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Study of a Fabry-Perot Resonator

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***Abstract*—**A laser is essentially an optical oscillator consisting of a resonant cavity, an amplifying medium and a pumping source. In semiconductor diode lasers, the cavity is created by the boundary between the cleaved face of the semiconductor crystal and air, and has reflective properties as a result of the differing refractive indices of the two media. For a GaAs-air interface a reflectance of 0.3 is typical and therefore the length of the semiconductor junction forms the resonant cavity. To prevent light being emitted in unwanted directions from the junction, sides perpendicular to the required direction are roughened. The objective of this work is to simulate the optical resonator Fabry-Perot and explore its main characteristics, such as FSR, finesse, linewidth, transmission and so on, that describe the performance of resonator.

***Keywords*—**Fabry-Perot Resonator, laser diode.

I. Introduction

A set of two or more mirrors arranged to cause light to propagate in a closed path is called an optical resonator or optical cavity. There are two basic types of optical resonators: the standing wave resonator and the traveling wave, or ring, resonator. The simplest resonator consists of two mirrors, and a ring resonator consists of at least three mirrors. Any given mirror may be characterized by its amplitude reflection coefficient r and its transmission coefficient t. In general these are complex-valued quantities [1]. Typically we are interested in the intensity reflectivity R and transmission T. Consider the simplest two-mirror resonator having a monochromatic laser beam incident on the input mirror. Of our interest is the intensity of light transmitted through the output mirror, and the intensity of light reflected from the resonator [2]. In this paper, we give a brief description of the Fabry-Perot resonator and also discuss its main characteristics.

II. Modeling Fabry-Perot Resonator

A Fabry-Perot resonator is a linear optical resonator which consists of two highly reflecting mirrors (with some small transmittivity) and is often used as a high-resolution optical spectrometer. One exploits the fact that the transmission through such a resonator exhibits sharp resonances and is very small between those [3].The incoming light makes multiple round-trip within this cavity and spring partly to each reflection, Different outgoing light rays interfere with each other, giving rise to an interference pattern of multiple wave consists of concentric rings fine [4].

The varying transmission function of a resonator is caused by interference between the multiple reflections of light between the two reflecting surfaces. Constructive interference occurs if the transmitted beams are in phase, and this corresponds to a high-transmission peak of the resonator. If the transmitted beams are out-of-phase, destructive interference occurs and this corresponds to a transmission minimum. Whether the multiply reflected beams are in phase or not depends on the wavelength (λ) of the light (in vacuum), the angle of the light travels through the resonator (θ), the thickness of the resonator (ℓ) and the refractive index of the material between the reflecting surfaces (n).The phase difference between each succeeding reflection is given by δ [5]:

$$\delta = \left(\frac{2\pi}{\lambda}\right) 2n\ell cos\theta \quad (1)$$

If both surfaces have a reflectance R, the transmittance function of the resonator is given by:

$$T = \frac{(1-R)^2}{1+R^2-2Rcos\delta} = \frac{1}{1+Q_R sin^2(\delta/2)} \quad (2)$$

where T is the transmission, R is the reflectivity of the mirrors and δ is the round-trip phase change of the light ray.

$$Q_R = \frac{4R}{(1-R)^2} \quad (3)$$

Q_R is the quality factor [4].

Maximum transmission (T=1) occurs when the optical path length difference (2n ℓcosθ) between each transmitted beam is an integer multiple of the wavelength. In the absence of absorption, the reflectance of the resonator R is the complement of the transmittance, such that T+R=1. The maximum reflectivity is given by:

$$R_{max} = 1 - \frac{1}{1+Q_R} = \frac{4R}{(1+R)^2} \quad (4)$$

And this occurs when the path-length difference is equal to half an odd multiple of the wavelength. The wavelength separation between adjacent transmission peaks is called the free spectral range (FSR) of the resonator, Δλ, and is given by:

$$\Delta\lambda = \frac{\lambda_0^2}{2n\ell cos\theta + \lambda_0} \approx \frac{\lambda_0^2}{2n\ell cos\theta} \quad (5)$$

The FSR is related to the full-width half-maximum (FWHM), δλ, of any one transmission band by a quantity known as the finesse:

$$F = \frac{\Delta\lambda}{\delta\lambda} = \frac{\pi}{2arcsin(1/\sqrt{Q_R})} \quad (6)$$

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This is commonly approximated (for $R > 0.5$) by:

$$F \approx \frac{\pi\sqrt{Q_R}}{2} = \frac{\pi R^{1/2}}{1-R} \quad (7)$$

where λ_0 is the central wavelength of the nearest transmission peak. $\delta\lambda$ is the width of a peak at half of its maximum intensity.

Finesse (F) is a factor given to quantify the performance of a resonator. Conceptually, finesse can be thought of as the number of interfering beams within the Fabry-Perot cavity. A higher finesse value, indicating a greater number of interfering beams, results in a more complete interference process and therefore higher resolution measurements. The primary factor that affects finesse is the reflectivity of the resonator mirrors because it directly affects the number of beams oscillating within the cavity; In addition to mirror reflectivity, factors that limit the finesse of a Fabry-Perot resonator include the mirror surface quality, as well as the vibrational and thermal stability of the resonator [6]. Like the finesse, the contrast factor is directly related to the reflectance of the Fabry-Perot mirrors, the contrast factor defined as a function of the reflectance of the mirrors is:

$$C = \frac{(1+R)^2}{(1-R)^2} \approx \frac{4R}{(1-R)^2} \quad (8)$$

Equations (7) and (8) indicate that the contrast factor and the finesse are closely related it should therefore be possible to define the contrast factor as a function of the finesse, the relation between these two parameters is as expressed in (9):

$$C = 1 + \left(\frac{2F}{\pi}\right)^2 \quad (9)$$

Equation (9) indicates that the contrast factor and finesse are directly proportional to each other [7].

III. Results and Discussion

The first step in the study of the Fabry-Perot resonator is to determine the transmission for the wavelength of the beam and the frequency. We study a Fabry-Perot resonator consisting of two plane mirrors of reflectivity (transmission) r_1 (t_1) and r_2 (t_2) given, separated by a distance ℓ. The optical index of the medium n between the two mirrors is 3.3; Table I gives the parameters used in our simulation.

TABLE I
THE PARAMETERS USED IN THE SIMULATION OF THE FABRY-PEROT RESONATOR

Symbol	Parameters	GaAs/AlGaAs
c_0	Speed of light	3×10^8 m. s^{-1}
n	Refractive index	3.3
ℓ	Thickness cavity	3 µm
R1	Reflectivity of the mirror 1	0.3 cm^2.S^{-1}
R2	Reflectivity of the mirror 2	0.3 cm^2.S^{-1}

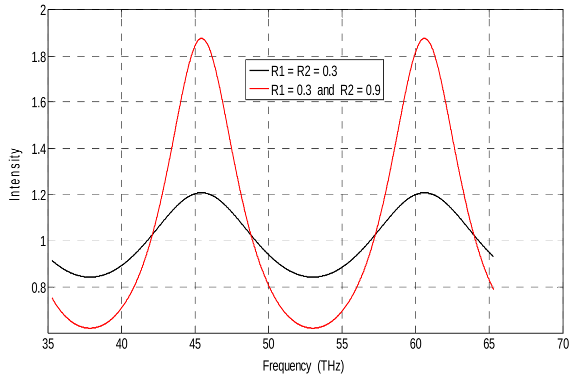

Fig. 1 Evolution of the transmitted intensity of the Fabry-Perot resonator as a function of frequency: a) Reflectivity mirrors similar (R1 = R2 = 0.3). b) Reflectivity mirrors of different types (R1 =0.3 and R2 = 0 .9)

Fig. 1 shows the transmitted intensity as a function of frequency when the reflectivity of the two mirrors of the same nature R1 = R2 = 0.3 and of different nature R1 = 0.3, R2 = 0.9 for photons inside the Fabry-Perot resonator of length ℓ = 3 µm and refractive index n = 3.3. The optical resonances are spaced by $\Delta\nu$= 15.15 THz. Furthermore, it is clear that the curve of the same reflectivity is lower than that of different reflectivity.

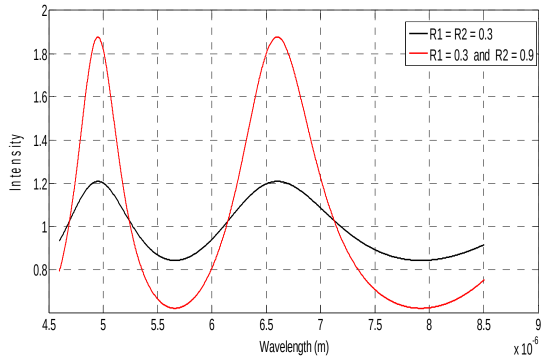

Fig. 2 Evolution of the transmitted intensity of the Fabry-Perot resonator as a function of wavelength: a) reflectivity mirrors similar (R1 = R2 = 0.3). b) Reflectivity mirrors of different types (R1 = R2 = 0.3 and 0 .9)

Fig. 2 shows the transmitted intensity of a Fabry-Perot depending on the wavelength of length ℓ =3µm, refractive index n = 3.3 and the emission wavelength λ=850nm corresponding to a frequency ν_0=35.27 THz, the optical resonator has mode spacing $\Delta\nu$=15.15 THz and $\Delta\lambda$=3653.3. In both cases, the reflectivity of the same nature R1 = R2 = 0.3 and of different nature R1 = 0.3, R2 = 0.9. In this latter, the total reflection R (where R=R1×R2) is equal to 0.27 which gives us an upper curve.

We will now study the behavior of the Fabry-Perot, in transmission and reflection.

Figs. 3 and 4 also show how the reflectivity of the surfaces affects the transmission. If the reflectivity is relatively low, the maxima in transmission will be broad. On the other hand, if the reflectivity is high, the maxima of transmission will be very narrow and sharp.

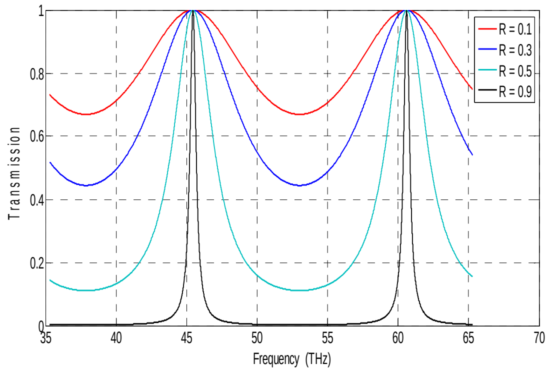

Fig. 3 Evolution of the transmission of the Fabry-Perot resonator as function of the frequency for different values of reflectivity

The transmission thus exhibits a periodic sequence of maxima, whose frequencies are separated by Δv, light will be transmitted by the resonator if its frequency lies within the frequency width δv of one of the resonance peaks, the resonance frequencies, determined by a maximum of the transmission, exhibit a frequency width that decreases as the mirror reflectivity and the loss factor are increased.

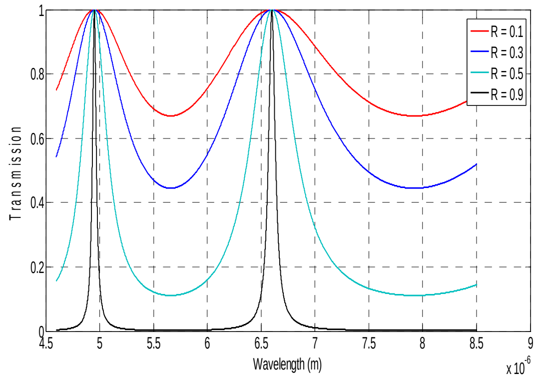

Fig. 4 Evolution of the transmission of the Fabry-Perot resonator as function of the wavelength for different values of reflectivity

Fig. 4 shows the evolution of the transmission of Fabry-Perot resonator depending on the wavelength for different values of reflectivity. And we note that the total transmission is allowed at certain wavelengths (in case the two mirrors have the same reflectivity) and the line width of the transmission decreases to increase the reflectivity. According to Figs. 3 and 4 we can conclude that the transmission peaks are even narrower than the value of R is large and the transmission not canceled when R becomes too low.

The reflectance of the mirrors component Fabry-Perot cavity can be determined by studying the transmission as a function of wavelength. In the case of GaAs lasers, the reflection coefficient of the mirrors is approximately 30 %.

Table II shows the characteristics of our resonator for mirror spacings $\ell = 3\mu m$, under conditions of normal incidence operation at λ = 850 nm. We take n = 3.3 so $\Delta v = c_0/2n\,\ell$ and $\Delta\lambda=\lambda^2 / 2n\,\ell$.

TABLE II
CHARACTERISTICS OF FABRY-PEROT RESONATOR WITH DIFFERENT REFLECTANCES AND SPACINGS

Reflectance (R)	Finesse (F)	Quality factor (Q_R)	$\Delta\lambda$ (nm)	$\delta\lambda$ (nm)	ΔV (THz)
0.1	1.10	0.4938	3653.3	3321.2	15.15
0.3	2.46	2.4490	3653.3	1485.1	15.15
0.5	4.44	8.0000	3653.3	822.8	15.15
0.9	29.8	360.00	3653.3	122.6	15.15

According to Fig. 3 and Table II we observe that when the finesse is large (F>1), it is clear that the spectral response of the resonator is sharply peaked about the resonance frequencies and the transmission is small such that for reflectivity of: 0.1, 0.3, 0.5, and 0.9 we found a finesse 1.10, 2.46, 4.44, and 29.8 respectively. So the transmission decreases with increasing the reflectivity as well for finesse we can conclude that the finesse F is even larger than the peaks are narrow, the response of the cavity is nearly sinusoidal and if we consider that we have two-wave interference. More the quality factor is important, more the number of round-trip in the cavity is high and the number of interfering waves is great and finesse is becoming important, In other words, more the quality factor is high, more the transmission is small and the resonance is piqued, more the resonator is preferment.

The interference pattern obtained has always concentric rings, but their size varies according to the distance between the two reflecting surfaces, and the wavelength of the light used. In fact, we see that only a few wavelengths are transmitted. The transmittance versus wavelength exhibits peaks separated from $\Delta\lambda$ and a width $\delta\lambda$. each wavelength corresponds to a ring system. And in the presence of several wavelengths can be compared these ring systems to measure wavelengths.

To better separate the different rings, it is interesting they are as thin as possible. One can show that this is equivalent to sharpen the peaks of the transmission, that is to say, to reduce $\delta\lambda$ from $\Delta\lambda$. Thus, a resonator of good quality presents a $\delta\lambda$ much smaller than $\Delta\lambda$. Because $\Delta\lambda$ is linearly proportional to the reflectivity of the resonator, the reflectivity must be high to achieve high $\Delta\lambda$. in particular we see that more than the reflectivity is higher, the emission wavelengths are separated, where the emission lines the most distant from each other facilitate the creation of a single-mode laser (a single emission ray) by resonator.

The finesse can be increased simply by increasing the reflectance of the mirrors. However, the consequence of this increase in reflectance results in the reduction of light transmitted by the Fabry-Perot Resonator. Figs. 5 and 6 show the effect of finesse on the transmission of the Resonator.

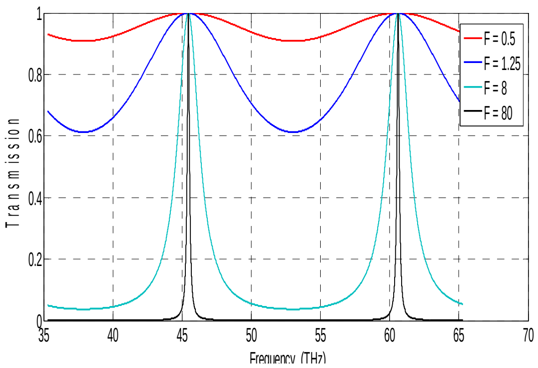

Fig. 5 Evolution of the transmission of the Fabry-Perot resonator as function of the frequency for different values of finesse

Fig. 5 shows the evolution of the transmission of the resonator according to frequency for different values of finesse. The transmission spectrum of a resonator will have a series of peaks spaced by the free spectral range. The width of each transmission peak decreases for higher finesse resonator, in fact; the finesse is defined as the ratio of the free spectral range to the full width half max.

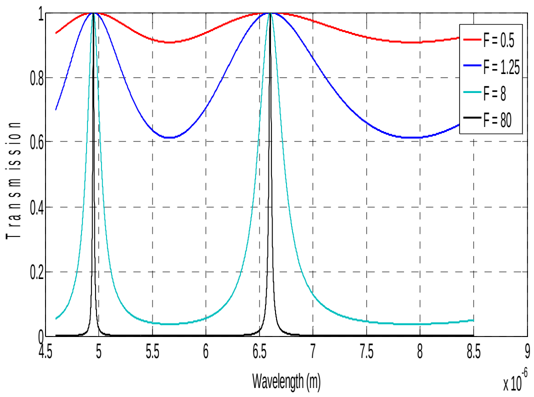

Fig. 6 Evolution of the transmission of the Fabry-Perot resonator as function of the wavelength for different values of finesse

Fig. 6 shows the evolution of the transmission of the resonator according to wavelength for different values of finesse. A resonator of high finesse F = 80 shows sharper peaks and lower transmission minima than a low finesse resonator F = 0.5. As F increases (due to increasing R) the fringe contrast increases, the transmittance minimum goes closer to 0, And the fringe thickness decreases.

And therefore, more the finesse is important, more the rings are fine. To increase the finesse, it is possible to make surfaces forming the cavity highly reflective. In fact, it can be shown, as illustrated by the curve below, the finesse increases with the mirror reflectivity.

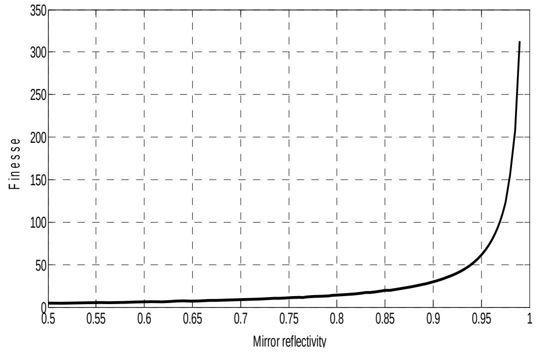

Fig. 7 Evolution of the finesse of the cavity as a function of the mirror reflectivity

Fig. 7 shows the variation of the finesse of the cavity as a function of mirror reflectivity. The finesse is a parameter that quantifies the strength of oscillation of a Fabry-Perot cavity. It is even higher than the reflectivity of the mirrors are high. As for a reflectivity of 0.95, finesse is about 61.2, so very high finesse factors require highly reflective mirrors. As we have already assumed, the finesse depends on the reflectivity R of the mirrors. The tendency now would be to bring the reflectivity R as close as possible to 1 to achieve a high finesse.

Finesse can be increased simply by increasing Fabry-Perot mirror reflectivity. However, this is not done without consequence because higher mirror reflectivity results in a reduction in the transmission of the incident light source.

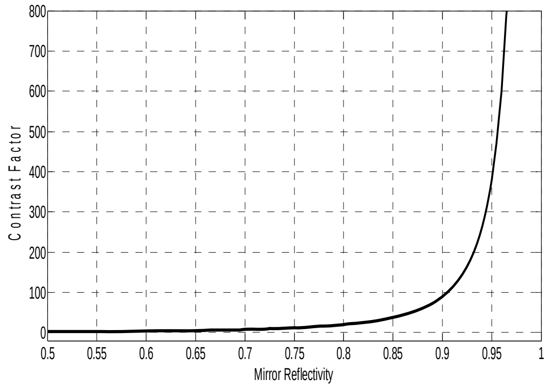

Fig. 8 Evolution of the contrast factor of the cavity as a function of the mirror reflectivity

Fig. 8 shows the variation of the contrast factor as a function of mirror reflectivity; we observe that the contrast factor increases with an increase in the mirror reflectivity.

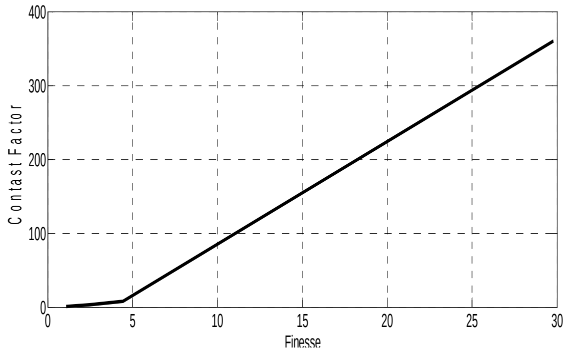

Fig. 9 Evolution of the contrast factor of the cavity as a function of the finesse

Fig. 9 indicates that the contrast factor and finesse are directly proportional to each other. We also show that a linear increase in finesse translated into a quadratic increase in the value of the contrast factor.

IV. Conclusion

Our modeling is employed to determine the important parameters that affect the performance and the efficiency when it is to design the Fabry-Perot resonator such as transmission intensity, Finesse, quality factor and the contrast Factor. We can conclude that the light is not transmitted is reflected by the resonator, at positions where the transmission is low, the reflectivity of the resonator is high. The finesse is an important parameter that determines the performance of a Fabry-Perot resonator. Conceptually, finesse can be thought of as the number of beams interfering within the Fabry-Perot cavity to form the standing wave. The primary factor that affects finesse is the reflectance R of the Fabry-Perot mirrors, which directly affects the number of beams circulating inside the cavity. Another important factor in the design of the resonator is the contrast factor which is defined primarily as the ratio of the maximum to minimum transmission. We also show that a higher finesse value indicates a greater number of interfering beams within the cavity, and hence a more complete interference process, and finally we found that the transmission decrease with increasing finesse (or contrast) which provides high optical quality factors, allowing the realization of resonators with very good performance.

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