

# Stochastic Modeling for Parameters of Modified Car-Following Model in Area-Based Traffic Flow

N. C. Sarkar, A. Bhaskar, Z. Zheng

**Abstract**—The driving behavior in area-based (i.e., non-lane based) traffic is induced by the presence of other individuals in the choice space from the driver's visual perception area. The driving behavior of a subject vehicle is constrained by the potential leaders and leaders are frequently changed over time. This paper is to determine a stochastic model for a parameter of modified intelligent driver model (MIDM) in area-based traffic (as in developing countries). The parametric and non-parametric distributions are presented to fit the parameters of MIDM. The goodness of fit for each parameter is measured in two different ways such as graphically and statistically. The quantile-quantile (Q-Q) plot is used for a graphical representation of a theoretical distribution to model a parameter and the Kolmogorov-Smirnov (K-S) test is used for a statistical measure of fitness for a parameter with a theoretical distribution. The distributions are performed on a set of estimated parameters of MIDM. The parameters are estimated on the real vehicle trajectory data from India. The fitness of each parameter with a stochastic model is well represented. The results support the applicability of the proposed modeling for parameters of MIDM in area-based traffic flow simulation.

**Keywords**—Area-based traffic, car-following model, micro-simulation, stochastic modeling.

## I. INTRODUCTION

MICROSCOPIC simulation has long been a topic of research. Numerous microscopic simulation models have been developed to advance the microscopic flow theory [1]. All microsimulation models require a set of parameters. In practice, there is no hard and fast rule to determine what values of parameters of models accurately represent an individual driver's behavior. Overall, in many cases, the model parameters are not estimated rigorously due to the limited availability of details vehicle trajectory data [2]. However, the importance of modeling estimated parameters in a microscopic simulation cannot be overlooked.

This paper focuses on the stochastic modeling for parameters of MIDM in area-based traffic. In area-based traffic which is frequently termed as non-lane based traffic in the literature of traffic flow, modeling drivers generally ignore the lane markings and perceive the entire road space while progressing longitudinally. The traditional car-following (CF) for longitudinal and lane changing (LC) for lateral movements

models are not directly applicable in this traffic regime. Recently such system has gained interest, and numbers of the simple and modified models are proposed for modeling the area-based traffic. For instance, [3] refers a simulation framework for modeling area-based heterogeneous traffic flow, [4] refers the strip-based space discretization framework for modeling the driving behaviors of non-lane based mixed traffic and the latent leader acceleration model is proposed in [5] for modeling driving behavior in weak lane discipline.

The driving behavior in area-based heterogeneous traffic has significantly different from the lane-based traffic. The subject vehicle frequently changes its lateral position in different lanes while progressing longitudinally. The dynamics of such vehicles define the variability of the state of traffic (speed and density) being experienced by the subject vehicle. Such variability influences the lateral and longitudinal movements of the subject vehicles, i.e. it follows the current direction of motion or move laterally (refer to [6] for details of modeling the area-based traffic in two steps approach in sequential order such as area selection using a discrete choice framework and vehicle movement modeling with a modified CF model). Firstly, a discrete choice modeling framework is developed for area-based traffic flow in terms of alternative selection to microscopically capture the dynamic of the subject vehicle in presence of the other mixed vehicles in its choice space from the visual perception area. The choice space of the subject vehicle is divided by numbers of realistic radial cones considering the possible moving directions of the subject vehicle in the next time step that forms the alternatives for his decision. The modeling framework consisted of alternatives from choice space of the driver's visual perception area, attributes of the alternatives and modeling the selection of an alternative. Secondly, the vehicle following behavior model is developed to simulate the next position of the subject vehicle along the direction of the selected alternative. The intelligent driver model (IDM) [7] is modified to incorporate such driving maneuverability in area-based traffic condition, and the model is known as the MIDM.

Based on the information that is available in data, the realistic bounds for each parameter of MIDM are defined for an individual vehicle from randomly selected subject vehicles. A univariate dataset is developed by calibrating the parameters of MIDM. This paper focuses on the modeling of parameters of MIDM. The purpose of this paper is to provide a robust simulation framework for the parameters of a microscopic simulation model in area-based traffic.

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## II. MODIFIED INTELLIGENT DRIVER MODEL

The acceleration profile from IDM model is defined as a continuous function of the speed, spacing and relative speed of a subject vehicle (S) to the leading vehicle (L) by (1)

$$a_S = a_{max} \left[ 1 - \left( \frac{v}{v_d} \right)^\delta - \left( \frac{g^*(v, \Delta v)}{g} \right)^2 \right] \quad (1)$$

where  $a_{max}$  is the maximum acceleration of vehicle (S),  $v_d$  is the desired speed of the vehicle (S),  $g$  is the spacing between a subject vehicle to a leading vehicle,  $g^*$  is the desired minimum gap (space headway), and  $\delta$  is an acceleration exponent (model parameter). The desired space headway is measured using (2)

$$g^*(v, \Delta v) = g_0 + g_1 \sqrt{\frac{v}{v_d}} + vT + \frac{v\Delta v}{2\sqrt{a_{max}b}} \quad (2)$$

where  $g_0$  represents minimum spacing in congested traffic; the term  $g_1$  represents nonlinear jam distance,  $T$  is for safety time headway, and  $b$  for the desired deceleration. The IDM brakes stronger than comfortable deceleration when the gap becomes too small which basically makes the model collision-free.

The driving behaviors in area-based heterogeneous traffic condition are significantly different from the lane-based traffic. The driving behavior in such regime induced by the presence of other individuals in the choice space from visual perception area of individual driver and captures the vehicle to vehicle interactions. In constraint driving situation, a subject vehicle is constrained by the potential leaders and leaders are changed over time. The choice space is partitioned into a defined number of radial cones. In each of these directions, a possible leader can be identified from a set of potential leaders. The leading vehicle induces an attractive interaction on the subject vehicle. Therefore, a subject vehicle acceleration profile can be derived from the corresponding leader acceleration which is basically described by a stimulus-response framework. For a given leader (L), the spacing ( $g$ ) between a subject vehicle (S) and a leader (L) is described by (3)

$$g = r \cos \Delta\theta \quad (3)$$

which is the vector projection of a leader along specified alternative, and  $\Delta\theta$  is the angle between position vector  $r$  and the direction of alternative (refer to [6] for more details about variables definition and measurement). The subject vehicle reacts to stimuli coming from the chosen leader. The relative speed is modeled as stimuli for subject vehicle defined by the difference between vector projection of leader's speed and vector projection of subject vehicle's speed along the direction of selected alternative (refer to [6] for details about variables) as in (4)

$$\Delta v = v_L \cos \Delta\psi_L - v_S \cos \Delta\psi_S \quad (4)$$

where,  $v_S$  and  $v_L$  represent a resultant speed of a subject (S)

vehicle and a leader (L), respectively;  $\Delta\psi_S$  and  $\Delta\psi_L$  represent angles in between the direction of an alternative occupied by a leader (L) and the direction of  $v_S$  and  $v_L$ , respectively.

The IDM which is (1) is then modified based on the specifications of (3) and (4); called MIDM and defined as in (5)

$$a_S = a_{max} \left[ 1 - \left( \frac{v_S \cos \Delta\psi_S}{v_d} \right)^\delta - \left( \frac{g^*(v_S \cos \Delta\psi_S, \Delta v)}{g} \right)^2 \right] \quad (5)$$

where

$$g^* = g_0 + g_1 \sqrt{\frac{v_S \cos \Delta\psi_S}{v_d}} + v_S \cos \Delta\psi_S T + \frac{v_S \cos \Delta\psi_S \Delta v}{2\sqrt{a_{max}b}} \quad (6)$$

The desired speed ( $v_d$ ), maximum acceleration ( $a_{max}$ ), desired deceleration ( $b$ ), safety time headway ( $T$ ), linear jam distance ( $g_0$ ), non-linear jam distance ( $g_1$ ) and acceleration exponent ( $\delta$ ) are calibrated parameters for MIDM in (5).

## III. METHODOLOGY

The modeling approach is developed for two regimes of stochastic modeling. One is a parametric distribution from an aggregated point of graphical and statistical analysis. The other approach is a non-parametric distribution for the parameters of MIDM which are not well suited for parametric distribution.

The following basic steps are considered for stochastic modeling for the parameters of MIDM in area-based traffic:

- Step1. Develop an estimated dataset for parameters of MIDM - The real vehicle traffic trajectory data are used to estimate the parameters of MIDM. The raw trajectory data [8] are used to develop a database for estimated parameters for the car. MIDM has a set of six parameters with a constant acceleration exponent ( $\delta = 4$ ). The parameters are estimated over randomly selected individual subject vehicle from the aforementioned data. A univariate dataset is developed by calibrating the parameters of MIDM where each parameter can be modeled by a probability distribution. For this, we consider a vector of 130 randomly selected vehicles from area-based heterogeneous traffic in the aforementioned data.
- Step2. Modeling with parametric distribution - Each calibrated parameter of MIDM builds up a continuous dataset. The goodness of fit that means how well data fit with a specific parametric distribution is described graphically and statistically. The quantile-quantile (Q-Q) plot (refer to [9] for details about the graphical representation of data using Q-Q plot) for each estimated parameter is used to represent graphically for the fitness of data with a specific distribution. The Kolmogorov-Smirnov (K-S) test (refer [10] for details about the K-S test) is performed for several commonly used probability distributions to find the suited model statistically for an individual parameter.
- Step3. Modeling with non-parametric distribution - The

probability density function (PDF) and cumulative density function (CDF) can be estimated from the empirical data by using a non-parametric distribution such as kernel distribution. Such kernel distribution is a function of a kernel density estimator, a smoothing function, and a bandwidth value.

IV. RESULTS AND DISCUSSIONS

A. Parametric Distribution

The Q-Q plot for each estimated parameter is used to represent graphically for the fitness of data with a specific distribution. The normal, gamma, and Weibull distributions are considered to compare empirical quantiles from data with the quantiles of specific theoretical distributions. The

nonlinearity of the points in normal Q-Q plot for all parameters of MIDM except desired speed indicates a departure from normality (refer to Fig. 1). The linearity of point pattern from the Q-Q plot is justified with reference line. For desired speed, the left end of point pattern from normal Q-Q plot is above and the right end of pattern is below from the reference line that indicates a short tail at both ends. Since the point patterns are curved with positive slopes for other parameters, the normal distribution is not properly fitted with them. Similarly, the nonlinearity of the points in Gamma and Weibull Q-Q plots for each parameter of MIDM except desired deceleration has measured the departure from such distributions (refer Fig. 2 and Fig. 3).

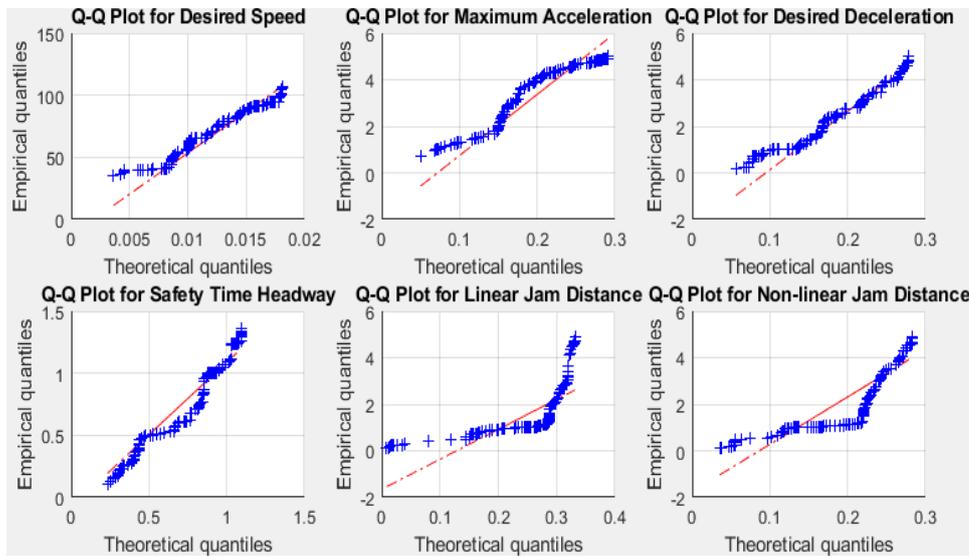


Fig. 1 Normal quantile-quantile (Q-Q) plots for estimated parameters of MIDM

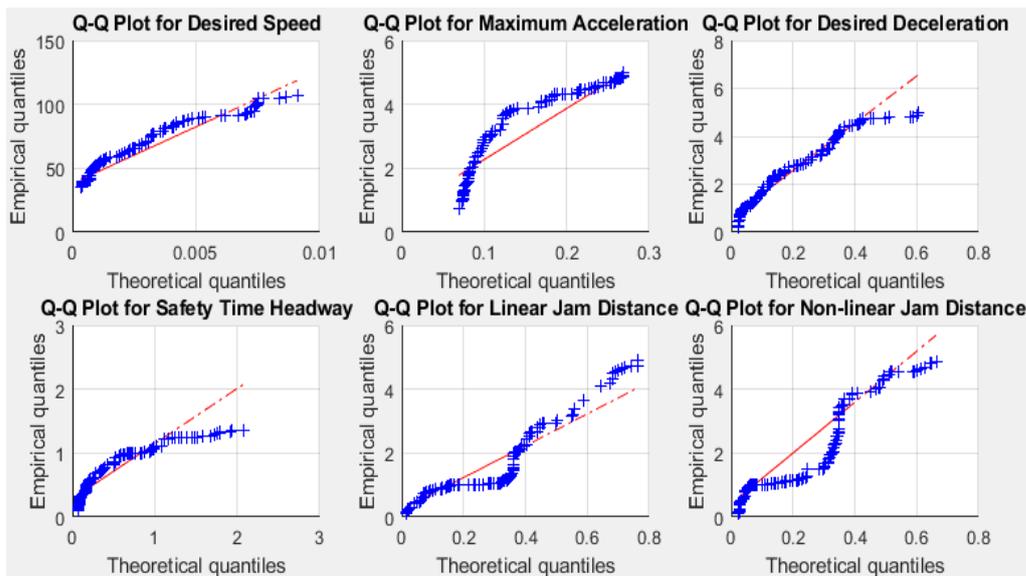


Fig. 2 Gamma quantile-quantile (Q-Q) plots for estimated parameters of MIDM

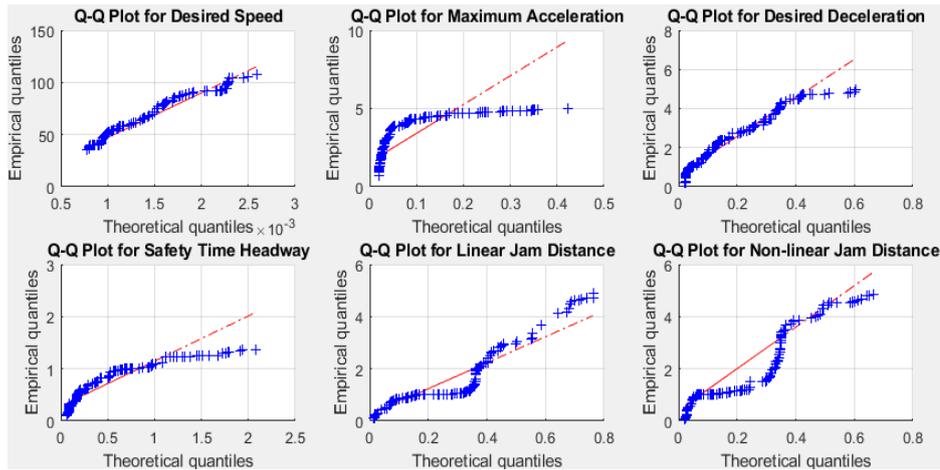


Fig. 3 Weibull quantile-quantile (Q-Q) plots for estimated parameters of MIDM

TABLE I  
A LIST OF K-S TEST ON ESTIMATED PARAMETERS OF MIDM

Distribution	K-S Statistics	K-S Test Performance, $\alpha = 0.05$ (reference value)					
		Desired speed	Maximum acceleration	Desired deceleration	Safety time headway	Linear jam Distance	Non-linear jam distance
Normal	p	0.056	0.0038	0.0325	0.029	6.67E-05	1.00E-04
	ks	0.1159	0.1538	0.1245	0.1262	0.1972	0.1932
	cv	0.1178	0.1178	0.1178	0.1178	0.1178	0.1178
Log normal	decision	Accepted	Rejected	Rejected	Rejected	Rejected	Rejected
	p	4.01E-115	2.58E-92	6.09E-60	5.38E-90	1.45E-36	7.31E-44
	ks	0.9979	0.8932	0.7189	0.8819	0.5602	0.6141
GEV	cv	0.1178	0.1178	0.1178	0.1178	0.1178	0.1178
	decision	Rejected	Rejected	Rejected	Rejected	Rejected	Rejected
	p	8.04E-06	2.74E-08	2.22E-06	1.85E-05	5.20E-12	2.49E-11
Gamma	ks	0.2164	0.2613	0.2274	0.209	0.3172	0.3077
	cv	0.1178	0.1178	0.1178	0.1178	0.1178	0.1178
	decision	Rejected	Rejected	Accepted	Rejected	Rejected	Rejected
Weibull	p	0.0317	9.23E-04	0.2419	0.0058	0.0427	0.0486
	ks	0.1249	0.1702	0.0888	0.1484	0.1203	0.1182
	cv	0.1178	0.1178	0.1178	0.1178	0.1178	0.1178
F	decision	Rejected	Rejected	Accepted	Rejected	Rejected	Rejected
	p	0.0724	0.0021	0.1222	0.0111	0.0351	0.0197
	ks	0.1116	0.1608	0.1024	0.1398	0.1404	0.1319
Student's t	cv	0.1178	0.1178	0.1178	0.1178	0.1178	0.1178
	decision	Accepted	Rejected	Accepted	Rejected	Rejected	Rejected
	p	6.13E-114	2.20E-25	4.75E-11	3.40E-21	7.57E-06	6.30E-08
Exponential	ks	0.9928	0.4655	0.3037	0.4247	0.217	0.2552
	cv	0.1178	0.1178	0.1178	0.1178	0.1178	0.1178
	decision	Rejected	Rejected	Rejected	Rejected	Rejected	Rejected
Exponential	p	1.57E-113	4.93E-63	7.82E-47	4.00E-33	1.54E-37	1.84E-36
	ks	0.991	0.7373	0.6348	0.5328	0.5677	0.5594
	cv	0.1178	0.1178	0.1178	0.1178	0.1178	0.1178
Exponential	decision	Rejected	Rejected	Rejected	Rejected	Rejected	Rejected
	p	3.02E-20	2.08E-07	1.30E-04	1.27E-05	4.17E-06	1.49E-04
	ks	0.4148	0.2462	0.1906	0.2124	0.2221	0.1893
Exponential	cv	0.1178	0.1178	0.1178	0.1178	0.1178	0.1178
	decision	Rejected	Rejected	Accepted	Rejected	Rejected	Rejected

The K-S test is performed for several commonly used probability distributions to find the suited model statistically for an individual parameter. The test is performed based on test statistic (ks), significance level ( $\alpha = 0.05$ ), p-value and critical value (cv). A list of the K-S test is presented in TABLE I. The K-S test statistic is less than the critical value, and p-value is greater than significance level  $\alpha = 0.05$  for desired speed in normal and Weibull distributions and desired

deceleration in Gamma and Weibull distributions but all other cases it is higher. Therefore, it is hypothesized that desired speed and desired deceleration can be modeled by a parametric distribution and other parameters can be modeled by a non-parametric distribution.

*B. Non-Parametric Distribution*

Based on the graphical and statistical analysis provided, the

desired speed and desired acceleration follow some parametric distributions. The maximum acceleration, safety time headway, linear and non-linear jam distance cannot be modeled accurately by using a parametric distribution. In this case, the PDF and CDF can be estimated from the empirical data of maximum acceleration, safety time headway, linear and non-linear jam distance by using a non-parametric distribution such as kernel distribution. To understand the effects of different kernel smoothing functions on the shape of the estimated PDF from maximum acceleration, safety time headway, linear and non-linear jam distance are illustrated in Fig. 4-Fig. 7, respectively. Finally, for simplicity, the normal smoothing function is considered for kernel CDF to compare with empirical CDF for mentioned parameters. The illustrations in Fig. 8 - Fig. 10 indicate that non-parametric distribution is properly fitted with maximum acceleration, safety time headway, linear and non-linear jam distance of MIDM.

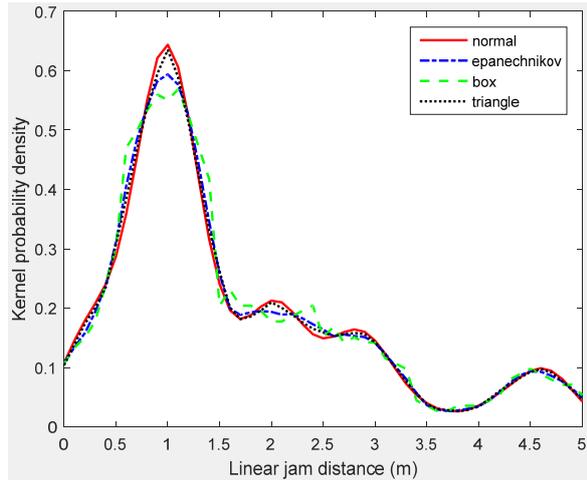


Fig. 6 Kernel density curves for different smoothing functions for linear jam distance

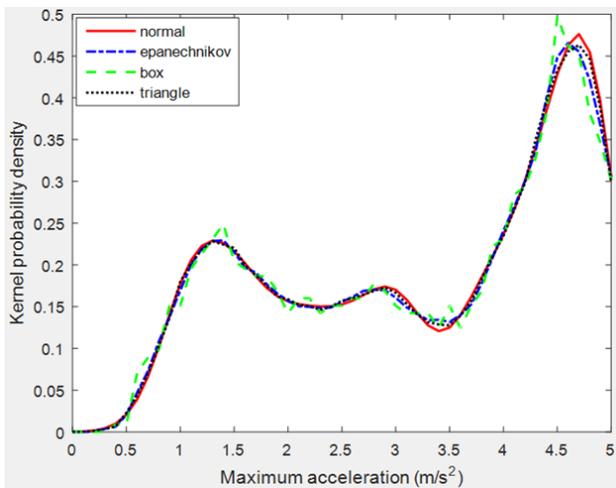


Fig. 4 Kernel density curves for different smoothing functions for maximum acceleration

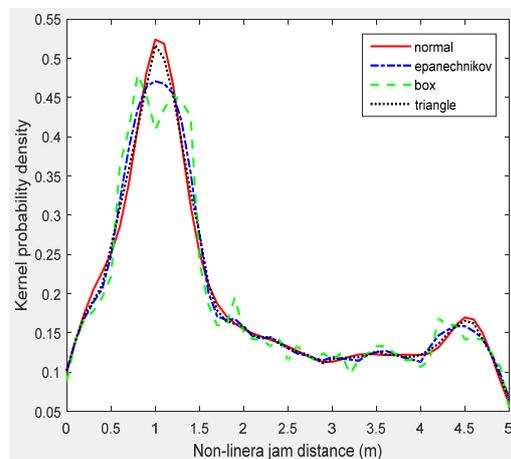


Fig. 7 Kernel density curves for different smoothing functions for non-linear jam distance

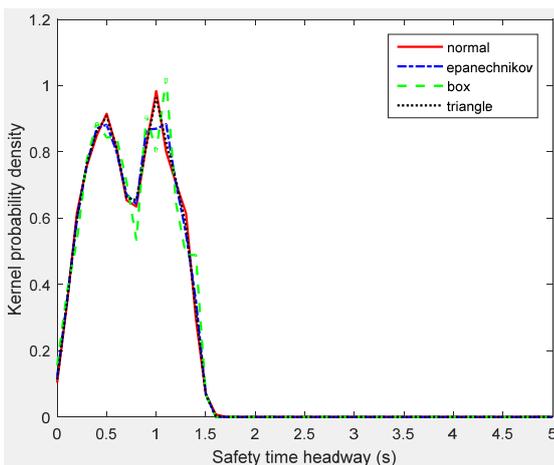


Fig. 5 Kernel density curves for different smoothing functions for safety time headway

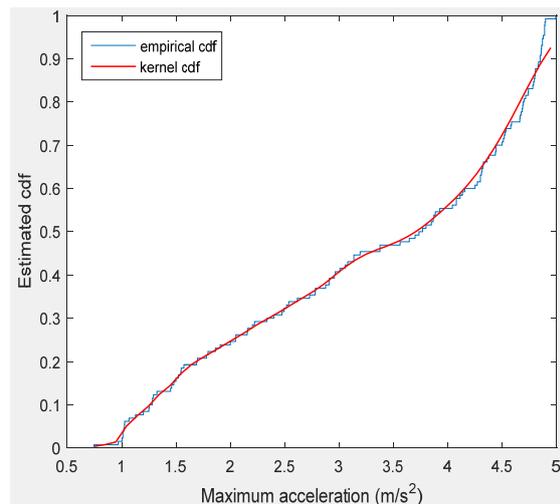


Fig. 8 The goodness of fit measured by kernel CDF with empirical CDF of maximum acceleration

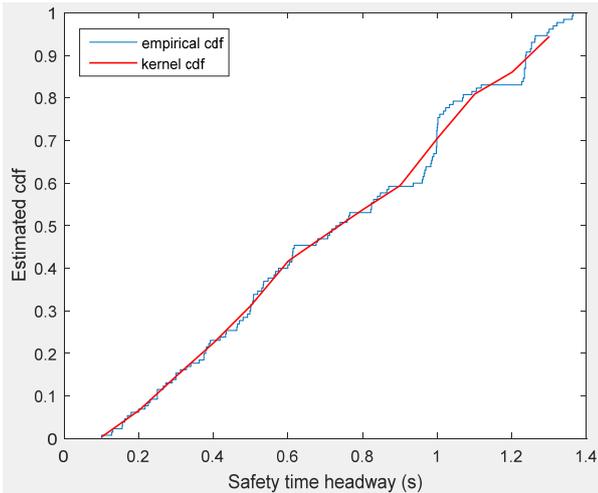


Fig. 9 The goodness of fit measured by kernel CDF with empirical CDF of safety time headway

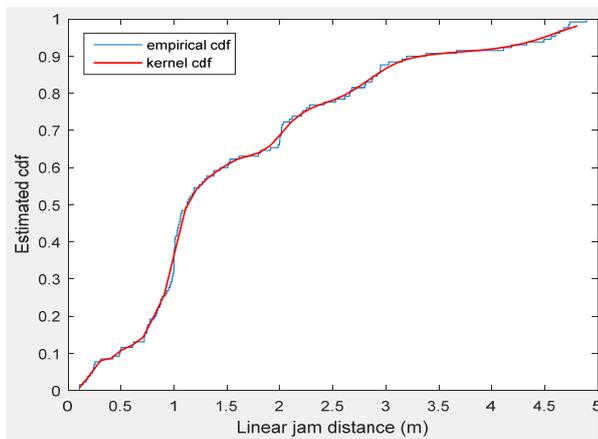


Fig. 10 The goodness of fit measured by kernel CDF with empirical CDF of linear jam distance

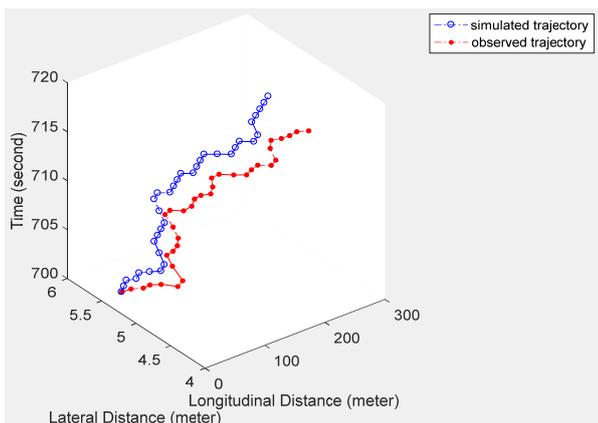


Fig. 11 Simulated and observed trajectory for a Car-I

randomly selected four cars (Car-I, Car-II, Car-III and Car-IV) using microsimulation of MIDM. The simulated trajectory generates the lateral and longitudinal distance errors due to the simulated position of a subject vehicle from MIDM. The simulated and observed 3D trajectories for Car-I, Car-II, Car-III and Car-IV are shown in Fig. 11-Fig. 14.

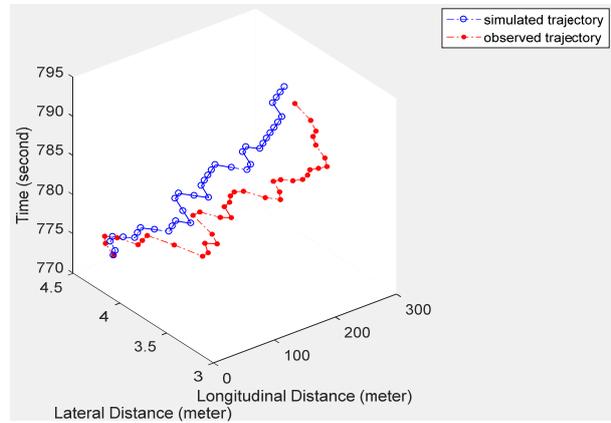


Fig. 12 Simulated and observed trajectory for a Car-II

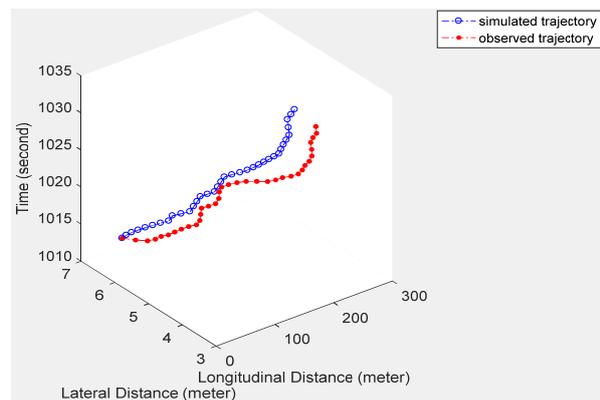


Fig. 13 Simulated and observed trajectory for a Car-III

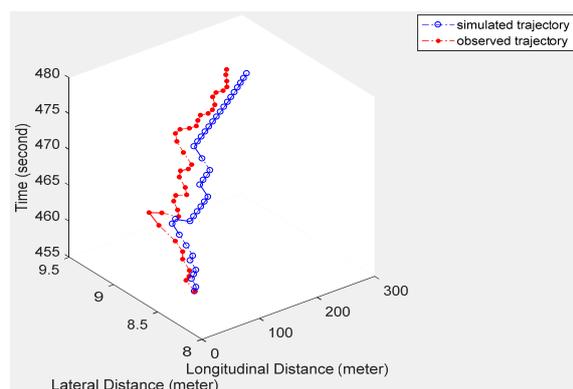


Fig. 14 Simulated and observed trajectory for a Car-IV

### V. MODEL VALIDATION

The performance of the proposed stochastic modeling for parameters of MIDM in area-based traffic is tested for

The performance of MIDM is measured by the root mean square error (RMSE) (refer [11] for details about the RMSE) of the deviation of the simulated trajectory with that of

observed trajectory. The RMSE is 5.75 m, 6.56 m, 6.62 m, and 6.68 m for Car-I, Car-II, Car-III and Car-IV, respectively. The simulated results from microsimulation of MIDM perform well for area-based traffic flow. However, the proposed stochastic modeling for parameters of MIDM can be implemented in area-based traffic flow modeling.

## VI. CONCLUSION

The parametric and non-parametric distributions are considered to model the estimated parameters of MIDM in area-based traffic. The nonlinearity of the points in theoretical Q-Q plots for estimated parameters of MIDM indicates a departure from the theoretical distributions. The point patterns of Q-Q plots are curved with positive and negative slopes indicate estimated data for parameters are not properly fitted with theoretical distributions.

The linearity of the points in normal Q-Q plot for the estimated desired speed of MIDM indicates data is fitted with a normal distribution. Similarly, the linearity of the points in Gamma and Weibull Q-Q plots for the estimated desired deceleration parameter of MIDM indicate estimated parameter can be modeled with them.

The K-S test statistic is less than the critical value for desired speed in the normal distribution and desired deceleration in Gamma and Weibull distributions but all other cases it is higher. Therefore, it is hypothesized that desired speed and desired deceleration can be modeled by a parametric distribution and other parameters can be modeled by a non-parametric distribution.

The PDF and CDF are estimated from the empirical data of maximum acceleration, safety time headway, linear and non-linear jam distance by using a non-parametric kernel distribution. The results indicate that non-parametric distribution is properly fitted with estimated maximum acceleration, safety time headway, linear and non-linear jam distance parameters of MIDM.

The stochastic modeling for all estimated parameters of MIDM can be further implemented for a microscopic simulation in area-based traffic. The findings are from data available which is the main limitation for such type of modeling for parameters of MIDM in area-based traffic conditions.

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