International Journal of Electrical, Electronic and Communication Sciences
ISSN: 2517-9438
Vol:11, No:1, 2017

Statistical Modeling of Mobile Fading Channels Based on Triply Stochastic Filtered Marked Poisson Point Processes

Jihad S. Daba, J. P. Dubois

***Abstract*—**Understanding the statistics of non-isotropic scattering multipath channels that fade randomly with respect to time, frequency, and space in a mobile environment is very crucial for the accurate detection of received signals in wireless and cellular communication systems. In this paper, we derive stochastic models for the probability density function (PDF) of the shift in the carrier frequency caused by the Doppler Effect on the received illuminating signal in the presence of a dominant line of sight. Our derivation is based on a generalized Clarke's and a two-wave partially developed scattering models, where the statistical distribution of the frequency shift is shown to be consistent with the power spectral density of the Doppler shifted signal.

***Keywords*—**Doppler shift, filtered Poisson process, generalized Clark's model, non-isotropic scattering, partially developed scattering, Rician distribution.

I. Introduction

A pulse-echo delay-Doppler measurement system such as an active sonar or radar measures an environment of interest by illuminating it with electromagnetic or acoustic radiation. The illumination field scattered by objects is collected by a receiver, which processes the scattered signal to determine the presence and characteristics of the mapped objects. As a result of radial target motion toward or away from the pulse-echo sensor, the Doppler Effect induces a shift in the scattered waveform's carrier frequency [1]-[3].

In this work, we derive stochastic models for the Doppler frequency shift in non-isotropic scattering channels for mobile fading environments.

II. Stochastic Modeling of Scattering Channels in the Presence of a Line-of-Sight

We study propagation problems encountered in the use of EM or acoustic transmission taking place mainly by way of scattering from the surfaces of the surrounding objects and by diffraction over and/or around them. The transmitted signal at a carrier frequency f_c reaches the pulse-echo system via a number of paths in addition to a dominant direct line-of-sight (LOS).

Each line path (or k-th scatterer) can be treated as a random phasor $a_k e^{j\phi_k}$, where the amplitude a_k contributes to the attenuation or energy absorption and the phase ϕ_k depends on the varying path lengths, changing between 0 and 2π, with the latter occurring when the path length changes by a full wavelength. The LOS component has amplitude strength V_0, which itself could be random. Hence, we can reasonably assume that the phases are uniformly distributed over $[0, 2\pi]$ and statistically independent of the amplitudes a_k and V_0. The total backscattered received complex signal is $\rho = \sum_{k=1}^{N} a_k e^{j\phi_k}$ and is driven by an underlying triply stochastic filtered marked Poisson point process [4]-[7]. In the case of fully-developed scattering noise, a large number of scatterers is present in the channel, and the central limit theorem (CLT) implies that ρ is asymptotically circularly Gaussian. Even for statistically dependent scatterers, ρ would still be asymptotically Gaussian hinging on the fact that the sequence of scattering random variables satisfies the α-mixing property or the conditions of the Lindeberg-Feller CLT. In this case, the delay-spread function is modeled as a zero-mean complex Gaussian process. The envelope $\Gamma = |\rho + V_0|$ and power γ^2 obey Rician & modified Rician (2 freedom-degrees) laws, respectively [8]-[11]. Figs. 1 and 2 illustrate the geometry of the scattering model as a random walk in the complex plane.

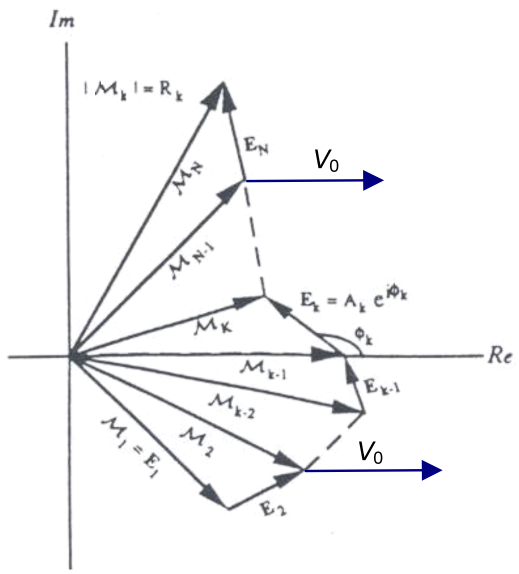

Fig. 1 The scattering model as a random walk in the complex plane

J. S. Daba and J. P. Dubois are with the Electrical Engineering Department, University of Balamand, Lebanon (e-mail: jihad.daba@balamand.edu.lb).

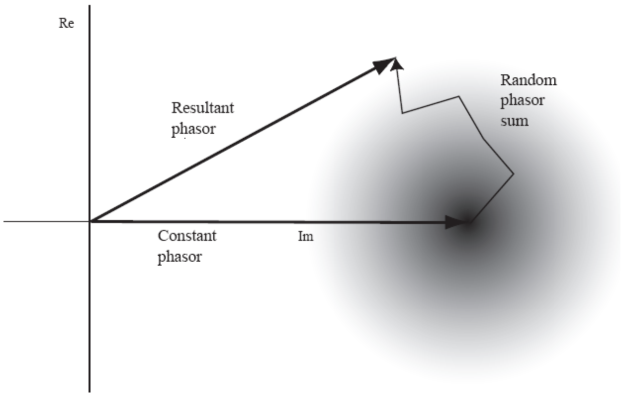

Fig. 2 Geometry of the scattering model

III. Statistics of the Doppler Frequency Based on a Generalized Clark's Model

In the classical Clarke's channel model [12], isotropic scattering is the main assumption, leading to a uniform distribution for the angle of arrival (AOA) of the multipath components at the mobile station. However, non-isotropic scattering are encountered in many mobile radio channels. Such generalized scattering characteristics strongly affect and alter the correlation function and power spectrum of the complex envelope at the mobile receiver [13]. In this section, we devise in a new model that captures more general scattering characteristics and generalizes the Clarke's model.

The expression of the received signal must also include the effects of motion induced frequency. Different arrival angles ψ_k from each k-th scattered wave will cause the transmitted signal to be received at different frequencies. The Doppler shift of each wave component $\xi_{D_k} = f_D \cos(\psi_k)$, where $f_D = (v/c) f_c$ is the maximum Doppler shift in frequency, v being the relative velocity between target and sensor. The arrival angles are i.i.d, and obey the distribution:

$$p_\psi(\theta) = \frac{P_T}{2\pi(K+1)}\left(1 + 2\pi K \delta(\theta - \theta_0)\right), \qquad (1)$$

where the Rician K-factor = $\dfrac{\text{Specular power}}{\text{Diffuse power}} = \dfrac{V_0^2}{P_{dif}}$, P_T is the total incoming power and θ_0 is an arbitary offset angle. The induced Doppler effect causes a random frequency shift $\nu_c = f_c + \xi_D$ due to the random spatial distribution of the scatterers. The received faded signal can be expressed as

$$\tilde{s}(t) = \gamma \cos(2\pi \nu_c t + \phi), \gamma \perp \phi \perp \nu_c. \qquad (2)$$

For a dense-scatter channel model with a LOS, the PDF of the Doppler frequency shift is

$$p(\nu_c) = \frac{1}{K+1}\left(\begin{array}{l} \dfrac{1}{\pi\sqrt{(\nu_c - (f_c - f_D))(f_c + f_D - \nu_c)}} \\ + K\delta(\nu_c - f_c) \end{array}\right) \times \qquad (3)$$
$$\times\, I_{(f_c - f_D, f_c + f_D)}(\nu_c)$$

with asymptotes at $\nu_c = f_c \pm f_D$ and an impulsive Dirac delta function at f_c.

We note that the expressed PDF is consistent with the power spectral density of the Doppler-shifted signal under a generalized classical Clark's model [12] as depicted in Fig. 3. Accordingly, the PDF of ν_c "fits" the characteristics of the power spectrum $S_D(f)$ of the Doppler signal. This "characteristical fitness" is in the sense that $S_D(f)df \propto p_{\nu_c}(\nu_c)d\nu_c$. As shown in Fig. 3, the PDF fits the following characteristics: (1) It is centered at f_c and is zero outside the limits of $f_c \pm f_D$; (2) Areas of large probabilities correspond to areas of larger power; (3) The PDF at $f = f_c \pm f_D$ is infinite, that is, Doppler components arriving at exactly 0^o and 180^o have an infinite distribution. The latter observation does not indicate model's divergence since the probability of exact angle arrivals at 0^o and 180^o is zero.

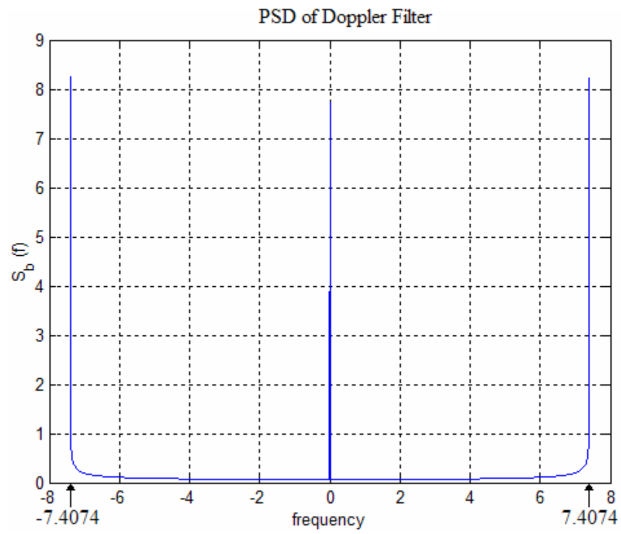

Fig. 3 PSD of the Doppler signal under the generalized Clark's model

In summary, the classical LOS Doppler model (generalized Clark) is identical to the diffuse model (Clark) to which a Dirac Delta function is superposed at the central carrier frequency f_c. The Dirac delta function at the carrier frequency is a mathematical idealism for an instantaneous event, that of a LOS being a straight direct path (a sector of angle 0). Our model is centered and peaked at the carrier frequency representing the more practical phenomena where the dominant LOS spans a narrow sector of infinitesimal angle

$\Delta\theta$: $\delta(f) = \lim_{\Delta\theta \to 0} \phi_{\Delta\theta}(f)$ where $\phi(.)$ is a test function and may be defined as $\phi_{\Delta\theta}(f) = (\Delta\theta)^{-1}\,\mathrm{Rect}(\Delta\theta)$.

IV. Statistics of the Doppler Frequency Based on a Generalized Clark's Model

A. Approximated Model for Partially Developed Fading

We first consider the PDF of fading power of a two-wave partially developed scattering noise. The distribution of such power is very difficult to model analytically, but can be approximated [14], [15]. For 2 scattering signal paths, the physical Rician *K*-factor is

$$K = \frac{\text{Specular LOS Power}}{\text{Diffuse Power}} = \frac{|V_0|^2}{A_1^2 + A_2^2}, \tag{4}$$

and the shaping PAR (peak-to-average ratio) parameter is

$$\Delta = \frac{\text{Peak diffuse power}}{\text{Average diffuse power}} = \frac{2A_1A_2}{A_1^2 + A_2^2}. \tag{5}$$

The PDF of the fading power is

$$p_v(v) \approx \frac{1}{|V_0|^2} e^{-\left(\frac{v}{|V_0|^2}+K\right)} \sum_{k=1}^{P} \alpha_k D\left(\frac{\sqrt{2v}}{V_0}; K, \Delta\cos\left(\frac{\pi(k-1)}{2P-1}\right)\right), \tag{6}$$
$$K \not\gg, \Delta \not\to 1$$

where

$$D(x;K\ ,\beta) = \frac{1}{2}\begin{bmatrix} e^{\beta K} I_0\left(x\sqrt{2K\ (1-\beta)}\right) \\ +e^{-\beta K} I_0\left(x\sqrt{2K\ (1+\beta)}\right) \end{bmatrix}, \tag{7}$$

and $I_0(.)$ is the zero-order modified Bessel function of the first kind. The exact series coefficients α_k and order P are tabulated in Table I. As a rule of thumb, the minimum order is chosen according to $O(P) = \lceil \Delta K/2 \rceil$, where $\lceil . \rceil$ is the ceiling or round up function.

TABLE I
COEFFICIENTS SERIES FOR THE APPROXIMATED FADING POWER PDF

P	1	2	3	4	5
	1				
	1/4	3/4			
	19/144	25/48	25/72		
α_k	$\frac{751}{8640}$	$\frac{3577}{8640}$	$\frac{49}{320}$	$\frac{2989}{8640}$	
	$\frac{2857}{44800}$	$\frac{15741}{44800}$	$\frac{27}{1120}$	$\frac{1209}{2800}$	$\frac{2889}{22400}$

We note that the pdf expression yields accurate representations over the most useful range of K and Δ parameters. The deviation between the approximate and exact PDFs will become significant only if (1) K >>, that is, the coherent specular power is much larger than the diffuse power, and (2) $\Delta \to 1$, that is, the amplitudes of the diffuse scatterers are relatively equal in magnitude.

In particular, the PDF exhibits the following proper limiting behavior: (1) for $K = 0$, the envelope is Rayleigh distributed and the power is exponentially distributed; (2) for $K > 0$, $\Delta = 0$, the envelope is Rician distributed and the power follows a modified Rician distribution; (3) for $K\Delta < 2$, the envelope follows a Rician-like shape; (4) for $K < \min\left(\frac{2}{\Delta}, \frac{1}{\sqrt{1-\Delta^2}} - 1\right)$, the envelope follows a Rayleigh-like shape.

B. PDF of the Doppler Shift

To obtain the PDF of the Doppler frequency shift, we make the following substitutions in the PDF of the fading power:

$$V_0 = \sqrt{f_c}\, e^{j\Delta\theta}, \Delta\theta \sim U[-f_D, f_D], \tag{8}$$

which implies that $|V_0|^2 = f_c$ as expected since it is a LOS component. This can be illustrated geometrically on the IQ diagram (in-phase and quadrature) as a the random walk in the complex plane consisting of a coherent specular LOS component which spans an infinitesimally small sector $\Delta\theta$ and is superposed to two randomly phased diffuse waves.

The model parameters are given by:

$$A_1 = 0.5\left(\sqrt{f_c + f_D} + \sqrt{f_c - f_D}\right), \tag{9}$$

$$A_2 = 0.5\left(\sqrt{f_c + f_D} - \sqrt{f_c - f_D}\right), \tag{10}$$

$$K = \frac{|V_0|^2}{A_1^2 + A_2^2} = \frac{4f_c}{\left(\sqrt{f_c + f_D} + \sqrt{f_c - f_D}\right)^2 + \left(\sqrt{f_c + f_D} - \sqrt{f_c - f_D}\right)^2} = 1 \text{ (0 dB)}, \tag{11}$$

$$\Delta = \frac{2A_1A_2}{A_1^2 + A_2^2} = \frac{f_D}{f_c} = \frac{V}{\lambda f_c} = \frac{V}{\delta c} \ll,\ 0 < \delta \le 1, \tag{12}$$

where δ is a percentage of the speed of light at which the EM wave travels in a given medium (eg. 0.7). Since $\Delta \ll$ and $K = 1 > 0$, the PDF of the Doppler frequency shift approaches a modified Rician distribution as $\Delta = V/\delta c \to 0$.

V. Results

Figs. 4-6 depict the simulated PDF curves for different values of the controlling parameters V_0^2, K, and Δ.

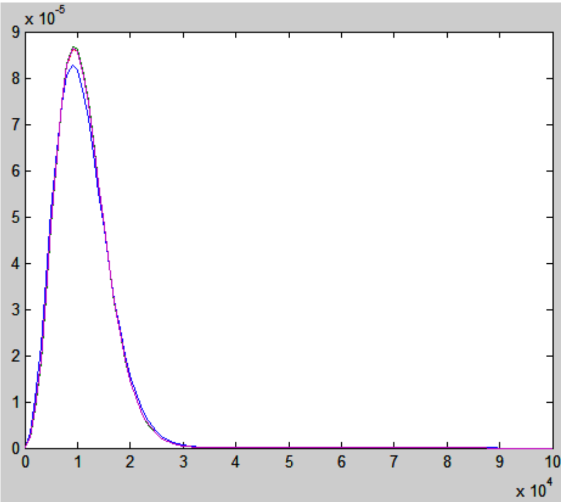

Fig. 4 The PDF for $V_0^2 = 1000$, $K = 10$, $\Delta = 0.2$

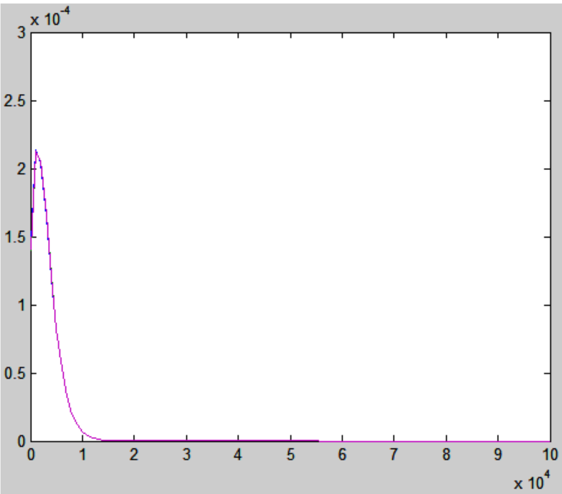

Fig. 5 The PDF for $V_0^2 = 1000$, $K = 2$, $\Delta = 0.2$

VI. Conclusions and Future Work

In this paper, we derived stochastic models for the statistical distribution of the Doppler frequency shift in the presence of a direct LOS based on a generalized Clark's model and on a two-wave partially developed scattering models. We showed that the PDF under the latter model approaches a modified Rician distribution. Analytical expressions for the PDF of the Doppler frequency shift are crucial in the design of optimal signal detection in mobile communication systems.

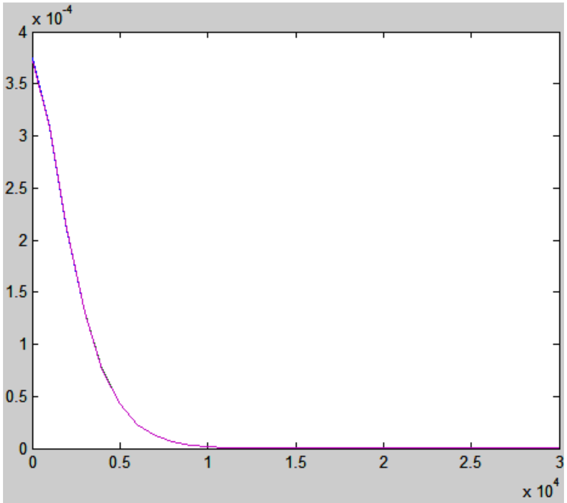

Fig. 6 The PDF for $V_0^2 = 1000$, $K = 1$, $\Delta = 0.2$

For future work, we will analyse the statistics of the post-detection Doppler signal in the presence of a direct LOS, having previously shown that the statistical distribution follows an elliptic-K function in the absence of a direct LOS [8].

References

[1] R. Prasad and L. Munoz, *WLANs and WPANs Towards 4G Wireless*, Artech House, MA, 2003.
[2] T. Rappaport, *Wireless Communications*, Prentice Hall. NY, 1996.
[3] D. Snyder and M. Miller, *Random Point Processes in Time and Space*, NY: Springer-Verlag, 2002.
[4] J. Daba and P. Jreije, "Advanced Stochastic Models for Partially Developed Speckle," *International Journal of Electrical and Electronics Engineering*, Vol. 3, No. 3, pp. 183-187, 2009.
[5] J. Daba and M. R. Bell, "Statistics of the Scattering Cross Section of a Small Number of Random Scatterers," *IEEE Transactions on Antennas and Propagation*, vol. 43, no. 8, pp. 773-783, August 1995.
[6] A. Abdi, S. Nader-Esfahani, J. Daba and M. R. Bell, "Comments on Statistics of the Scattering Cross Section of a Small Number of Random Scatterers," *IEEE Transactions on Antennas and Propagation*, vol. 48, no. 5, pp. 844-845, May 2000.
[7] J. Daba, "Advanced Models for Partial-Speckle Noise," *5th International Conference on Computer, Electrical, and Systems Science, and Engineering* (*CESSE 2008*), organized by the World Congress on Science, Engineering, and Technology, Vienna, Austria, August 2008.
[8] J. Daba, "Poisson Modulated Stochastic Model for Partially Developed Multi-Look Speckle," *American Conference on Applied* Mathematics, Harvard University, Cambridge, MA, USA, March 2008.
[9] J. Daba, "Scattering Statistics of Doppler Faded Acoustic Signals Using Speckle Noise Models," *IEEE International Conference on Direct and Inverse Problems of Electromagnetic and Acoustic Wave Theory*, Lviv, Ukraine, Sept. 2003.
[10] J. Daba and M R. Bell, "Statistics of the Scattering Cross Section of a Collection of Constant Amplitude Scatterers with Random Phase," West Lafayette, IN, *Purdue University, School of Electrical Engineering Technical Report*, TR-EE 94-25, July 1994.
[11] J. Daba, *Detection and Estimation in Speckled Images Based on Marked Point Process Speckle Noise Models*, Ph.D Dissertation, Purdue University, West Lafayette, IN, USA, August 1994. Published by University Microfilms International, Ann Arbor, Michigan, USA.
[12] J. S. Daba, J. P. Dubois, "Statistical Modeling of Local Area Fading Channels Based on Triply Stochastic Filtered Marked Poisson Point Processes," *International Journal of Electrical, Computer, Energetic,*

Electronic and Communication Engineering, Vol. 9, No. 7, pp. 652-657, 2015.

[13] R. H. Clarke, "A statistical theory of mobile radio reception," *Bell System Technical Journal*, 47, No. 6, July-August 1968, pp. 957-1000.

[14] A. Abdi, J. Barger, and M. Kaveh, "A Parametric Model for the Distribution of the Angle of Arrival and the Associated Correlation Function and Power Spectrum at the Mobile Station," *IEEE Transactions on Vehicular Technology*, Vol. 51, No. 3, May, 2002.

[15] G. D. Durgin, T. S. Rappaport, and D. A. de Wolf, "New Analytical Models and Probability Density Functions for Fading in Wireless Communications," *IEEE Transactions on Communications*, vol. 50, no. 6, pp. 1005-1015, June 2002.