

# Straightness Error Compensation Servo-system for Single-axis Linear Motor Stage

M. S. Kang, D. H. Kim, J. S. Yoon, B. S. Park, and J. K. Lee

**Abstract**—Since straightness error of linear motor stage is hardly dependent upon machining accuracy and assembling accuracy, there is limit on maximum realizable accuracy. To cope with this limitation, this paper proposed a servo system to compensate straightness error of a linear motor stage. The servo system is mounted on the slider of the linear motor stage and moves in the direction of the straightness error so as to compensate the error. From position dependency and repeatability of the straightness error of the slider, a feedforward compensation control is applied to the platform servo control. In the consideration of required fine positioning accuracy, a platform driven by an electro-magnetic actuator is suggested and a sliding mode control was applied. The effectiveness of the sliding mode control was verified along with some experimental results.

**Keywords**—Linear Motor Stage, Straightness Error, Friction, Sliding Mode Control.

## I. INTRODUCTION

PROBABLY, linear motor stage is the most preferable device among various motion generation systems in advanced manufacturing industries because of their unique advantages. Since traditional mechanical transmission mechanisms have unavoidable nonlinear dynamic characteristics such as friction, backlash, slip, etc., fine positioning is still remained as a challenging subject in this indirect drive field. On the other hand, direct drive linear motor stage needs no such power transmission. The main features of linear motor stages include high force density achievable, low thermal losses, more exact positioning, longer life, less maintenance, fewer moving part, high positioning precision and accuracy associated with the mechanical simplicity[1-2].

However, the performance of linear motor stages is unavoidably limited by geometric errors such as straightness, flatness, yaw, pitch, roll errors [3]. Since stage is moving along sliding guide without losing mechanical contact, these errors are hardly dependent upon machining accuracy and assembling skills. Thus for high precision linear motor stage, high precision machining and assembling are necessary. Certainly,

higher precision machining and assembling requires higher manufacturing expense. Moreover there is limitation on realizable machining and assembling accuracy. Typically, for long stroke stage, it is extremely difficult to achieve precision machining and assembling accuracy.

Motivated from these, we propose an add-on platform servo system which is mounted on slider of linear motor stage and the platform moves relative to the slider in the straightness error direction so as to cancel the straightness error. This is based on the facts that the straightness of the slide guide is dependent upon slider's traveling position and the error is repeatable [5]. From some straightness error measurements for the linear motor stage subjected, we confirm these position dependency and repeatability of the straightness error. An electro-magnetic actuator is chosen to drive the platform because electro-magnetic drives provide several advantages over conventional indirect drive [4]. Since the platform is sliding on a mechanical slide surface, friction is incorporated during control. The effect of friction on servo-systems is noticeable especially at low velocity motion. Commonly referred to as stick-up, intermittent motion can lead to overshoot and limit cycling [5].

Various friction compensation techniques that are effective in ameliorating the effect of friction at low velocity have been proposed for a servo-system. These are high-gain PD-control [6], PID-control [7], friction compensation [8], adaptive control[9], neural network[10], sliding mode control, etc.

Sliding mode control is widely used in control of uncertain systems, mainly due to its robust characteristics with respect to parameter uncertainties and external disturbances [11]. Various methods based on sliding mode control framework have been applied to precision motion control systems in the consideration of model uncertainties and friction disturbances [11-14].

In this paper, we applied a sliding mode control to achieve robustness and desired disturbance attenuation and tracking performances. The system parameter uncertainties caused from linearization of electro-magnetic actuating force were analysed to establish reachability to sliding surface. The effectiveness of the sliding mode control was verified along with some experimental results.

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## II. LINEAR MOTOR STAGE AND STRAIGHTNESS ERROR COMPENSATION SYSTEM

Fig. 1 shows a single-axis linear motor stage and the straightness error compensation servo-system mounted on the slider of the linear motor stage. The straightness error is defined by deviational motion of the stage in the Y-direction. Since the stage moves along the slide guide without losing mechanical contact, the straightness error is hardly dependent upon machining accuracy of slide guide and assembling accuracy.

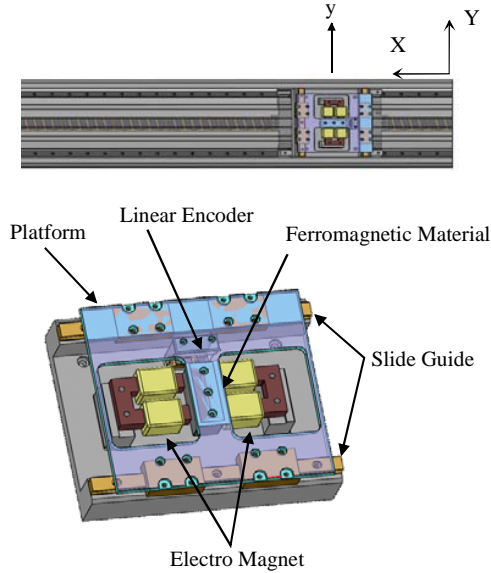


Fig. 1 Schematic of linear motor stage and straightness error compensation system mounted on slider

The straightness error of the linear motor stage shown in Fig. 1 was measured by means of a laser displacement sensor. The error was measured while the slider moves back and forth along the slide guide, X-direction in Fig. 1. From repeated measurements, we found the straightness error is position dependent along the X-axis, and the error is repeatable at the same position with small negligible error. A typical measured straightness error is shown in Fig. 2. The stroke of the stage is  $1,760\text{mm}$  and the error was measured while the slider moves with maximum acceleration  $200\text{mm/s}^2$ , constant velocity  $300\text{mm/s}$  and maximum deceleration  $-200\text{mm/s}^2$  sequence. The first half shows the time history of the measured error while the slider is moving forward, and the last half moving backward. Obviously, the error is symmetric with respect to the center line. From Fig. 2, the maximum straightness error is determined to be  $22.4\mu\text{m}$ .

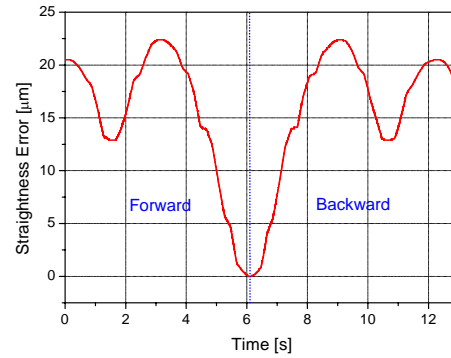


Fig. 2 Measured straightness error of the linear motor stage

In Fig. 1, the straightness error compensation system consists of a platform, mechanical slide guide, a linear encoder of  $0.1\mu\text{m}$  resolution, and an electro-magnetic actuator. The actuator is fixed rigidly on the slider.

The equation of motion of the platform system is given by:

$$M\ddot{y} = f_c + f_d \quad (1)$$

where  $M$  and  $y$  denote the mass and the displacement of the platform, respectively.  $f_c$  is control force and  $f_d$  is friction force.

The control force generated by a paired-electro-magnetic actuator can be represented by:

$$f_c = \frac{\mu_0 N^2 A}{4} \left[ \left( \frac{I_o + i_c}{y_o - y} \right)^2 - \left( \frac{I_o - i_c}{y_o + y} \right)^2 \right] \quad (2)$$

where  $\mu_0$  is the permeability of air,  $N$  is the number of coil turn, and  $A$  denote the area of pole.  $y_o$  and  $I_o$  are the nominal air-gap and the bias coil current, respectively.  $y$  and  $i_c$  denote the air-gap deviation from the nominal air-gap and the control current, respectively.

The attractive force in (2) can be rewritten as

$$f_c = K_y y + K_i i_c, \quad K_y = \bar{K}_y + \Delta K_y = (1 + m_y) \bar{K}_y, \quad K_i = \bar{K}_i + \Delta K_i = (1 + m_i) \bar{K}_i \quad (3)$$

$$\bar{K}_y = \frac{\mu_0 N^2 A I_o^2}{y_o^3}, \quad \bar{K}_i = \frac{\mu_0 N^2 A I_o}{y_o^2},$$

$$\Delta K_y = \left\{ \frac{1}{(1 - y^2/y_o^2)^2} - 1 \right\} \bar{K}_y$$

$$\Delta K_i = \left\{ \frac{3 \frac{y^2}{y_o^2} - \frac{y^4}{y_o^4} + \frac{y}{y_o} \frac{i_c}{I_o}}{(1 - \frac{y^2}{y_o^2})^2} \right\} \bar{K}_i \quad (4)$$

where  $K_y$  and  $K_i$  denote the position stiffness and the current stiffness, respectively.  $\bar{K}_y$  and  $\bar{K}_i$  are their nominal values and  $\Delta K_y$  and  $\Delta K_i$  are their uncertainties, and  $m_y = \Delta K_y / \bar{K}_y$  and  $m_i = \Delta K_i / \bar{K}_i$  are the ratios of the position stiffness uncertainty and the current stiffness uncertainty to their nominal values, respectively.

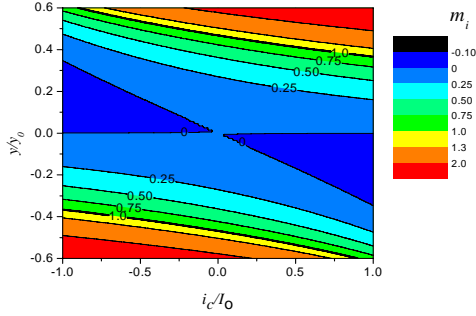


Fig. 3 Contour map of  $m_i$  v.s.  $y/y_o$  and  $i_c/I_o$

From (4), contour map of  $m_i$  is shown in Fig. 3. Clearly,  $|m_i| \leq 1$  within the whole range of control current when  $|y/y_o| \leq 0.35$ .

Substituting (3) into (1) gives the system dynamic equation as follows:

$$\begin{aligned} M\ddot{y} - \bar{K}_y y &= \bar{K}_i K_a u + f_t \\ f_t &= m_y \bar{K}_y y + m_i \bar{K}_i K_a u + f_d \end{aligned} \quad (5)$$

where the power amplifier model,  $i_c = K_a u$ , is included.  $K_a$  is the gain of the amplifier.  $f_t$  denotes the total disturbance including uncertainties and friction force.

It is clear from (5) that the control objective is to stabilize the system and to attenuate disturbance such that the platform tracks the desired position command accurately.

### III. SLIDING MODE CONTROL

The system dynamic equation in (5) can be represented by the expanded state space equation given as follows:

$$\dot{x} = Ax + Bu + m_i Bu + B_r r + B\eta \quad (6)$$

where  $x$ ,  $u$ ,  $\eta$ , and  $r$  are the state vector, control input, total disturbance, and position reference input, respectively. The state vector and the system matrices are defined by

$$x = [x_1 \ x_2 \ x_3]^T = [-\int edt \ y \ \dot{y}]^T \quad (7)$$

with

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & \bar{K}_y / M & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ K_a \bar{K}_i / M \end{bmatrix}, \quad B_r = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix},$$

$$\eta = \frac{\Delta K_y x_2 + \Delta K_i K_a u + f_d}{K_a \bar{K}_i}$$

where  $e$  denotes the error between the reference input and the response  $y$ , i.e.  $e = r - y$ . The integration of the position error is included as a state variable in order to eliminate the steady state position errors by means of a form of state feedback control.

Obviously, the nominal plant  $(A, B)$  is controllable and the uncertainties and the external disturbance satisfy the matching condition.

Let define a sliding surface  $\sigma$  as

$$\sigma(t) = Sx, \quad S \in R^{1 \times 3} \quad (8)$$

The sliding mode control is to drive the system in (7) to the sliding plane,  $\sigma(t) = 0$ , and sustain thereafter. An appropriate control  $u$  is chosen to satisfy the condition,  $\sigma \dot{\sigma} < 0$ .

Let the control input  $u$  be formulated as

$$u(t) = -(SB)^{-1} \{SAx + SB_r r + \phi \sigma\} - \rho (SB)^{-1} \text{sgn}[\sigma] \quad (9)$$

where  $\phi$  and  $\rho$  are positive constants to be designed.  $\text{sgn}[\cdot]$  is the sign function.

Applying the control, (9), gives the sliding mode dynamics as  $\dot{\sigma}(t) + \phi \sigma(t) = 0$ . To establish the reachability of the control in (9),  $\sigma \dot{\sigma}$  is given as

$$\begin{aligned} \sigma(t) \dot{\sigma}(t) &= -(1 + m_i) \phi \sigma^2 - \\ &\quad \sigma \{ \rho (1 + m_i) \text{sgn}[\sigma] + m_i (SAx + SB_r r) + SB\eta \} \end{aligned} \quad (10)$$

From (10),  $\sigma \dot{\sigma}$  is negative when  $|m_i| < 1$  and  $\rho$  satisfies the following inequality

$$\rho > \frac{|m_i| |SAx + SB_r r| + |SB\eta| + \gamma}{1 - |m_i|}, \quad \gamma > 0 \quad (11)$$

Thus, the closed-loop responses reach the sliding surface asymptotically and ideal sliding motion takes place thereafter.

If we define the vector as  $S = [\omega_n^2 \ 2\zeta\omega_n \ 1]$ , then the sliding motion is given by

$$\ddot{y} + 2\zeta\omega_n \dot{y} + \omega_n^2 y = \omega_n^2 r \quad (12)$$

Note, from (12), the nominal system guarantees zero steady state error for constant reference input. Consequently, we can choose the vector  $S$  to have desired closed-loop dynamics of

nominal model, and  $\phi$  to have required closed-loop sliding mode dynamics.

#### IV. SIMULATIONS AND EXPERIMENTS

In order to check the necessity for reachability,  $|m_i| < 1$ , in Fig. 3, sliding mode response are simulated with two initial conditions,  $y(0) = 0.6y_o$ ,  $u(0) = 0$  and  $y(0) = 0.2y_o$ ,  $u(0) = 0$ . Obviously, the first initial condition  $y(0) = 0.6y_o$ ,  $u(0) = 0$  does not meet the necessity.

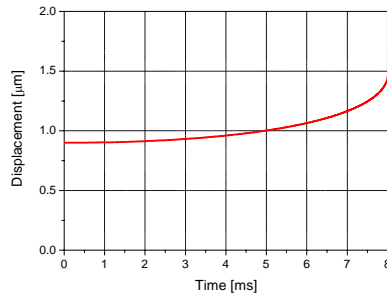


Fig. 4 Response for first initial condition

As can be seen in Fig. 4, the response monotonically diverged as expected. On the other hand, the second initial condition,  $y(0) = 0.2y_o$ ,  $u(0) = 0$ , satisfies the necessity as can be illustrated in Fig. 3. Fig. 5.a exhibits the sliding function with respect to time in this case. Where the discontinuous control input  $\rho$  is chosen large enough to meet (11). The sliding surface is reached at  $t = 0.26s$ , and ideal sliding motion takes place after then. The displacement of the platform is shown in Fig. 5.b. After reaching the sliding surface, the response dynamics is governed by (12). From some successive simulations with different initial conditions, we confirm that the contour map in Fig. 3 illustrates well the necessity for reachability.

To evaluate the feasibility of the proposed system and the performance of the controls, linear control and sliding mode control (SMC) were applied. The vector  $S$  in (8) was designed to have a closed-loop system with a damping ratio of  $\zeta_1 = 0.8$ , a natural frequency  $\omega_n = 35Hz$ , and  $\phi_1 = 100$ . Thus the closed-loop system has poles at  $s_{1,2} = -175.9 \pm j131.9$  and  $s_3 = -100$ .

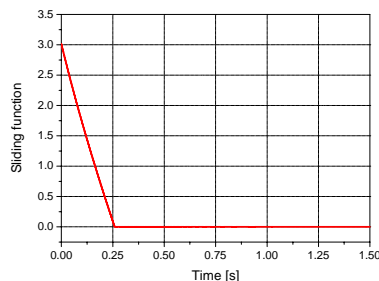


Fig. 5.a Sliding function for second initial condition

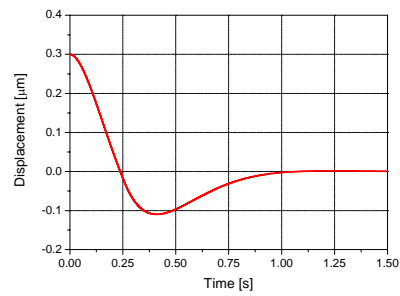


Fig. 5.b Response for second initial condition

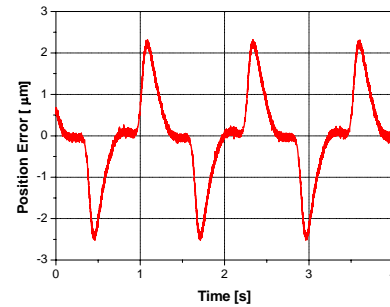


Fig. 6 Position error of platform by linear control

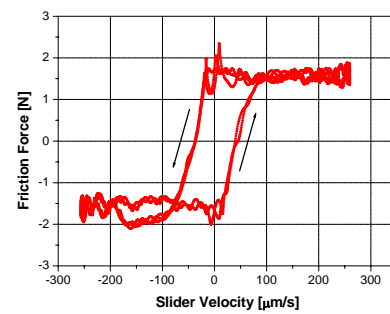


Fig. 7 Estimated friction force w.r.t. platform velocity

All controls were implemented on a digital computer equipped with a DSP (digital signal processing) board, 12-bit A/D converters, and 12-bit D/A converters. Throughout the experiments, the sampling frequency was kept at 2kHz. To evaluate the tracking performance of the controls to varying reference inputs, linear control and sliding mode control were applied. The linear control refers to the linear component of the SMC given in (9). Fig. 6 shows the position error measured from the linear control when the reference input is a single harmonic of the amplitude  $50\mu m$  and the frequency 0.8Hz. This response is a typical one exhibiting stick-slip motion caused by friction. By substituting the system parameters, estimated velocity and acceleration data, and the measured current data, the friction force is calculated from (1) and (2). The estimated friction force with respect to platform velocity is shown in Fig. 7. This shows a typical hysterical behavior of stick and slip frictions.

To investigate the feasibility of the straightness error compensation system, a comparison was made between the compensation error responses obtained from experiments with the linear control and the SMC. The nonlinear control input  $\rho$  has been chosen to be large enough so as to satisfy (11). During the control experiments, multiple sets of time history data sequence of the straightness error given in Fig. 2 were delivered as the reference position input. As shown in Figs. 8 and 9, the compensation errors were reduced by applying the SMC. The peak-to-peak compensation error were calculated to be  $1.77\mu\text{m}$  and  $0.4\mu\text{m}$  for the linear control and the SMC, respectively. Since the response falls within the range  $|y/y_o| \leq 0.00013$ , from Fig. 3, the current stiffness uncertainty satisfies the necessity  $|m_i| \leq 1$  sufficiently for reachability to the sliding surface.

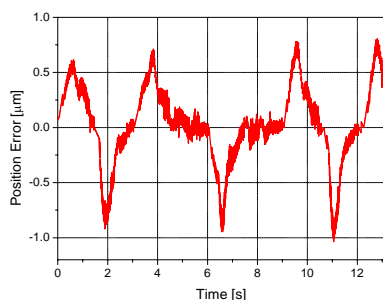


Fig. 8 Straightness compensation error by linear control

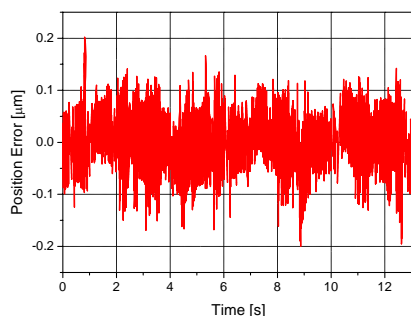


Fig. 9 Straightness compensation error by SMC

From the above experimental results, we can conclude that the SMC was effective in reducing the position error of the platform servo system, and the proposed straightness error compensation system is a feasible device to improve straightness error of linear motor stages.

## V. CONCLUSION

In this work, we proposed an add-on platform servo-system driven by an electro-magnetic actuator to improve straightness error of linear motor stages. Based on position dependence of the straightness error of the slider at each traveling position of the slider, a feedforward compensation control was suggested. In the consideration of the friction disturbance of the platform

and nonlinear uncertainties of electro-magnetic actuator, a sliding mode control was applied. The control-input related uncertainties were analyzed and reachability to sliding mode surface was verified. The experimental results demonstrated that the sliding mode control was effective to track reference position input in the presence of friction and model uncertainties, and the proposed straightness error compensation mechanism is a feasible device to improve straightness accuracy of linear motor stages.

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