International Journal of Mechanical, Industrial and Aerospace Sciences
ISSN: 2517-9950
Vol:3, No:2, 2009

Stress Analysis for Two Fitted Thin Walled Cylinder with High Angular Velocity

A.V. Hoseini, A. Bidi, M. H. Pol, M.Jalali azizpour

***Abstract*—**In this paper stress and strain for two rotating thin wall cylinder fitted together with initial interference and overlap are computed. Also stress value for variation of initial interference is calculated.

At first problem is considered without rotation and next angular velocity increased from 0 to 50000 rev/min and stress in each stage is calculated. The important point is that when stress become very small in magnitude the angular velocity is critical and two cylinders will separate. The critical speed i.e. speed of separation is calculated in each step.

***Keywords*—**Thin walled cylinder, high angular velocity, two fitted thin walled

I. Introduction

THE contact problem is strongly nonlinear and need to high Memory in computer to be solved. There is two complicated subjects in contact analysis first the contact region is not defined recognized before analysis. I.e. depending on load material and boundary condition. Second, it is needed to consider the friction in contact problems and so several models is considered.

Each contact problem may be considered rigid-flexible or flexible- flexible (1). In the rigid – flexible model one or several surfaces are considered to be rigid i.e. they have very higher stiffness compared with the flexible body. In usual when a soft material is in contact with a hard body the contact is rigid-flexible such as metal forming process but commonly the contact is flexible-flexible and two bodies have the same stiffness such as threaded components.

II. Geometry and Model Development

There are three models for contact analysis modeling:

a) node- node element
b) surface- node element
c) surface- surface element

In the analysis the surface node elements for modeling is selected. CONTACT48 element in two dimensions is proper [1]. (5) The problem is axisymetric so only one section is considered for analysis. Boundary condition for axisymmetric element must be used it is essential that when two bodies in analysis are separated the stiffness matrix will be singular so the problem must be solved by one of these methods:

a) drawing the model in contacted position
b) using weak springs for bodies to be connected
c) solving the problem dynamically

In this problem two cylinders contacted together. In order to achieve good converging solution, small values for contact stiffness KN for beginning are chose and gradually increased in each steps.

A. Governing equation and formulation

The compatibility equation for stress around a cylinder is [2]:

$$\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\right)\left(\frac{\partial^2\phi}{\partial r^2}+\frac{1}{r}\frac{\partial\phi}{\partial r}+\frac{1}{r^2}\frac{\partial^2\phi}{\partial \theta^2}\right)=0 \tag{1}$$

For symmetric condition, stresses depend on only to r. Therefor

$$\left(\frac{d^2}{dr^2}+\frac{1}{r}\frac{d}{dr}\right)\left(\frac{d^2\phi}{dr^2}+\frac{1}{r}\frac{d\phi}{dr}\right)=\frac{d^4\phi}{dr^4}+\frac{2d^3\phi}{rdr^3}-\frac{1}{r^2}\frac{d^2\phi}{dr^2}+\frac{1}{r^3}\frac{d\phi}{dr}=0 \tag{2}$$

A solution for equation (2) is [3]:

$$\phi = A\log r + Br^2\log r + Cr^2 + D \tag{3}$$

Equilibrium equations in polar coordinate are [3]:

$$\frac{\partial\sigma_r}{\partial r}+\frac{1}{r}\frac{\partial\tau_{r\theta}}{\partial\theta}+\frac{\sigma_r-\sigma_\theta}{r}+R=0 \tag{4}$$

$$\frac{1}{r}\frac{\partial\sigma_\theta}{\partial\theta}+\frac{\partial\tau_{r\theta}}{\partial r}+\frac{2\tau_{r\theta}}{r}+S=0 \tag{5}$$

If body force is being negligible, parameter R&S in equations (4) and (5) are zero .With assuming axisymetric condition, the stress components will be [3]

$$\sigma_r=\frac{1}{r}\frac{\partial\phi}{\partial r}=\frac{A}{r^2}+B(1+2\log r)+2C \tag{6}$$

$$\sigma_\theta=\frac{\partial^2\phi}{\partial r^2}=-\frac{A}{r^2}+B(3+2\log r)+2C \tag{7}$$

For the case a plate with a hole in center, the radial and tangential stresses are

$$\sigma_r=\frac{A}{r^2}+2C \tag{8-a}$$

$$\sigma_\theta=-\frac{A}{r^2}+2C \tag{8-b}$$

F. A. Author is a member of faculty in Islamic Azad University of Iran (Shoshtar branch).

He is with Ahvaz manufacturing technology Research Center (ACECR) too. (Corresponding author to provide phone: +989163125714; e-mail: savh@ymail.com).

S. B. Author, Jr., is with Engineering Center of IKCO, Tehran, Iran (e-mail: beh_eng@yahoo.com).

T. C. Author is with Ahvaz manufacturing technology Research Center (ACECR). (Corresponding author to provide phone: +989122257173; e-mail: m_h_pol@ modares.ac.ir).

Also (8) shows the stresses for cylinder subjected to internal or external pressure wehere $(\sigma_r)_{r=a} = -P_i$ and $(\sigma_r)_{r=b} = -P_o$ are boundary conditions and P_i and P_o are internal and external pressures respectively. After calculating constants, the stresses are determined as

$$\sigma_r = \frac{a^2 b(P_o - P_i)}{b^2 - a^2}\frac{1}{r^2} + \frac{P_i a^2 - P_o b^2}{b^2 - a^2} \tag{9-a}$$

$$\sigma_\theta = -\frac{a^2 b(P_o - P_i)}{b^2 - a^2}\frac{1}{r^2} + \frac{P_i a^2 - P_o b^2}{b^2 - a^2} \tag{9-b}$$

For a cylinder that is fit pressed there is contact pressure between two bodies so assigning as p the relation is (2)

$$\sigma_{it} = -p\frac{b^2 + a^2}{b^2 - a^2} \tag{10-a}$$

$$\sigma_{ot} = p\frac{c^2 + b^2}{c^2 - b^2} \tag{10-b}$$

Where σ_{it} and σ_{ot} are internal and external tangential stresses in inner and outer surface respectively.

If the initial interference is δ, because of compatibilities of deformation relation ($\delta=\delta_i-\delta_o$), the pressure p at contact position can be obtained so

$$p = \frac{E\delta}{b}\left(\frac{c^2 - b^2}{2b^2}\right)\left(\frac{b^2 - a^2}{c^2 - a^2}\right) \tag{11}$$

Then from (10) and (11), stresses can be calculated.

For a rotating disk or cylinder the equilibrium equation is

$$\frac{d}{dr}(r\sigma_r) - \sigma_\theta + \rho\omega^2 r^2 = 0 \tag{12}$$

Therefor for case of rotating disk with a hole in center can be calculated (6)

$$\sigma_r = \frac{3+\nu}{8}\rho\omega^2\left(b^2 + a^2 - \frac{a^2 b^2}{r^2} - r^2\right) \tag{13-a}$$

$$\sigma_\theta = \frac{3+\nu}{8}\rho\omega^2\left(b^2 + a^2 + \frac{a^2 b^2}{r^2} - \frac{1+3\nu}{3+\nu}r^2\right) \tag{13-b}$$

The stresses for thin walled cylinders from equilibrium equations are [3]

$$pd_i = 2t\sigma_t \Rightarrow \sigma_t = \frac{pd_i}{2t} \tag{14}$$

For two fitted thin walled cylinder with high Angular velocity, two solutions should be compounded.

A. *Input data for Parameters of cylinders*

Length of each cylinder	$L_1 = L_2 = 40cm$
Radius of each cylinder	$r_1 \approx r_2 \approx 75mm$
Cylinder thickness	$t = 1mm$
Initial interference	*e=0.1 mm, 0.2 mm*
Overlape	*5cm,10cm*
Cylinder materials:	*aluminum*
Element and nodes combination	Fig. 1

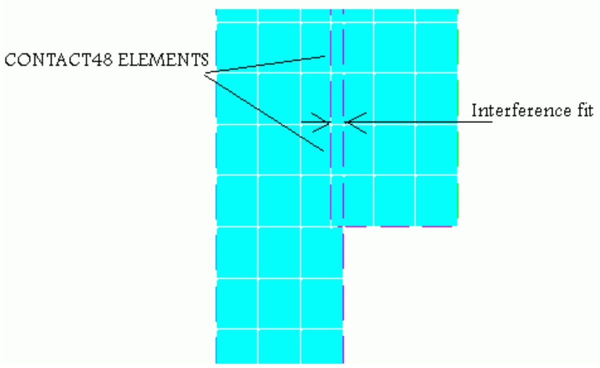

Fig. 1. element and mesh combination

III. Conclusion

The stress variation along cylinder thickness shows that at the line of contact there is a sudden change in stresses.

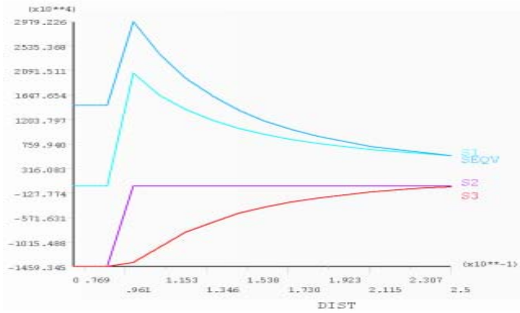

Fig. 2. Principle and von misses stress curve along section

Location of maximum stresses as shown in fig. 3 is in end of each cylinder.

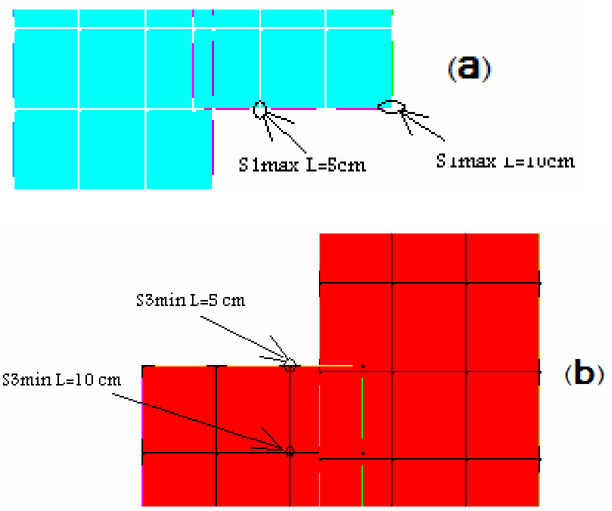

Fig. 3. Location of maximum (a) radial (b) tangential stress

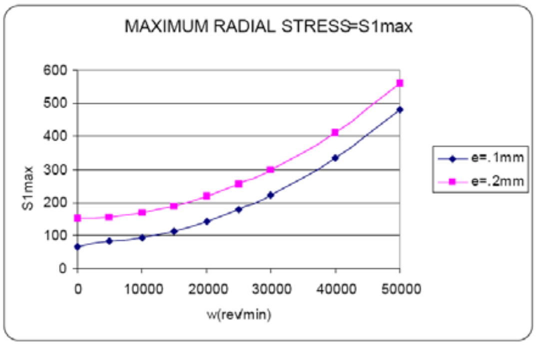

Fig. 4 Variation of maximum radial stresses versus speed of rotation for overlap=10 cm

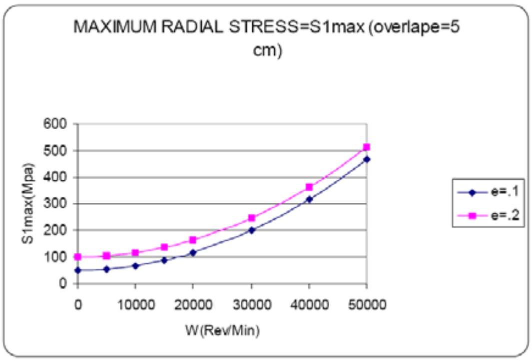

Fig. 5 Variation of maximum radial stresses versus speed of rotation for overlap =5 cm

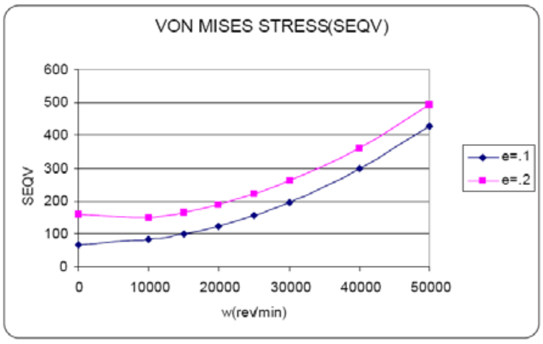

Fig. 6 Variation of maximum von misses stress versus speed of rotation for overlap=10 cm

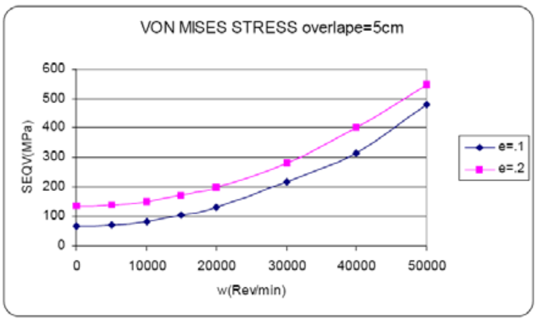

Fig. 7 Variation of maximum von misses stress versus speed of rotation for overlap=5 cm

IV. Conclusion

a) In all cases stresses are proportional with square of speed as assigned in equation (13).
b) As shown at figures (6) and (7) Von Mises stress variation is the same as radial stress and varies with speed parabolic.
c) As known in classic strength of materials book increasing initial interference will increase the speed of separation (or critical speed)

References

[1] Ansys Structural analysis guide.
[2] Popav, Mechanics of materials, 4th edition.
[3] Thimoshenko, Teory of elasticity.