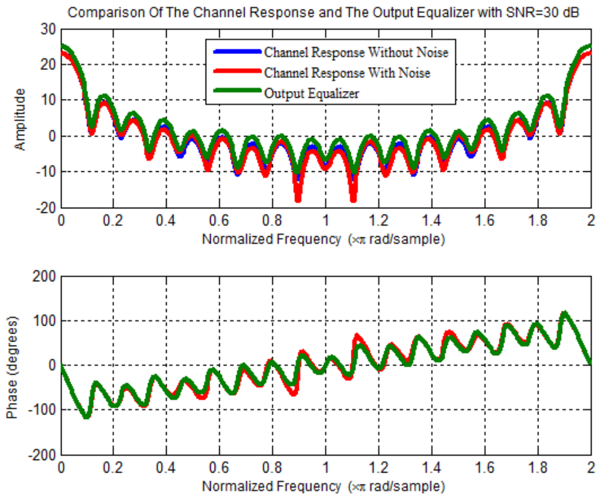

Fig. 9 Comparison of the channel response Bran A and the sortie equalizer with the MMSE criterion in the SNR=30 dB cases

V. Comparison of Results

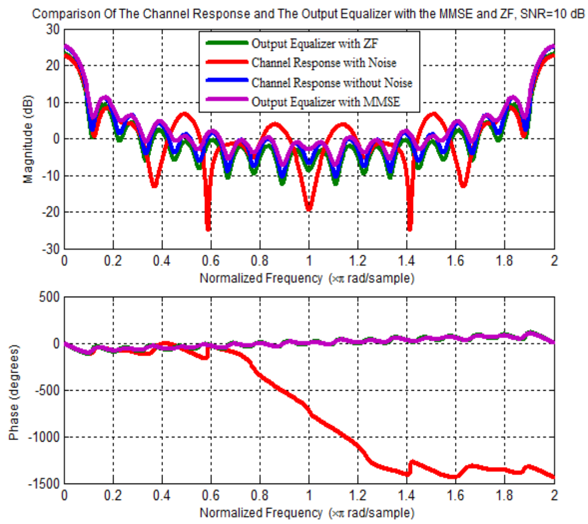

Fig. 10 Comparison of the channel response Bran A, and the sortie Equalizer with the MMSE and ZF criterion in the SNR=10 dB cases

From the simulation results, we see that the equalizer obtained by the criterion of MMSE is better than that provided by the criterion ZF, due to the effective inclusion of noise.

VI. Performance of the LMS and RLS Algorithm

In this section we will make a comparison between the two algorithms of the LMS adaptive equalization and RLS are studied previously for that. Consider the channel Proakis (B) [2], and a modulation amplitude states 4 (4-ASK), with equalization coefficients 9.

It was found by applying the algorithm of the equalizer coefficient values for SNR=50 dB and the two ZF and MMSE criteria:

TABLE I
Coefficients Calculated by the Algorithm of the Equalizer with the MMSE Criterion

Coefficients EQM : h_E				
0.0652	-0.1480	0.2814	-0.4215	1.4793
-0.4228	0.2832	-0.1524	0.0705	

TABLE II
Coefficients Calculated by the Algorithm of the Equalizer with ZF Criterion

Coefficients ZF : h_E				
0.0816	-0.1640	0.2958	-0.4346	1.4921
-0.4365	0.2986	-0.1698	0.0883	

A. Performance of the LMS Algorithm

The values of the coefficients h_E calculated by the LMS algorithm at the last iteration are:

TABLE III
The Coefficients Calculated by the LMS Adaptation Algorithm with $u = 0.0053$

0.0606	-0.1416	0.2679	0.2679	1.4707
-0.4099	0.2667	-0.1381	0.0603	

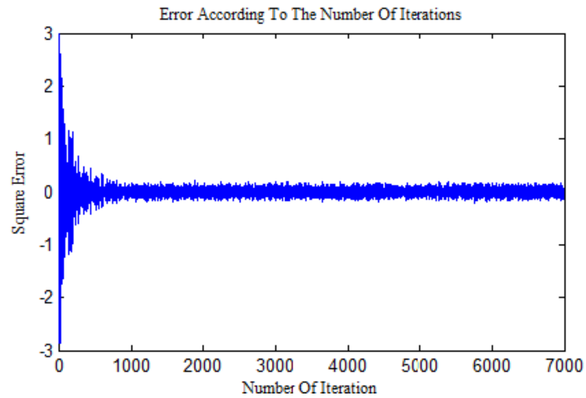

Fig. 11 The variation of the error based on iteration number M=7000

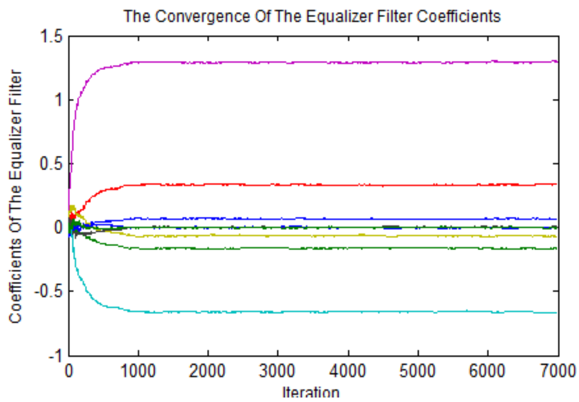

Fig. 12 The convergence of the equalizer filter coefficients with no convergence of the LMS, u= 0.0053

From Fig. 11 we see that the error signal e_k is low when the number of iterations is important (M=7000). And from Figs. 12 and 13 we notice that for a low pitch results in slow convergence. A strong will not lead to closer than results obtained by the algorithm equalization with criterion MMSE (Tables I and III). The LMS allows every moment to "update" the equalizer filter coefficients in proportion to the estimation error e_k. In case of variations of the channel, the equalizer will be able to adapt more rapidly than the constant u is large.

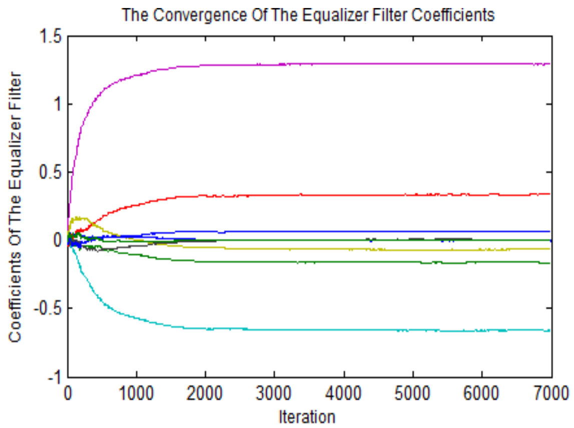

Fig. 13 The convergence of the equalizer filter coefficients with no convergence of the LMS, u= 0.002

B. Performance of the RLS Algorithm

The values of the coefficients h_E calculated by the RLS algorithm adaptation at the last iteration are:

TABLE IV
THE COEFFICIENTS CALCULATED BY THE ADAPTATION ALGORITHM WITH RLS, u= 0.0053.

0.0634	-0.1536	0.3318	-0.6546	1.2883
-0.0626	0.0028	-0.0042	0.0055	

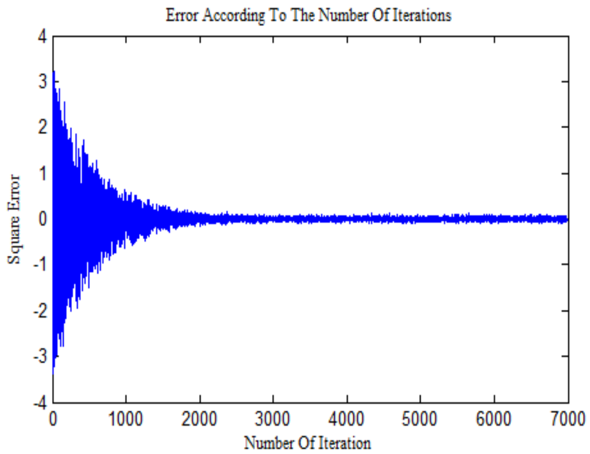

Fig. 14 The variation of the error e_k against the number of iterations M=7000 with u= 0.0053

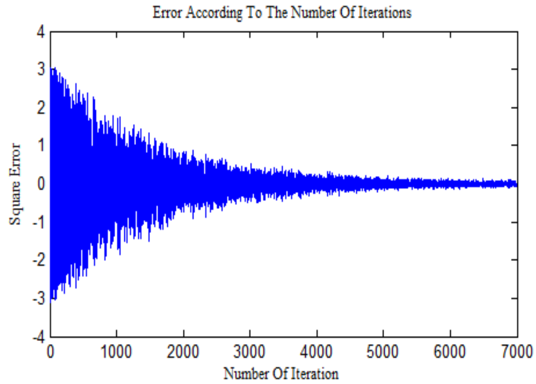

Fig. 15 The variation of the error e_k against the number of iterations M=7000 with u= 0.002

Figs. 14 and 15 show different results with u= 0.0053 and u= 0.002, we note that the estimate of the error e_k is tends to rapidly to low values when the number of iterations M and u are stronger. Then filter the RLS algorithm is performed correctly, it means that all influences of the noise were suppressed.

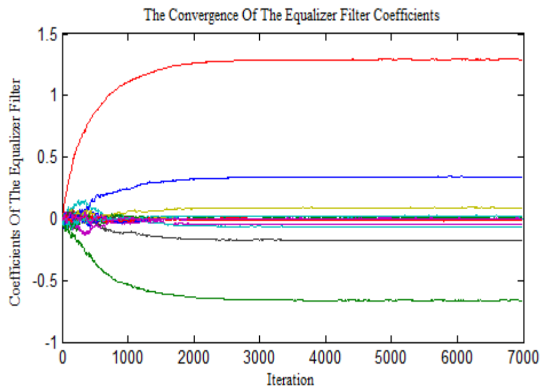

Fig. 16 The convergence of the equalizer filter coefficients with u= 0.0053

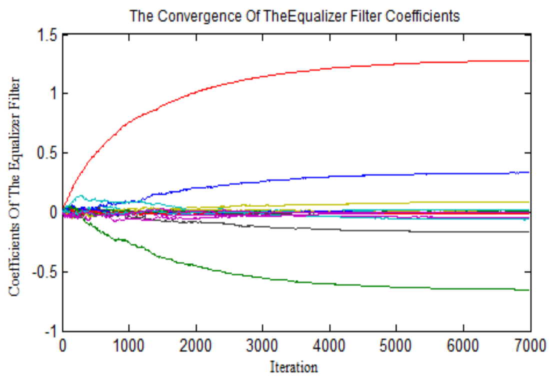

Fig. 17 The convergence of the equalizer filter coefficients with u= 0.002

The curves in Figs. 16 and 17 show the variation of the filter coefficients depending on numbers of iterations, we find that for a low pitch, slow convergence is obtained.

C. Comparison between the LMS and RLS

From simulation results, we see that the LMS converges quickly compared to the RLS algorithm because only one control parameter (the u adaptation) and will lead to results closer to that obtained by the algorithm of the equalizer with the MMSE criterion (Tables I, III, and IV). There is another difference between the LMS and RLS is in the term $P(k)$, which allows you to update various coefficients and gives the superiority of the RLS algorithm in terms of speed of convergence but time is running slower.

VII. CONCLUSION

In this paper we presented three algorithms, the first algorithm to equalize the channel Bran A; with the two criteria ZF and MMSE, and the other two algorithms for estimating the parameters of the equalizer filter adjust the channel and reduce the error signal.

Simulation results show that the algorithm of the equalizer is able to equalize the channel Bran A with the MMSE criterion, due to the effective inclusion of noise. Thus the adaptive LMS filter algorithm converges quickly with respect to the RLS algorithm because of the adaptation step, another difference between the LMS and RLS is within the term $P(k)$, which gives a superiority of RLS algorithm in terms of speed of convergence but time is running slower.

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